ENGINEERING PROBLEMS MANUAL

The quality of the materials used in the manufacture of this book is governed by continued postwar shortages.

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Forest C. Dana, C.E.
PROFESSOR OF GENERAL ENGINEERING
IOWA STATE COLLEGE

Lawrence R. Hillyard, M.S.
ASSOCIATE PROFESSOR OF GENERAL ENGINEERING
AND ENGINEERING PERSONNEL OFFICER
IOWA STATE COLLEGE

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ENGINEERING PROBLEMS MANUAL

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PREFACE TO THE FOURTH EDITION

It is now nine years since this manual has been revised. During this period the civilized world has fought and won the Second World War. These have been years in which unprecedented demands have been made upon the facilities and staffs of the engineering colleges. The urgent need for trained and semi-trained men and women compelled the educational institutions to experiment with high-pressure, accelerated courses and curriculums, and to utilize the services of inexperienced personnel.

The teaching staff in Engineering Problems at Iowa State College was fortunate in having had previous experience in handling unexpectedly heavy enrollments and in training teachers without classroom experience. For many years (since 1926) it had been the regular practice to have a new teacher assist in a "pilot" section taught by one of the experienced instructors. The former took notes regarding teaching methods, class problems, instructions, and the classroom discipline. Uniform time schedules for problems and study assignments were prepared, and generous use was made of mimeographed material. In this way a continuity in standards and techniques was obtained. When the war demands came and volunteer enlistments or the draft began to call the younger men away, there was an effective teacher training process already available. This same method of teacher training is today proving its worth once more, in quickly preparing a large number of new teachers to handle the vastly increased enrollments.

The "Engineering Problems Manual" is now 20 years old. This revision is based upon many such war-inspired experiments and represents the pooled experience of several instructors. Old problems have been examined to see if they truly served their purpose. New problems were collected and each one tested several times before being accepted. Many experiments were made in teaching methods. Various presentations of study prob-

lems and classroom or printed instructions were tried in order to secure the clearest comprehension by the students.

To a large extent blackboard instructions have been abandoned in favor of printed or lithographed materials. A workbook[1] was prepared so that many drawings, tables, and other stock forms would be ready for the student to work upon. The workbook is intended to supplement the material in the "Engineering Problems Manual," illustrating with full-size pages various forms and problem illustrations that are not suitable for small page reproductions. Since one of the purposes of Engineering Problems is to train students in making calculations, it seemed advisable to gain computing time by furnishing them with prepared work sheets so that they would spend less time in ruling forms. The past year's work has shown that the decision was a wise one.

The insistance upon neat, well-organized calculation sheets paid large dividends during the war. Many former students have indicated that this training in clearness and accuracy was one of the most valuable things that they carried into the armed forces. For such reasons the specifications have been retained in much the same form as before, but they have been brought in line with existing national codes.

Many of the new experiments, such as the introduction of curve fitting in the Curtiss-Wright Engineering Cadette Program, gave surprisingly good results. Curve fitting has been incorporated, therefore, as a part of the work in third-quarter freshman Engineering Problems. Some notes on this topic are now a part of this text and the workbook. The work in derived curves and graphical calculus has also been tested in this freshman course with good success. It can, however, be taught in parallel with any of the calculus courses.

This edition of the "Engineering Problems Manual" represents, therefore, the results of many years of experience in developing a series of courses intended to give the student the maximum amount of practical training in engineering calculations in the time allotted to the work.

The authors wish to acknowledge the hearty cooperation of David King, Richard Hoverter, Wayne Moore, and Robert Lyon in the development of the work in the ESMDT courses, which

[1] DANA, FOREST C., and LAWRENCE R. HILLYARD, "Engineering Problems Workbook," Wm. C. Brown Co., Dubuque, Iowa.

is represented here by various problems. We wish to extend our thanks to Harald Birkness who tried various methods in curve fitting, nomography, and other material in the Curtiss-Wright Engineering Cadette Program.

Forest C. Dana
Lawrence R. Hillyard

Ames, Iowa,
November, 1946.

PREFACE TO THE FIRST EDITION

For several years many engineering schools have been experimenting with special courses for freshmen and sophomores. Some of these are merely illustrated lecture courses requiring little, if any, effort on the part of the student, their only purpose being to show something of the field covered by engineering. Other courses are designed with the intention of giving considerable training in the development of good habits of work and study. Courses of this type are motivated by the use of practical engineering situations, and close attention and persistent effort are demanded of the student. Most of these courses seem to have been given the name "Engineering Problems" although the range of purpose and subject matter is exceedingly broad.

Many of these courses owe their origin to the pioneer work of Prof. Charles C. More and Prof. William E. Duckering at the University of Washington. About 1915 they introduced and developed new methods of teaching Mechanics. The new type of instruction was so successful that freshmen courses were organized under the name, "Engineering Problems," and were conducted on the same plan as the Mechanics classes. During the World War both men were stationed at Camp Humphries, Virginia, and their new teaching methods were used in the Mechanics courses in the Engineer School. As a result of the publicity given to this method by the "Mann Report," several colleges began experimenting with similar courses, particularly the Engineering Problems courses for freshmen.

Since 1919 all engineers at Iowa State College have been required to take a series of Engineering Problems courses based upon practical engineering situations. The courses at Ames were developed by Professor Duckering who came here directly from Camp Humphries. The work has been growing, and for some time it has been tending toward a closer cooperation between the Mathematics, Physics, and Engineering groups. The problems are intended to give training along many lines, but all are presented in an engineering setting and, as far as

possible, one which is familiar to the average student. Many of them make use of the principles of physics and mechanics. They are arranged so that the mathematical tools required will be those the student is then studying in his mathematics courses. Machine shops, power plants, drafting rooms, surveys, or any other engineering activities are drawn upon for the settings. Therefore, the Engineering Problems courses are not primarily physics or mathematics but a coordination of both in an engineering atmosphere. Thus the interest of the worth-while student is gained, and he sees more of the purposes of the fundamental sciences. The young man who has no aptitude for engineering has an opportunity to discover that fact early in his college career and can get into more congenial lines of effort with minimum loss of time.

This book has been prepared for use in the courses given at Iowa State College where they coordinate with algebra, trigonometry, and calculus. Enough tabulated material has been included to give the information needed in solving the drill problems, but no attempt has been made to make this a handbook. The manual is not planned as a conventional textbook to be used for assigned readings and problems. It is expected that the principles of Chaps. 4–10 will be introduced by class discussions and the notes will be used merely for reference when solving problems. This book, therefore, is not a text on mathematics, physics, or elementary mechanics but rather a student notebook covering the work offered in the Engineering Problems courses at Iowa State College.

The chapter on specifications sets forth an efficient and effective method for obtaining neat, legible, well-organized computation sheets. The emphasis is placed upon good workmanship in both the solving and the recording of problems rather than upon "quantity production." Most students have little or no idea as to how computations should be arranged and do not appreciate the value of systematic methods. The young engineer should be given this training as early as possible, as it will help him throughout his college career as well as in later life.

The chapter on drill problems gives many typical situations and will suggest others to student and instructor. Numerical data have been given in some of them. The blanks will allow the instructor to use his own material or to use data supplied by

members of the class, and the student should write the values into the proper spaces. The problems range from the very simple to those having many details. Even the most complex, however, is not too difficult for the average student if he will be patient and reason his way through step by step. Both pupil and teacher should not forget that mental growth is not possible in the absence of difficulty and effort. Hard problems, therefore, should be accepted by the young engineer as a challenge to his mental ability and perseverance.

The work on Calculus with the problems showing its engineering applications is used in the sophomore course. This course is given in the third quarter of the second year, after students have had two quarters of both calculus and physics. At this time they are regularly scheduled for the third quarter of calculus and the statics work in mechanics. This Problems course is, therefore, primarily one of review and drill on previous work but with an occasional look ahead into hydraulics and strength of materials.

Considerable emphasis is placed on the geometrical applications and relationships. Engineers can usually see through their difficulties and often simplify their solutions, if they know how to translate algebraic equations into physical quantities or the physical measurements into algebraic laws.

The authors are glad to acknowledge their indebtedness to the work done by Prof. C. C. More at the University of Washington and Prof. W. E. Duckering.

Professor Duckering's booklet, "Notes and Problems for Engineering Problems Classes," which he prepared while at Ames, has been in constant use up to the time the content of the courses was changed. We have found his form of presenting the notes on basic principles to be very satisfactory and have used much the same plan in presenting this material.

We also wish to thank Prof. J. W. Woodrow, of the Physics Department, Iowa State College, for his valuable suggestions and reading of the manuscript on Basic Principles.

FOREST C. DANA
ELMER H. WILLMARTH

AMES, IOWA,
July, 1927.

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ENGINEERING PROBLEMS MANUAL

CHAPTER 1

PURPOSES OF ENGINEERING PROBLEMS COURSES

1.1 Living in a Technical Era.

For many years the phrase "the American way of life" has been used as a concise name for the technical civilization that has been developed in both the United States and Canada. This rich culture has been based upon a constantly growing body of technical knowledge, skills, and control over materials and processes. So truly has ours become a technological civilization that there is hardly an area in the production of goods or services in which the knowledge and skills of the engineer are not needed somewhere along the route between the raw materials and the consumer.

For this reason an increasing demand has arisen for engineers and technicians. New vocational fields are opening to the engineer. Often the jobs are those that once could be filled by men who had had a more or less limited education but who were skilled in the mechanics of their work from long experience. Today an increasing number of employers are finding that these same jobs now demand the services of men with training not only in the art of their profession but also in its fundamental theory. This means that there are constantly widening areas where the engineering graduate is needed. The need for better trained technical workers in the lower levels during the period of the Second World War resulted in the United States government sponsoring many short, high-pressure training courses such as the EDT and ESMDT courses.[1] Later came the

[1] "Engineering Defense Training Course" and "Engineering, Science and Management Defense Training Courses," Federal Security Agency, U. S. Office of Education.

Navy's V-12 program and the Army's ASTP courses for the military forces.[1] Since then many colleges have revised their curriculums, streamlining them to help meet this growing demand for trained, competent men.

1.2 More Accurate Knowledge Is Essential.

This growth in the need for better educated men and women in technical work is tied in closely with the growth in fundamental science. The areas in which trial-and-error methods can be used with safety are shrinking rapidly. Empirical formulas, developed experimentally, are giving way to more exact, rational concepts and mathematical formulas. Rough-and-ready ways of manufacturing goods have had to be replaced by high-precision methods, close inspections, and better materials. When wage scales are rising, employers must save labor as well as material if they are to stay in business; hence, time and motion studies must be made for nearly every task in order to determine standard times and standard methods of procedure, the aim being to discover the most economical procedure.

1.3 The Upgrading of Jobs.

All this means that an upgrading of jobs is constantly under way, and the end of the process is not yet in sight. Many jobs that once could be filled satisfactorily by skilled workers now need men of subprofessional caliber; other jobs once suitable for the man with semiprofessional training now need the skills and knowledge of the graduate of a 4-yr engineering course. At the top there are various positions requiring men with the higher degrees: the master of science, the professional, or the doctor of philosophy. This upgrading has been going on not only in engineering but also in commercial fields as well. Today engineers are finding a welcome in such once strange activities as sales; life insurance; fire and general insurance; appraisals; management of retail chain stores, banks, and bond houses; and other fields formerly considered very remote from engineering.

The engineer needs more than a mind well stocked with facts, formulas, and design theory if he is to climb very high in engineering or any of these new lines of endeavor now opening to him. He not only must be competent in his special branch of

[1] Army Specialized Training Program.

engineering but must have acquired a knowledge of the money value of goods and services. He must have acquired a working knowledge of human nature, applied psychology, sociology, economics, and other subjects once considered the exclusive domain of the liberal arts college. The prospective employer must be convinced that the engineer not only has ability, knowledge, and certain skills, but also has courage, ingenuity, perseverance, a pleasant personality, dependability, high moral standards, integrity, and other desirable traits of character. Some students feel that character is no concern of the teacher's as long as the student does acceptable work. They are wrong, however, because character *is* important. Employers ask more questions about a student's traits of character than they do about grades. Grades count, but they are always on file at the office of the registrar, whereas a man's character rating is on file only in the minds of those who know him. That the engineering profession acknowledges the importance of character, integrity, and responsibilities to others is shown by a study of the "codes of ethics" adopted by the various national engineering societies.

1.4 Competition Is Keen.

The foregoing comments indicate that there is probably an informal, inconspicuous competition among men for desirable jobs and among the employers for qualified men. This is true. An unseen sifting or screening process is always going on so that in a few years the general trend of a graduate's career can be seen. All men, engineers or others, tend to fall into several well-defined groups in respect to their skills, knowledge, and traits of character. At the bottom are the lazy and shiftless. Too inert to learn to drive themselves, their chief function in life seems to be that of consumers of food, fuel, and clothing provided by the energy of others. Always failures, economic parasites, they move from job to job blaming everyone but themselves for their condition. Next come the dodgers, the ones who go through life and never learn how to face difficulties. They fear unfamiliar situations, the new, the unusual. They dodge responsibilities and promotions that would compel them to leave familiar routines. Frequently they are fluent with excuses for their shortcomings and seldom take the blame for

their mistakes. They will not take the initiative, always relying upon someone else to start them off and give them detailed instructions for each major step. They are doomed to routine work; and although they can and often do fill very necessary jobs in the lower ranks, they never become leaders. Next above them are the ones that might fairly be called the cut-and-try artists. They often show surprising ingenuity and originality. They are not afraid of the new and unfamiliar and have ambition and perseverance. They frequently make mistakes, but they do get things done. Learning by observation, by trial and error, they develop hit and miss techniques that serve them well as long as they do not run into situations that demand the application of exact theory and scientific methods. The most effective group, however, is the one that engineers should help to fill. To the traits of the previous group they add broader vision; trained, well-stocked minds; judgment; and balance. They know how to analyze a problem or project, and with their training they gradually fit themselves for positions of great responsibility. They do not shun difficulties but attack them with courage and vigor, regarding each new experience as an opportunity to learn something worth while. Such men are outstanding for their ability to grasp quickly the essentials of a new situation, to break a problem down into its component parts, and to reach a logical, workable solution of the difficulty. The competent engineers are marked by their knowledge of the fundamentals that underlie their profession and by their ability to apply these basic rules to the new, ever-changing demands of the present-day world.

1.5 Economy of Time and Effort Is Imperative.

So large is the body of knowledge, technical, social science, and humanistic, that must be crowded into a 4- or 5-yr engineering course that economy of time and effort are imperative if a student is to do better than mediocre work. A similar need for efficiency is found in many engineering offices. Efficiency in study, thinking, figuring, designing machines and processes, in all the varied activities of the engineer is attained only by conscious effort. Very seldom do students in our high schools and junior colleges ever receive any coaching in good habits of work and study. It seems to be assumed that everyone is

born with the knowledge of efficient ways to prepare lessons, to attack and solve problems, and how to record his work in a neat, legible, workmanlike manner. Such knowledge and skills are not inherited, however, and are acquired only through proper environment, coaching, and determined effort on the part of the student or worker. Since the young men who enter the engineering colleges so seldom have received even the slightest definite guidance in the organization of calculations or ever have had to meet an engineer's standard of neatness and general workmanship, many of them find the first year or two in college to be a trying, often discouraging time.

They have not been taught to use laborsaving ways of doing their work or preparing lessons. Many find that they have to "unlearn" things that they have been taught. For example, the American Standards Association, the Federal Bureau of Standards, and similar organizations have sponsored modern codes of symbols, abbreviations, and various notations to be used in mathematics, physics, chemistry, and other branches of science. But the texts found in high schools and grade schools are still using and teaching notations long out of date. Samples are the "shilling" (inclined bar) fractions and the colon method of showing proportion. The latter notation ($a:b::c:d$ or $a:b = c:d$) was discarded by a national committee[1] of teachers of mathematics in secondary schools as early as 1921, and a standard engineering handbook[2] spoke of it in 1916 as one "which is now passing out of use." It should, therefore, be abandoned by the engineers of today.

Speed and economy of effort, effective study habits, and rapid preparation of problems and reports are possible only for the person who has been trained and drilled in these skills. Since such skills have become so highly essential for one entering upon the truly arduous courses of study leading to one of the engineering degrees, increasing numbers of colleges and universities have been introducing a new type of course into the freshman and sophomore curriculums. Various names have been chosen for these training courses, but "engineering problems" seems to be the most popular.

[1] "The Reorganization of Mathematics in Secondary Education," *U. S. Dept. Interior Bur. Education Bull.* 32, p. 68, 1921.

[2] MARKS, "Mechanical Engineers Handbook," 1st ed., McGraw-Hill, 1916, p. 113.

1.6 Definition of the Name Engineering Problems.

Engineering students and others who come in contact with the courses under this name naturally want to know their subject matter, their aims, and how the work is conducted. Everyone knows that a man cannot do his best work if he does not perceive the objective of his efforts or have any particular interest in what he may be doing. With interest comes cooperation, so that the student no longer remains passive, waiting for the teacher to do all the work, but instead takes an active part in the learning process. This is of vital importance because all true education is self-education; the development must come from within, not without. What, then, are engineering problems courses?

Engineering problems courses are courses in which the student is coached in the development of accurate, effective, efficient work and study habits while he is actually solving and recording problems under the supervision of the teacher.

1.7 Aims cf the Courses.

Since these courses are primarily concerned with the forming of desirable habits of mind and hand instead of the memorizing of a specific body of factual knowledge, several rankings of the various goals suggested above are possible. The goals outlined below are in general accord with the emphasis placed on them by most instructors.

a. Accuracy.
b. Efficiency.
c. Good workmanship.
d. Engineering attitude.
e. Subject matter.

In the remainder of the chapter these various aims will be discussed in more detail in order to show the reasons for certain instructions and the interrelationship among the goals.

1.8 Accuracy in Calculation.

There is really only one reason in engineering work for making computations: They are made to use. Not only must they be accurate but also they must be efficiently obtained, complete, readily checked, and as precise as the data justify. The average

student wastes much time and effort and gets needlessly lower grades than he should because he is inaccurate in his figuring. Sometimes he appears to have drifted onto an evil path that leads to failure. He makes one or more arithmetical mistakes on a problem, and it comes back with little or no credit. He concludes hastily that he *does not* understand the problem. He makes more blunders in arithmetic and decides that he *cannot* understand the problem. A few students seem to reach the point where they appear to decide that they *will not* understand the problem, no matter how much effort the teacher puts forth to clear up their troubles. Now, with a mind firmly locked shut, defeat is inevitable. Thus the habit of inaccuracy saps confidence and leads to future difficulties.

The obtaining of correct results involves four main steps:

a. **Correct reasoning,** which in turn, depends upon the basic theory underlying the problem and its statement in mathematical form. That is, the computer must be using correct theory and mathematical tools to start with.

b. **A systematic method cf attack.** This means breaking a problem into simple steps, logically arranged, and the avoidance of so-called short cuts. There is no virtue in the ability to set up a problem in the form of a long, involved formula filled with many unit complications and abstruse calculations. On the contrary, experienced men in industry are constantly urging the college teachers to tell the young engineer to form the habit of breaking his work into simple steps, each one easy to check and easy to use. He should arrange his work so that there is enough detail to leave a well-marked trail leading from the data to the answer. Intermediate answers serve as frequent landmarks that enable the computer and the checker to pause and verify the route taken and the results obtained to that point. Students often express resentment when textbook authors give sample problems and when about halfway through the solution say "Hence it is obvious that . . . " when the conclusion is anything but obvious. Why are these students guilty of the same offence both in their college problems and their professional work?

c. **Correct use of the basic branches of mathematics** such as arithmetic, algebra, and trigonometry. This does not mean a deep grasp of all the intricate phases of mathematical theory. Far from it, because it is lack of skill in the simple phases of

the various branches of mathematics that causes the most grief. In trigonometry, for example, the essential formulas for solving any plane triangle can be typed on a postal card.

d. **Accuracy in simple arithmetic** is the most important of all the skills required. Blunders in addition, subtraction, multiplication, and division are the cause of far greater loss of time and money, of effort and material, than mistakes in theory. The blunders come from momentary lapses of attention, interruptions, and a subconscious feeling that this phase of the work is subordinate and unworthy of close attention and sometimes from plain incompetence in arithmetical operations. Such losses of time and effort may be reduced by the aid of mechanical helps such as tables, charts and graphs, slide rules, and computing machines. Engineers should also form the habit very early in their careers of checking and rechecking all their work. The man who accepts and uses his first, unverified result is a gambler, not an engineer. Retracing operations is *not* checking. A different method, a varied grouping of values, and another computing tool should be used. Then last, but far from least, the engineer should sit back, look at his numerical answers with the skeptical eye of common sense, and ask if they look reasonable. Many tales are told of bad blunders causing serious losses because no one used "horse sense." Far too many men are like the workman who was reproved for a blunder and, when told to use common sense, replied: "But sir, common sense is a rare gift of God and I have only a technical education."

1.9 Efficiency in Work and Study.

Students (and workers in industry, too) are constantly laboring under the pressure of skimped and unfinished tasks simply because they are inefficient in the handling of their jobs. Work is put off and often left undone, but the only excuse offered is "I have more work ahead than I can do." In most instances this is not true; these workers waste time because they are unsystematic; use long, indirect methods; and in general, fail to use the best available tools for getting results. They lose time and waste energy in aimless, frantic attempts to work faster. Then they make more mistakes in judgment and execution and end by doing inferior work. They worry about their troubles, and by worrying they simply magnify the whole problem.

These men resent bitterly the suggestion that they are poor organizers and do not really know their own jobs. In truth, however, poor planning and the lack of systematic methods are foremost causes of inefficiency. Instead of making use of standardized methods of preparing assignments, of attacking and solving problems, of organizing reports and term papers, of developing a program for handling the routine tasks, they use hit-and-miss, spur-of-the-moment methods. They treat each new task that is put before them as if it were an unusual, special situation, instead of looking for the underlying features common to other problems solved in the past. In some cases, of course, this inefficient approach to the task is, as suggested in the preceding topic, simply due to the fact that the worker has a poor foundation in the basic skills and in the knowledge of computation. There is no remedy for this situation except that of getting down to work and learning how to figure.

Faulty judgment in the choice of mechanical and mathematical tools is also a source of wasted time. The use of unduly precise calculation tools can cost much in time and labor with no gain in accuracy. Some men wait until they can get a computing machine when the task could be done in a very few seconds by using cut-longhand or a slide rule. Others regard logarithms as a cure-all. One mining engineer, for example, said that he always used a seven-place table of logarithms, even for four-figure data, because he wanted his answers correct to five or six figures. He never realized nor could he be convinced that no seven-place table could ever create six-figure precision from four-figure data or that he was grossly inefficient because other methods would give four-figure precision in far less time. In other instances a graphic solution of satisfactory precision can be obtained in a fourth to a tenth of the time required by analytical methods. Sometimes a switch from one branch of mathematics to another will save time. In computing the length of the steel members in a certain design of crane, for example, a switch, about halfway through, from trigonometry to analytic geometry will save considerable time and labor.

Another group of time wasters are the formula worshipers. Some men make a fetish of formulas, especially in fields where they are not sure of themselves; consequently, they spend more time hunting a magic formula than they would need to analyze

the problem piece by piece using simple, familiar methods and calculations. The principal virtue of formulas, if virtue it is, lies in the fact that by them men can avoid the labor of thinking.

A prime time-wasting habit of far too many people is the "scratch-paper habit." Insufficient records due to the use of scratch paper are a frequent cause of delay and of mistakes. Many people think that it takes too long to make clear, permanent entries of calculations; so we find men (from the citizen laboring over an income tax report to an engineer designing a multimillion-dollar project) putting their arithmetical work on scraps of paper that are thrown away when the answers are obtained. If the scraps are saved at all, they are usually so mixed up and shuffled that they are useless in checking back to find errors. Later, when it is necessary to verify results or to change the procedure in some way, the entire calculation must be made all over again. Quite a few commercial organizations have found it necessary to forbid the use of scratch paper in any form. Some set aside part of the page for arithmetical work as suggested in the specifications that are given in Chap. 3. Others put the analysis and main solution on one color of paper and the arithmetical calculations on paper of a different color. Still other firms insist that all calculations be made in permanently bound books. In some cases not only the computer's signature is required on each page of the workbook, but the signature of two witnesses. Such books then become legal evidence, admissable in court, and may be of the utmost importance in patent cases and other lawsuits. The chief engineer of a large oil company was asked if they permitted the use of scratch paper. His answer was "Positively *no!*" He went on to say:

The one point which I wish to emphasize, and I think I am expressing the opinion of industrial engineers in general, is that there is no place for "scratch paper" in engineering calculations. In the first place "scratch paper" which is to be thrown in the waste paper basket usually means sloppy work. I do not wish to accuse young engineers of being dishonest, but the psychological effect of retaining all calculations to be filed cannot help but encourage neater mechanical effort and clearer thinking. Furthermore, the filing of all calculations has a decided practical advantage. The tools with which we work are changing rapidly. A piece of equipment designed on the best available data of today may be open to criticism a year hence. As our fundamental data

increase in volume or become more exact quantitatively, we not only design new equipment on this basis, but often modify existing equipment to bring it up to date if such changes are economically justified. If the original calculations, initialed and dated, are available, it is a much simpler and shorter job to revamp these calculations than to prepare a complete new set.

Another advantage of retaining calculations is to definitely place the responsibility for the original design on the man who made the calculations and the one who checked his figures. We make every effort to keep errors at a minimum, but if they do creep in, it is most helpful to know who made the error.

1.10 Good Craftsmanship Is an Asset.

Professional pride in good craftsmanship is not a useless frill to be developed or not as an engineer sees fit. It has very real values, both tangible and intangible. The doors of opportunity and advancement have opened sooner and wider to many a young engineer just because his work was outstanding in its clean-cut, competent appearance. In addition to monetary rewards there is a more personal, inner satisfaction known only to the man himself when he can lay aside a completed task with the knowledge that he has done his best and that his best is a truly professional piece of work. Reaching such a goal is not easy even for the best of men, and some find that they have to break life-long habits of doing slovenly work before they can acquire the desirable habits.

Experienced men say that there is a close correlation between the appearance of a computation sheet and the mental habits of the computer. The president of a large manufacturing concern once said "There is a close connection between slovenly thinking and slovenly records." He indicated that he was not interested in having either one in his organization. Another man wrote:

> Sloppy notes are usually associated with a mediocre mind. An intelligent, clear-thinking engineer keeps neat and legible notes. In most instances we find that there is a close connection between slovenly thinking and disorderly records.

These men are entirely justified in their statements, because it is practically impossible for anyone to produce neat, systematic, well-arranged original records if his thinking has not

previously been organized into logical, systematic form. Various checkers of engineering problems papers have repeatedly commented that there is no evidence that doing neat work slows down the worker. On the contrary, they say that the men who do the most work and with the largest number of correct answers are generally the neatest and most careful as to the appearance of their papers.

Basically, slovenliness in the preparation of reports and calculations is a form of dishonesty as well as a mark of incompetence. It is of the same nature as carelessness in the making and operating of machines. A careless mechanic ruins materials and wastes time; a careless engineer may cause the loss of time, materials, even human life. The man whose motto in life is "Why bother? That is good enough. It will get by" is as much a cheat as the worker in a shop or on a construction job whose life is built around the same low standard. Such a man should never be intrusted with responsibilities or be put into positions of trust. A careless, indifferent worker resents the necessity of putting a workmanlike finish on any sort of job. A man with such an outlook on his work will seldom go far or long continue in engineering. He will find the preparation of high-caliber drawings, estimates, reports, or calculations most distasteful and will probably gravitate to employment that is less exacting than a high-standard engineering office.

When a man truly wishes to improve the quality of his work from the standpoint of appearance, there are several steps that he can take. Since most of these have little to do with his skill with pen or pencil, there is no excuse for not improving the appearance of his work. The most important aspect has been discussed in connection with accuracy and efficiency in calculation. That is:

a. He should break his work into a series of simple operations arranged in a logical pattern that leads, without backtracking, from data to conclusion.

b. All numerical values should be so clearly formed that no critic could find an excuse to misread them even intentionally.

c. A habit that helps greatly in producing competent-appearing papers is that of devising and consistently *using* standardized sheet rulings, placements of analyses, and groupings of calculations and the clear emphasizing of answers to all calculations.

Such standard arrangements justify themselves in various other ways as well.

d. Another highly important element in the production of good records is that of preparing them for permanence. The worker should be especially careful with pencil work and be sure that grade of lead, pressure on pencil, and type of paper are such that smearing, fading, and tearing are not likely to occur with reasonable handling. The records should be made on the assumption that they must speak for themselves many years after the maker has left the organization or, perhaps, passed to his future reward. A representative of a well-known dealer of engineering supplies reported that several firms that he had just visited were being forced to spend many hundreds of dollars to recopy design records, calculations, and drawings simply because the individuals making them had failed to bear down on their pencils hard enough to keep the pencil marks from smearing and losing legibility as the papers rubbed against each other in the storage files. There is no excuse for a computer's turning out work that someday must be copied because he would not use enough pressure on his pencil.

e. The computer can greatly improve the general appearance of his work by using well-formed lettering, preferably the American Drafting Standard types, instead of script. When script is used, however, it must be completely legible. Ambiguous script is just as indefensible as illegible figures.

f. Long, formal reports on projects, designs, and processes should, of course, be neatly typed on bond paper.

1.11 The Engineering Attitude.

The group of habits of mind and spirit that are characteristic of so many members of the engineering profession is so noticeable that it has been named "the engineering attitude." It is essentially the emotional reaction of the engineer to the world of men, forces, materials, and conditions that affect his efforts. Its possession is almost imperative for any large measure of success, and the men who do not develop it find engineering too highly exacting to suit them. The man who resents difficulties, the quitter, the whiner, the leaner, the bluffer, gravitates sooner or later to a less demanding way of making a living.

There are many essential elements in this character pattern

which has become known as the typical engineering attitude. They are so interdependent that an accurate ranking of them is not feasible, but integrity, honesty, fairness should undoubtedly come first. The great engineering societies have recognized this through the adoption of their "codes of ethics." These codes serve as guides to engineers in their relations with their employers, the public, and their fellow engineers. Basically they are a restatement of the principle so concretely given in the Golden Rule.

The engineer soon becomes philosophical about difficulties, unfamiliar or unpleasant situations. He quickly learns that they are normal conditions in the engineering world and that it is folly for him to get upset or angry over them. He learns to "play the game," to adopt the never-say-die attitude and, when one plan fails, to devise another solution to the difficulty. One writer[1] when discussing the learning process wrote:

> Uncertainty, confusion, doubt, hesitancy are the sources from which thinking takes its start and the spur that urges us forward to reflection. . . . If you wish to increase the power of thought and to develop the ability to do clear and reflective thinking, embrace every opportunity to handle problems. Plan your work in problems. Put yourself in a situation where you have to take the initiative. Assume responsibility, incur risks, be liable for something. If you want to learn to think, get into trouble! Until you become involved in a hazardous undertaking, until you are up against a real difficulty, you will never learn to think. The way to develop leadership is to take the lead. You do not first learn to think and afterwards apply that learning to the solution of a problem; you begin by handling problems, and the effective handling of the problem is the thinking.

The acceptance of difficulties as normal and then the ingenious surmounting of them naturally lead to the development of the traits of self-reliance, originality, and initiative. The engineer must become a "self-starter," self-reliant and ingenious in finding ways of getting results. It has been said that he is the man who, when told that something is impossible, goes out and does it. It has been due in great measure to these traits that modern industry has reached its present level. The student who is always asking a classmate to lift him over a difficulty or

[1] McClure, "How to Think in Business," McGraw-Hill, 1921, pp. 27 and 30.

a hard problem is headed the wrong way. He may secure an answer to be handed to his instructor, but he has learned little or nothing and certainly has toughened no moral fibers.

Perhaps by training, perhaps through the development of the foregoing traits of character, the engineer also eventually shows what may be called the judicial frame of mind. That is, he learns to withhold decisions until the facts are known, not to act on snap judgment as do so many men and women. This judicial temperament is almost forced upon him by the very nature of the physical world with which he works. He cannot decide questions of design, loads on structures, their strength, or the speed and power of machines on the basis of prejudice, of desire, of politics, of wishful thinking in any form. The pertinent facts—and facts alone—can be a safe foundation for a true, sound engineering project. Many millions of dollars have been wasted in North America because the men who had the final authority based their decisions on some political expediency, some unproved economic theory, or the selfish desires of some pressure group instead of on a carefully studied engineering analysis.

There is one phase of their duties that engineers frequently dodge, and as a result they are sometimes refused true professional recognition of the type given to members of the bar, the ministry, or the medical profession. The engineer has all too frequently left the civic duties to others and has been reluctant to take part in community activities whether political, recreational, charitable, or educational. He could often help the people reach sounder decisions on public undertakings if he would recognize the fact that he should be a good citizen as well as a good engineer. He should, therefore, strive to become well balanced on what is called the cultural side of his life as well as on the technical. He should acquire some familiarity with the economic and sociological aspects of life in his community and nation.

1.12 The Subject Matter of the Problems.

In order to form the habits of accuracy and efficiency, in order to develop the traits of character discussed above, a body of knowledge must be chosen for the subject matter of such a course as engineering problems. In order to cultivate skill in

any line of effort it is a matter of common knowledge that we must have material upon which to work. Readings and lectures on such subjects as "How to Study," "Use of the Mind," "How to Read," "How to Organize Problems and Reports" are all the acme of futility unless the suggestions can be applied immediately and in a concrete way to lessons and problems in preparation at that moment. The student must have something to study, to analyze, to record, a problem calling for the application of basic principles, if he is to see any real need for the training that has been planned to fit him for the practice of engineering. Engineers are concerned not only with the discovery of truth through research and analysis but also with the application of truth in such ways as will serve the needs of mankind. The engineer is seldom interested in scientific knowledge merely for its own sake; he does not work in a cloister but in the rush and scramble of an active, growing world, facing new situations and surmounting new difficulties every day. His profession is the one that has the responsibility of translating theories and formulas from the mental world of ideas to the realm of physical reality. He most frequently asks when considering a new idea "Will it work?" "Can we use it?" "Can we afford it?" Because so much of his work consists of investigations that sooner or later take some mathematical form and all are intended to serve useful ends, the engineering problems courses are based upon the solution of various types of problems calling for the application of several basic branches of mathematics. The problems, therefore, serve as the framework upon which the coaching and practice outlined in the preceding topics can be built.

The engineering student has to choose his major group of studies from a group of specialized courses of study. In each there are numerous subdivisions, but underlying all of them are the basic sciences common to all branches of engineering. With such a large body of knowledge to choose from when a training course like engineering problems is being built, it becomes necessary to limit the choice to a few typical fundamental topics. The problems are drawn from the shop, the drafting room, design offices, the surveyor's field books, and other sources. The engineering problems are not planned to give a thorough drill in one or more of these subjects but rather

to pick out some of the most valuable topics in several branches and develop a technique for each that meets the requirements laid down in Topics 1.7–1.9. The student must become aware of the fact that techniques that work in one situation will generally work in others and that good work and study habits can be developed for each type of task. The subject matter varies considerably in the colleges offering engineering problems but most frequently concerns the engineering applications of mathematics and mechanics.

1.13 Conclusion.

Engineering is a profession only when its practitioner is a professional man with character and with an outlook on his work and community that measures up to that of other well-known professions. Without this attitude, this approach to his lifework, he is little more than a skilled technician, only a few rounds above the skilled mechanic. Engineering becomes a trade to those who are trade-minded. The true engineer, however, develops a pride in his profession. He qualifies himself for this high calling by always doing his best in his work, by continuing his studies all his life, by broad reading, by learning to express himself with both the written and spoken word, and by taking an interest and an active part in the affairs of his community, its schools, its churches, its community life, yes, even its government and the administration of its financial affairs. His work is always outstanding for its neatness, its conciseness, its honesty, and its thoroughness, whether the task be great or small.

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CHAPTER 2

PREPARING FOR THE LONG PULL

2.1 Getting Set.

In the language of various sports we have phrases that well express the purpose of this chapter. The sprinter must "get set on his marks"; the golfer must have "the proper stance"; and the baseball pitcher must "wind up" before he pitches the ball. Careful preparation for the task in hand is an essential step in the successful completion of any undertaking. The college program for engineers is such a training period for men who wish to become members of the engineering profession.

Sometimes it is advisable to have an introduction to the training period. Thus colleges have tried to aid the new student to find himself by introducing such programs as Freshman Days and orientation courses. Even though a student has grown up beside the campus, there are, nonetheless, many things he does not know in regard to college life and requirements. New students will find that there are things that the official counselors never mention and problems are encountered that are not discussed in any textbook.

2.2 Getting Along with College Professors.

One of these highly important studies never mentioned in college catalogues is one that might be called "Professors: A Study of Their Nature and Behavior." This study is always offered by the students themselves, every term, as an informal discussion course, usually over coffee and doughnuts, "cokes," or "malts." It is not a bad idea at all, because students have to associate with professors for four or five years. Learning how to get along with the teachers and to become acquainted with them is not always easy, but the rewards for the effort are very real.

The first of these rewards is that the student who is sincerely interested in his chosen profession can get much more out of a course when he knows something of his teacher's background,

his experience, his personality. The student may discover that his teacher really knows his subject and that his suggestions and corrections are not based upon mere opinion or other men's writings. If he knows that his teacher is competent in his field, the student will perhaps make allowance for odd and intriguing mannerisms. No campus is truly memorable unless it has known and loved a teacher who was remembered by alumni because he had some odd mannerism, some quirk of fancy, something unconventional as a distinction around which campus tales could be built.

It is a fact that a sincere student can profit from really getting to know his teachers. He will discover that they have met many of the problems that he is facing. He will come to a realization that regardless of age his teachers are on the same road that he has chosen to travel. The main difference is that they started earlier, and now they are reaching back trying to coach him so the path will be less rugged. He can learn something about his older teachers if he will look in the college library. If they have become known in their particular fields of activity, the student will find a brief biography in one or more of the biographical dictionaries.[1]

2.3 The Art of Apple Polishing.

The ancient art of "apple polishing," sometimes known by other names, is one that is sadly neglected in these modern times. It should be cultivated more; it is one way of getting acquainted with teachers. The experienced professor recognizes it readily, gets a little fun out of it and perhaps a feeling of being appreciated. The student profits, too. He may not be able to talk his way to a better grade, but he may discover that his professor is human and a potential friend. Before the student knows it, he may be liking his teacher, enjoying the subject, and really earning the desired higher grade.

2.4 How Strangely He Speaks.

One of the first experiences that really bothers the new student is that of taking notes in lectures. Even though the lecture

[1] "Who's Who in America," A. N. Marquis Co.; "Who's Who in Engineering," Lewis; "American Men of Science," American Association for the Advancement of Science; and other such directories.

system is not so widely used in engineering colleges as it was once, it still has its place in education. There is a lot of knowledge that has not yet reached the pages of a textbook. It is too new. If the student is to receive up-to-the-minute instruction in the most recent discoveries in science and engineering, it will have to reach him in lecture form.

Students usually have to take their own notes in these lectures, and it is not always easy. Poor lecturers fall into several classifications, and each is a challenge to a student, for he must manage somehow to take a set of usable notes. For example, there is the confidential speaker who talks so low that he cannot be heard over three rows away. Even worse is the one who slurs his words together, mumbling along in a monotone that defies unscrambling. Everybody knows the grunter who seasons his talk with so many ahs and ughs that it sounds like "pig Latin."

The "blackboard lecturer" uses the board primarily as an audience, not as a means of illustrating points. Students get his words by echo, if at all. Some of these blackboard artists are also ambidextrous; such a man will keep the eraser working so close to the hand holding the chalk that his body hides the few marks on the board.

Everyone has experience with the dry-as-dust lecturer who has the deplorable ability to take the most fascinating topic and phrase it in the driest, dullest manner possible. Delivered in an expressionless monotone, without a single highlight of wit or phrasing to redeem them, such lectures are as tiresome as the reading of a legal document. The pathetic fact is that many of these lecturers really possess a prodigious amount of knowledge of their special subjects, but they soon extinguish any spark of interest that a listener may once have had. Any of the speakers mentioned may also be a "hobby rider." He is the owner of a private hobby or experience, not at all related to the subject in hand, but nonetheless he manages to drag it in so often that he seldom completes the assigned lesson.

Among the lecturers who can be understood but who still fail to hold the attention of a class are the "readers." Apparently forgetting that his students have long since learned to read, this man stands before a class and reads each assigned part of the text in full, usually with little or no amplification or explana-

tion. If he also wrote the book, this period becomes one of agony for his victims. It is useful chiefly for catching up on sleep.

The "entertainer" is the lecturer who endeavors to be well liked; and so tries to secure the interest of his students by assuming the role of a popular humorist. His lectures may be well spiced with stories and clever sayings so that his classroom resounds with laughter, but as a rule his subject suffers in the popularizing process.

Of all the lecturers who can be classed as substandard, however, the intellectual snob is the worst. He is the one who is so conscious of his own superior knowledge, so impressed with the vast importance of himself and his narrow little field of information that he has only contempt for those to whom he unwillingly lectures. Undergraduates bore him, and he quickly shows his feelings. His slurs and sarcastic answers to questions soon kill any possible interest. His office is usually securely locked to make sure that no brash student can find him.

Students are not slow to detect all of the above kinds of lecturers, and they might add others to the list. They will find that each type furnishes a problem in note taking and much ingenuity must be shown in building up a set of notes, especially if the record must be handed in or be used in preparing for examinations.

Remember that far greater attention is required to secure good notes when the lecture is dry and uninteresting than when the lecturer knows how to keep his audience awake. Be charitable to the speaker, and remember that very few speakers, whether they are college teachers or practicing engineers or scientists, have ever studied the art of public speaking. It is true that anyone who is called upon to address an audience, either in a classroom or at an engineering convention, should be able to speak clearly and in an interesting manner, but even now it is hard to persuade students to take a course in public speaking. The professional reputation of many an expert has suffered because he made such a pitiful exhibition of himself when giving a talk to some group. This is no doubt one reason that the lecture system is in such evil repute as an educational device.

The attitude that a student takes toward the difficulties of note taking and other more or less intangible obstacles encountered

under difficult teachers may very well be a forecast of his later success or failure in the business world. If he sees a difficulty as a challenge, as something to be overcome without "griping," if he can only realize that Dr. McClure (see page 14) is right in regarding a difficulty as an opportunity, then he need never fear being in that large company of failures who blame everyone but themselves while claiming that the world owes them a living.

2.5 Students from A to F.

Each time that a class assembles for its first meeting of a term the students start "sizing up" the teacher in charge. Not only is he under close scrutiny for some time for his teaching ability and knowledge of his subject, but they study his appearance, his mannerisms, and his speech habits. They hunt for all the signs that they hope will enable them to catalogue the teacher. They seldom stop to realize that these personality studies are mutual. From his first meeting until the last the teacher is constantly endeavoring to determine character traits, working habits, and the strong and the weak points of each student. He does so because experience has proved to him that scholastic achievement is interlocked with personality traits. He also watches for these things because someday he may be asked by a prospective employer to appraise the student for a possible job and the employer is interested in character as much as he is in grades.

The attitude that a man takes toward his tasks is a decisive factor in determining his success and the ease with which he does his work. This is shown in the classroom in a very striking manner. Students classify themselves in one of several well-marked groups within a very few weeks. Dean William E. Duckering has given an excellent description of the various groups in the following paragraphs, which are taken from his booklet "Notes and Problems."

At every step in the analysis it must be remembered that the most effective progress is that which the class makes in the gradual surmounting of successive difficulties leading to a useful conclusion. It is important that the student get the feeling that engineering is a combination of technical training and information with good sound judgment and clear thinking. No amount of mere knowledge can take the place of the ability to face fearlessly an unanalyzed situation,

to separate the important from the unimportant, and then to apply the few fundamental ideas which are the basis of every solution.

During the analysis of a study problem, the instructor gets a good view of the relative capacities of the members of the class. Engineering is not merely a garment which may be draped over any skeleton which happens to be exposed successfully to the vicissitudes of a four-year technical course; it is dependent upon a certain attitude and state of mind in the face of difficulty, as well as on the ability to understand and assimilate technical training and instruction. The class naturally resolves itself into four main groups:

First, there is the small but powerful group which responds to the call and begins to *play the game*. Some of these will ask leading questions, but they do not shrink from relying on their own resources, and their efforts are long sustained and earnest.

Second, there is the larger group which makes an attempt to unravel the problem, but finding itself in unfamiliar circumstances falls back on more direct questions. Though bewildered in the early stages, they do not lack courage. Once they get the idea that the problem is really "up to them," they respond with sincere effort. Not so quick as the first group, they are often safer in their judgment and in the long run become thoroughly reliable and effective.

Third, there is the main body which in the classroom is not in the habit of doing any real thinking. Mere copyists in the main, they lack originality even in their mistakes; but gradually they learn enough of the contents of books to pass examinations, enough of the sayings and idiosyncrasies of the instructor to avoid his displeasure, enough of the mechanical phases of the course to do medium quality work, and eventually by sheer repetition become familiar with sufficient knowledge to develop into routine men filling subordinate positions.

Fourth, there is the group of quitters who lie down. Always mentally "tired" and inert, they balk at any effort to induce them to stand on their own mental legs, and they respond to difficulty with the cries of "I do not know anything about that," "I cannot handle that problem," "I never have had that before."

This horizontal classification according to attitude, in which the student voluntarily chooses his own level, cuts across another classification which is based upon natural ability interwoven with preparation. Natural ability and thorough preparation will push the student to the front of whatever division of attitude he chooses, but unless he really desires engineering activity and unless he possesses a certain amount of engineering aptitude, he will pay the price grudgingly, will dislike the effort involved, and will eventually drift into other fields of endeavor.

The above analysis of classroom attitude is not intended as a goad

to whip up laggard minds or reluctant students, but as a reminder to those who are really serious at heart that the true joy of work lies in growth. The goal sought for measures the aim, and the goal achieved measures the outer success; but the manner in which the struggle is made measures the permanent inner growth of the man.

As indicated in these comments, students reveal far more than they realize by their classroom attitude and the way in which they approach, solve, and record problems. They sometimes wonder how it is that an instructor can form such close estimates of their interest, ability, and aptitude for engineering work. The spirit in which a student works and the problem sheets that he submits tell a plain story to the teacher. The student should know and keep it clearly in mind that *he rates himself;* the instructor merely records this rating.

2.6 Time and Patience Are Needed.

A reading of the previous chapter shows that the young man who hopes to become an engineer must start the formation of various habits of mind and hand at the same time that he is trying to memorize many facts and specifications. He will be seriously handicapped in his studies and his later professional work unless he does have the knowledge, skills, and habits appropriate to his calling. It should be obvious that the traits and habits that are the sign of the competent man are of slow growth. Long and persistent effort must be made to develop them if the young man aspires to positions of leadership. He must realize that no college course nor any collection of degrees can make him into a "finished" engineer. An engineer's education is never completed, and he should continue his professional reading and studies until he is ready to retire from engineering work.

College courses do, however, give him the opportunity to start the development of the traits and skills that he should have if he wishes to succeed. The student must remember that the most his instructors can do is to give him a vision of the ideal toward which he is striving, to coach him, and to guide him toward his goal. An instructor may inspire the student; he may impart much information; he may show him better working methods but when he has done this, he has done all that he can for the pupil. The student must do the rest; he must carry on alone.

No one else, fellow student or teacher, helps him by solving his problems for him. No one can get an education by proxy.

2.7 How to "Flunk out."

Unless he has been especially fortunate in his preparatory-school training, the average student finds that at least his first term in an engineering college is a rather trying and troubled time. He discovers that not only has he gained new freedoms but he has at the same time acquired a set of responsibilities. Most students sincerely want to do a good job with their studies. They are genuinely disturbed when they have continued trouble with their lessons. Eventually they work out more or less effective methods of study but often spend an undue amount of time and energy on their tasks. There are far too many students, however, who, having average ability or better, either become discouraged and quit school or else are failed and dropped by the college.

The causes of nonsuccess may be few and fairly easy to correct, or they may be due to a number of things, some of which call for long-continued effort if the handicaps are to be overcome. Some of the more common causes of scholastic trouble are as follows:

a. **Procrastination.** This is the chief cause of trouble. Many students have the habit of postponing the preparation of assignments, reports, or problems until the last minute. This invariably results in hasty, half-done work. It is wise, therefore, to get lessons ready ahead of time so that the material can have a chance to organize itself in the mind more or less unconsciously.

b. **Lack of interest.** Far too many young men and women are in college not because of any desire for an education but because it "seems to be the thing to do." Others want a diploma but have never really made up their minds as to what they want from life. With no central interest, they work halfheartedly, never doing their utmost in any course. Often they shift around from one field to another or endeavor to take advantage of an elective system to build up "snap schedules."

c. **Laziness.** This is frequently coupled with procrastination as a cause of disaster. The lazy man either fails entirely or else learns to be highly efficient during his few, brief working periods.

d. **Inattention.** A very widespread cause of trouble, involving many types of students, is the failure to concentrate on the written or spoken word when assignments are made, lectures

given, or troubles discussed. Momentary mind wandering may easily lose the key word or statement, and the whole lesson may be pointless without it.

e. **Lack of good study habits.** As a rule very few students come to college well coached in work and study habits. The whole purpose of the engineering problems courses is to try to compensate for this lack if possible. There are very real differences in reading habits, in ways to prepare for examinations, and to attack and record problems, the best of which should be learned.

f. **Reliance on formulas.** It cannot be said too often that the ability to understand and solve problems *does not* come by memorizing formulas. In fact, more time is wasted and more blunders made on mechanics, chemistry, and physics problems from a frantic search to find or recall a formula than from any failure to understand principles. Formulas are not substitutes for thought, nor can they be used safely by blindly substituting data assumed to fit them.

g. **Too many activities.** Sometimes the low grades and classroom troubles stem from the fact that the student is simply "spread too thin." He has so many outside activities and interests that there is neither time enough nor energy enough left to prepare class assignments as they should have been prepared. As a rule it is the student with superior ability who falls into this trap. He is personable; he is alert; he is energetic; he is known to have the knack of getting things done. As a result he has far too many calls on his time and strength, and so his studies suffer.

h. **Overwork.** Students who are wholly or partially self-supporting are usually in the group of the most ambitious, conscientious, and hard-working students. Quite frequently, however, they fail to get what they should from their studies simply because they are overtired most of the time. They lose chances at honor grades and also much of the pleasure of the campus life simply because they have tackled a work and college program that would down a Samson. Then they often complicate the situation by worry, by too little rest and recreation, and usually by highly irregular eating habits. It is much wiser to avoid such excessive overloads even if a year or two longer is needed to get the coveted engineering degree. It has been demonstrated again and again that a student can carry a normal

schedule and a job that takes up to 12 or 15 hr. a week, but after that he should treat each hour on the job as the equivalent of an hour of laboratory work and start to lighten his study load accordingly.

i. **Inadequate rooming conditions.** Study and comfort conditions in student lodgings are frequently deplorable, some rooms being the equivalent of big-city slums. Some people seem to believe the ancient idea that a man will study best when he is physically miserable This is a theory that seems to have its origin in the "log-cabin, light-of-a-pine-knot" tradition. It was probably started by a romantic individual who never tried it, for it certainly is false. Although the newer college-built dormitories are usually well planned, there are many old ones that are little better than barracks. Probably the worst conditions are found in the lowest priced privately owned rooming places.

Noise, poor heat, inadequate ventilation, and primitive lighting add up to a long-time drain on human nerves and energy. Sometimes the only light source is an unshaded bulb pendent in the center of the room. The glare from such a light source inflicts a punishment on eyes that they were never meant to withstand. Study lights over the desk should be used in addition to the general illumination of the room. They should be powerful enough to give ample light on the work, shaded to prevent glare, and placed so they neither cast shadows on the work nor throw reflected light into the eyes. The desk or table should not be crowded and overloaded but have space for spreading out books, drawings, and writing pad. The study chair should be comfortable but not one to induce slouching. Room temperature might range from 66 to 74F, but most people study best at 68 or 70F.

Regular study periods are important, and managers of rooming places can perform a real service by establishing study hours and insisting on quiet during those times. Radios should also be banned at this time. Many students have a pernicious habit of running the radio while trying to study. They claim that they can listen and study, but the evidence is that this is not true. If they do not hear the program, why run the radio? If they find that they are ever conscious of the program, then that very fact proves that their attention has been called away from the lesson being studied. Constant flitting of the attention from book to

radio and back again is the reverse of concentration, and college studies, especially of a mathematical and technical nature, demand the closest attention. Radio and effective study are incompatible.

j. **Health.** Many students do not realize that they are achieving much less than their best because they are below par physically. It is hard to convince them that a sound, efficient body is as important to a student working in the classroom and laboratory as it is to the man or woman who hopes to make an average showing or better in any athletic sport.

Some people have been cheated by nature or accident but make good in spite of that fact simply because they have an inward, spiritual drive that will not let them use their misfortune as an excuse for quitting. But far too many students have only themselves to blame for health troubles. The causes vary; but as a rule, insufficient sleep and improper diet are the root causes of short tempers, "that tired feeling," and a craving for excitement. The average full schedule of preparation, recitations, and laboratory periods will require not less than 50 clock hours per week. This is a full work week in any line of endeavor, and the student needs as much sleep and rest as any office engineer.

Students are notorious for poor eating habits. Not only are they addicted to eating at odd hours, but they also show very poor judgment in choosing foods. Too many limit their fare to a few favorite dishes. Avoiding milk, fruits, and vegetables, they fill up on starchy foods, carbonated drinks, and candy bars. The snack eaten at four or five o'clock in the afternoon has ruined the appetite for many a well-balanced dinner served at six.

Too much smoking is also a drain on nerve and body energy whether the addict is willing to admit it or not. No student has ever yet been harmed by leaving both nicotine and alcohol alone. The use or nonuse of either is not basically a moral problem at all but simply a question of scientific fact. The vast amount of scientific evidence that has been collected furnishes overwhelming proof that neither drug is conducive to the best operation of the mechanism of the human brain or body, especially that of young people.

k. **Poor attitude.** The last of the causes of poor work to be mentioned here is one that, fortunately, is not encountered at the college level very often. Poor attitude is a name that can cover

many student responses but is used here to refer to that chronic state of resentment which manifests itself in many ways. The experienced teacher can usually detect the symptoms in a short time. Most of the victims of this cross-grained habit of mind do not reach college, because they will not knowingly submit to the disciplines inherent in a 4- or 5-yr study program. The person who has this habitual reaction to life is to be pitied, for he is really a sick person and his own nemesis. His state is hopeless unless he can be awakened somehow to realize that a porcupine has few bosom friends. Unless he can recover from the egotistic idea that his immediate world must circle about his desires and whims, he should forget about college. He should try to find an occupation where he is his own boss and has to please no one. The student who resents correction or coaching or who regards his teachers as enemies who delight in "picking on him" is thus complicating all his study troubles.

Another trait that is related to the resentful habit and is sometimes its cause is that of being supersensitive. This busy, hustling world, whether met in the classroom or in the professional fields, has no time to spend in coddling "sensitive plants" or in weaning a young man from the attitudes of babyhood. The person whose "feelings" are easily hurt will have them bruised many times by others who do not even know that the bruised one is thin-skinned and who certainly have no unkind intentions. Such sensitive folk should, therefore, either grow up or find a corner where they will not be bumped while the workers concentrate on their tasks. If a man (or woman) cannot accept without resentment the rules by which the adult world does its work or learn to discipline himself, he will sooner or later receive his disciplining from an utterly indifferent, impersonal public.

2.8 Characteristics of a Good Student.

There are seven principal characteristics that clearly mark the good student and enable the instructor quickly to identify him.

a. **He is courageous.**

He does not lose heart when he strikes a difficulty or an unfamiliar situation but, on the contrary, accepts the challenge and works his way through to a satisfactory solution of the problem.

b. **He is independent.**

He shows initiative and self-confidence. He has too much self-respect to copy the work of others.

c. **He is sincere.**

His interest in a subject is real, not feigned. He does not bluff or try to substitute talk for deeds.

d. **He is a good worker.**

He is more interested in his work than in the clock, even near the close of a period.

e. **He has a good attitude when corrected.**

Some students resent correction, evidently considering it a personal affront. Others are supersensitive and as a result are constantly having their feelings hurt. The good student has the right spirit and welcomes suggestions for bettering his work.

f. **He is systematic.**

His orderly computations and his clear sketches show his forethought and logical thinking. He concentrates on his work and plainly shows that he means business.

g. **He is neat in his work.**

His willingness to adhere to specifications and standards; his clear, well-formed lettering; and his properly arranged papers are a reliable indication of orderly mental and personal habits.

2.9 The Laws of Habit Formation.

If one is lacking certain of the traits of a good student but earnestly desires to acquire them, there are only two steps to take: abandon the old habits and develop the desired ones. Every man is a "walking bundle of habits," and the nature of his thoughts, his character, his actions, and even his outlook upon life are determined by the habits of mind and body that he has developed over the years. Bad habits cannot be cast aside without long-continued thought and effort, nor can good habits be assumed instantly, much as one would put on a garment. There are certain laws of habit that every man must obey if he would be successful in the attempt to develop latent powers. Bad habits, like weeds, just grow; good habits, like fine fruit, must be cultivated. Bad habits cannot be rooted out, and the space be left bare. They must be replaced by worth-while, controlled habits;

otherwise the weedy ones will soon be in command again. When any man attempts to form a new habit, he must obey four laws.

a. **Make a definite assertion of will.**
He must use his will power; mere wishing is not enough. Students who say "I cannot" after a few attempts usually mean "I do not care to make the effort."

b. **Make a positive beginning.**
Give the new habit the advantage of a correct and emphatic start. Study the best examples, and try to pattern after them. Kitson[1] says: "As you value your intellectual salvation, then, go slowly in making the first impression and be sure it is right." Unless this start is correctly made, the man may find that he is only substituting a new bad habit for an old one.

c. **Do not excuse any exceptions.**
Anyone will have scant success in "tapering off" on a bad habit or in trying to drift into a good one. The drunkard says, "I won't count this one," but his bad habit has a firmer grip than ever.

d. **Exercise the new habit.**
Give the new habit a "workout" at every opportunity. It will grow strong through use. Review its beginnings from time to time to make sure that no undesirable variations are creeping in; see that it is still according to plan. The way to kill any habit is to quit it. Habits, good and bad, die of neglect.

The entire learning process is essentially a habit-forming process, and for this reason the instructors in the engineering problems courses try to make conditions favorable for the starting of many new habits. The methods used in the classroom are based upon the laws of habit formation. The instructor's efforts are in vain, however, if a student does not care about improving or if he does not practice the new habits outside the classroom as faithfully as when under the direct supervision of the teacher. For example, a student should apply the ideas of neatness and systematic methods to all his studies, not to engineering problems alone. If he exercises the old habit of scribbling ten times as

[1] KITSON, HARRY D., "How to Use Your Mind," Lippincott, 1921, p. 62.

often as he does the new habit of being neat, what chance does he have of becoming habitually neat and orderly?

2.10 The Final Answer Is Not Enough.

New students often wonder why there is so much emphasis on workmanship in the engineering problems courses. They find not only that their results are important but that the instructor insists that they leave a plain trail all the way through the solution of a problem so that the reasoning as well as the numerical work can be checked with ease. The instructor also says that this record must be neat, systematic, and done in accordance with a detailed code of specifications for computations. The majority of students begin not only to adapt themselves to the system in a few weeks but also to see that the code is aiding them to think better, to solve problems with less fumbling, and to reduce mistakes. Their papers become neater and begin to take on a professional appearance. There are always some students, however, who have good minds but who, for various reasons, do not make satisfactory progress. Some of them would be misfits in engineering and should enter other occupations; but as a rule, all that most of them need to do is to trust their instructor and "play the rules of the game." The man who makes the best progress is usually the one who keeps an open mind, builds upon the advice of experienced men, and adapts the old to the new situations as they arise.

If a young man enters an engineering career with the erroneous idea that the final answer is the only thing that matters, he is due, sooner or later, to have some painful and embarrassing experiences. He may find that for him the building up of new computation habits is a long and sometimes tedious process. Then there is the lad who, apparently, never has learned to abide by the rules of any game or other activity. He seems to resent all requests to do things in a specified manner; and when he is given a definite instruction, no matter how reasonable it may be, he acts as though it were a personal affront. These nonconformists should by all means leave the field of engineering if they are not willing to change their attitudes. The engineer must make and do things according to specifications all his life. These specifications may be the laws of nature or the mere whim of the owner of a property, but obey them he must. Another group of

students who have their troubles in engineering problems classes is made up of those who seem to be constitutionally slovenly in all that they do, say, and think. Their records are dirty, scribbled, and without plan. They frequently show these same traits in their speech, dress, and personal habits. They consider it a waste of time to be neat or systematic or to regard the appearance of any work. Many of these men may have excellent minds, but they little realize how great a handicap their careless habits may prove to be when they seek a job. Personnel directors report that frequently the students who are hard to place in jobs because of unprepossessing personalities are from this group. Good workmanship is not just a frill; it is what employers expect. The men who are naturally neat will, of course, be the quickest to turn out satisfactory papers, but without doubt these other groups can profit most from the many suggestions and specifications in this manual.

2.11 The Two Kinds of Learning.

Most students and even some teachers are not aware of the fact that learning frequently consists of two distinct but related activities. One is mental: the acquisition of knowledge and the development of the reasoning ability. The other is manual: the training in various skills that usually involve muscle control. Knowledge and understanding may "dawn on one" suddenly, almost intuitively, but muscular skills are never acquired in that way. The development of a skill requires persistent and repeated practice, even though the mind may have complete understanding of the task itself. In some fields of endeavor only the mental qualities are important, and a man may succeed brilliantly even though he is sadly lacking in all skills requiring accurate muscular coordination. In other occupations mechanical skills are of the utmost importance, and the intellectual development is of secondary value. It used to be said of a certain model of automobile that the driver needed "brains in his feet." There is much truth in that for any make of car. A clumsy watchmaker, for instance, will soon be out of a job. He must be "finger-minded." The engineer, however, must have a balanced blending of the two abilities if he is to go very far in his profession. Not only must he have a sure knowledge of many facts and know how to use them accurately in reasoning through his problems,

but he must also become well trained in the various manual skills that are a part of his profession.

It is for these various reasons that the engineering problems courses have been put into the engineering curriculums of many schools. Not only are they planned to coach the student in the formation of good mental habits, but also one of their principal aims is to train the engineering student in the workmanship habits that are taken for granted by employers. Unless the student applies the principles of good workmanship to his other studies, however, he will not make very rapid progress in developing the traits discussed in this chapter. Even though a beginner's lettering may be crude for a time, it will clearly show whether or not he has his mind on it and is trying to do good work. Surely he can adhere to the instructions regarding sheet rulings, scribbling, scratch paper, and other specifications that have no reference to lettering. He should not attempt to memorize the chapter of specifications, but instead he should glance through it occasionally and, without fail, read carefully any specific items that are called to his attention. In this way he will learn the details gradually and not have to go through the laborious task of rote memorization. If he works faithfully, he will develop latent powers of which he was unconscious and eventually become a competent, reliable worker showing initiative and ingenuity in the handling of his assignment.

2.12 Method of Attacking a Problem.

Reasoning is purely a problem-solving process, and it is self-evident that the only way to develop the reasoning power is to solve problems. Only by exercising the muscles can physical strength be developed, and, in like manner, the ability to overcome difficulties is obtained by actually facing them and attacking them courageously. The student must welcome difficulties as opportunities to test his ingenuity, his initiative, his ability to cope with a new situation. Each new difficulty is a challenge. How will he meet it?

Because so few students know how to study their lessons, to read assignments, and to solve problems, some condensed suggestions on methods and mental attitudes will be given in a few topics to follow. There are numerous well-written books that discuss study methods, and the ambitious student will find many

valuable suggestions in any one of the volumes listed at the close of this chapter. The student who finds his college work very difficult should read at least one such book and apply its suggestions to his daily assignments.

As stated earlier, the man who uses hit-or-miss methods of solving his problems may achieve a certain crude proficiency, but such methods are not sufficient for the man who desires to master his vocation. It is plain that only by controlled and organized thinking can the solving of problems be put on an efficient and effective basis. It is worth while noting that there are but five main steps in the solution of any problem. These may be stated as follows:

a. **Is there a problem?**

One must be aware that there is a problem or difficulty before he can reason about it.

b. **What is the problem?**

The problem must be clearly stated. One must understand the statement of the problem and perceive its central question before he can start its solution. What is the central objective?

c. **How can the problem be solved?**

Call to mind all the fundamental principles that seem to have a bearing on the problem and all of the various methods of applying these principles. Collect all of the tools.

d. **Solve the problem.**

Choose the laws and principles that seem to apply, and get a result. Use the most logical tools.

e. **Is the answer correct?**

Look at it. Does it look reasonable? Use another method of solving the problem, and go through it again. Check the first answer.

One who tries the suggestions just given will find that he will be saved many false starts. Many students have no definite plan in mind when they attempt to solve a new problem. Frequently they have little or no idea of what they are trying to find. They copy a few numbers on their papers and "just start figuring" regardless of whether the numbers make sense or not. Others try to find a magic formula that will enable

them to dodge all the difficulties and the labor of thinking and yet give them an answer that the instructor will accept.

Problem solving loses much of its difficulty as soon as the computer abandons hit-and-miss methods of attack and slavish use of textbook formulas and adopts systematic ways of going about the task of getting answers. One of the greatest causes of mistakes and lack of understanding of problems is the blind use of long and involved formulas. Certain men seem to believe that the measure of their engineering knowledge is their ability to substitute data in a formula that they have found and then "to rattle the thing around" until some sort of an answer falls out. Such fellows are not thinkers but mere "handbook engineers" who are usually completely lost when they run into a problem that is new to them or one that differs in any respect from the type formula that they found in a handbook. Handbook engineers are regarded with contempt by all truly competent engineers and sooner or later wind up in minor, routine jobs. Symbol formulas are only a form of shorthand, useful in routine work but safe only in the hands of experienced computers. Beginners frequently use them to conceal ignorance or to dodge the labor of thinking something out for themselves. Someone has said that a symbol formula is "an incantation for getting answers without reasoning." In order to signify anything symbols must be translated into words in the mind of the user, and the words must stand for very definite, concrete ideas; otherwise the symbols are meaningless.

One excellent scheme for attacking a problem, especially a long, involved one, is to block out the solution, step by step, using words or symbols for the various quantities but introducing no numerical values. Time can be saved by providing space for the actual values to be entered later on. If the reasoning in this skeleton solution is found to be correct, it then becomes a routine matter to substitute the numerical data and do the figuring. This method often prevents blind-alley calculations and the determination of intermediate values later discovered to be of no use in getting the desired final answer.

Check lists of operations involved, steps to be taken, intermediate values wanted, etc., are often a help in organizing both the thinking and the solution, because check lists tend to keep the computer from forgetting some item. This is where the

use of standard forms and methods proves to be very helpful in the handling of involved calculations. Like well-designed machinery they leave the operator free to concentrate on the main task.

2.13 Learn to Read.

Experiments made in several universities seem to prove that many of the students have trouble with their studies because they are slow in reading. The lack of ability to learn from the written or printed page is a severe handicap, because the greater part of college instruction is given with the aid of textbooks and blackboard demonstrations.

There may be several reasons for poor reading; some will require special instruction to overcome, but others are unjustifiable excuses to offer for unsatisfactory work. The commonest cause is inattention. Get rid of distractions, such as a radio program; then put your mind on the job, and concentrate on what a writer is saying. The second great cause of error in reading is allied to the first. It is lack of observation. Many men "have eyes and see not; have ears and hear not." A third cause of trouble is a scanty vocabulary. Own at least one good dictionary, and use it daily. Look up the meaning of every word that seems unfamiliar. Also be sure that the author is not using a common word in an unusual sense.

One way of studying text assignments that seems to work with many students is the following:

a. Read the assignment rapidly to get the gist of the discussion.

b. Read it a second time slowly enough to get the details. Look up the strange words. Do not attempt, however, to memorize the material.

c. Now run through the assignment again, and underline the key sentences or ideas in each paragraph. Do not overdo the underlining; it loses its effect. If time permits, review the underlined material just before going to class.

One place where poor reading habits exact a heavy premium from a student is in reading lesson assignments. In too many cases the student gives the assignment a quick glance and lets it go at that. Later he finds that he has missed a critical instruction or bit of information. Fully 50 per cent of the engineering problems students who miss all or part of assigned class problems

do so simply because they start to compute something before they really know what they have been asked to do. When told to read an important paragraph of instructions in this manual before starting work, students show a strange reluctance to open the book. Instead they sit there and "beat their brains" for 10 or 15 min simply because they do not want to do 5 min reading. They seem to regard reference to the printed word as a waste of time. They will have to get over this attitude, because engineers have to consult books and periodicals constantly. The student who thinks that the teacher should tell him orally each detailed step to be taken in his lessons, experiments, and problems has the wrong conception of engineering. Most of the time the engineer is on his own. Any student who thinks that it is a teacher's duty to lead him step by step through or around all troubles is by temperament entirely unsuited for the practice of engineering. He will have to be content with a mediocre, routine job all his life.

It is plain, therefore, that every engineer must learn to read accurately, with reasonable speed, and without inner resentment of the fact that such reading is an absolute necessity for the professional man. The printed page is the most important device that mankind has for preserving and transmitting the knowledge and experience of others, living and dead.

2.14 Note Taking.

Engineering students do not have so much lecture work, fortunately, as those in some other courses of study, but even so they should know how to take usable notes from speeches and oral instructions. The majority of directions on the job are given rapidly and informally in oral form. Accurate notes are imperative if the young engineer is to fill his job in satisfactory fashion.

It is an art to take lecture notes in the classroom, a highly individual skill, and many students never become even passably proficient in it. There are a few aids for a student in his effort to make note taking more successful. If the lecture is an elaboration of subjects covered in the text, it is a fairly simple matter to correlate it with the key sentences in the text. Do not attempt to take down a lecture word for word or get all the minor details. Concentrate on what is being said in the lecture,

being on the alert for key words or sentences. Get the important points; then use them to help recall details soon after class, and complete the notes while the material is still fresh.

2.15 Examinations.

It is important for the engineering student to learn how to take examinations without becoming all "worked up" and filled with dread over the ordeal. Examinations are no novelty in the world outside the classroom. In most states no engineer can practice engineering independently unless he has passed a set of qualifying examinations. These tests are given by the state licensing board and may require two or more long days. Applicants for civil service jobs also will have to take examinations before being put on the eligible list. In some corporations examinations must be passed in order to qualify for higher rating and pay.

Tests, examinations, quizzes, and similar devices for measuring a student's grasp of new knowledge will probably exist as long as there are students. These testings need not be ordeals, however, if the student has made even a fair attempt to learn the subject. Many good students make their task more difficult because they have no systematic attack for such examinations.

The following plan for answering examination questions has been used with excellent success.

a. As soon as the questions are put before one, read over the entire lot rather quickly to get the range of topics.

b. Note the number of questions and the total time available. Deduct 10 or 15 min for reserve time to use on difficult questions; then divide the rest of the time evenly among the questions.

c. Now answer the questions in order; but if any one of the answers is incomplete when its share of time has been used, leave it and go to the next. Remember that a few minutes have been reserved for cleaning up such unfinished business.

Many students who have tried this plan find that frequently the answers to later questions seem to half formulate themselves while the conscious attention is on the actual writing of the answer to an earlier question.

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CHAPTER 3

STANDARDS FOR THE COMPUTING ROOM

3.1 Practice Makes the Master.

As pointed out in the preceding chapter the acquisition of the habit of neatness, the use of systematic methods, the adherance to standard forms, and the development of the other skills that are the mark of the competent worker are of prime importance to the ambitious man. The gain in working speed, the reduction in the number of mistakes, and the resulting feeling of confidence in personal ability are ample rewards for the conscious, painstaking efforts that must be made by the beginner. It is a trite saying, but true, that "Perfection is made up of trifles but perfection is no trifle." Such is certainly the case when the engineer, young or old, endeavors to raise his personal standards. The ability to do an excellent piece of engineering lettering comes by giving careful attention to the details of the form of each individual letter, not by giving the alphabet a sweeping glance. After a close study of the sample form should come the effort to reproduce it. Moreover, one successful letter does not mean that the skill has been acquired for all time, but practice, practice, and more practice is needed. The good swimmer, the low-scoring golfer, the big-league ball player, all who are better than average in any sport will be the first to say that coordination and muscle control come by practice and more practice. They will also say that there must be no letdown if the skill is to be kept at a high level.

3.2 The Function of Specifications.

The suggestions and specifications in this chapter may be compared to the books that have been written to help sportsmen and hobbyists perfect their skills. The suggestions must be followed if they are to have any value whatever; and as in most sports, the advice and criticism of a coach is also needed from time to time. He is your engineering problems teacher. He is simply a coach—the student has to be the performer. Speci-

fications are in universal use in the world around us. They range from the simple, single instruction to veritable volumes such as govern the construction of a public building. When the housewife says to the butcher "I want a 5-lb rib roast of young beef, Grade A," she has given a set of specifications that she expects the butcher to fulfill. A set of architectural specifications for the construction of a skyscraper is many times longer and more complicated, but on examination it will be found to be made up of single items differing only in kind from the housewife's order for a beef roast. Specifications are written to obtain certain standards of workmanship, to decide among two or more alternate products or methods, or to outline what must be done in order to meet certain natural laws. In the main, specifications are just a codifying of generally accepted practices. They are subject to revision from time to time and to changes as required to meet special situations.

In the industrial and commercial world it is taken for granted that specifications will be followed. It does not matter whether the work is done by an individual or a large corporation. Work not done according to specification may be rejected and need not be paid for. There have been countless lawsuits and unnumbered thousands of dollars lost because men have chosen to ignore specifications, have tried to cheat them, or have been too impatient to study them. Some men by temperament resent and try to dodge specifications merely because they are instructions that must be followed. Such men lead dissatisfied lives, however, for there are few people indeed who can ignore the requirements of the world in which they live and earn their livelihood. Because he will have to follow them and will in time be writing specifications, the young engineer should welcome the opportunity to gain experience in the application of specifications wherever he has the chance. The specifications that are given in this book are not aimless restrictions but will help the student become competent in organizing his work and at the same time give him practice in reading, understanding, and applying detailed instructions.

3.3 Origin of These Specifications.

The various details that are embodied in these standards for the computing room are based upon the experience and observa-

tion of a number of engineers. Many of them are commonly used in engineering offices; others are taken from certain national standards as developed by joint committees of engineering societies; still others are the result of classroom experiences; and some are based on suggestions made by students or instructors. Thus this chapter is a composite code derived from various sources. It has been found to be suitable for the engineering office as well as the classroom. This code is, of course, not the only possible one or necessarily the best for all offices, but it has been used in classwork and offices in essentially its present form for a long time and has been found to be of great practical value in producing neat, clear, explicit engineering records.

It is highly desirable for the young engineer to acquire, as early as possible, the various manual skills and mental habits that tend to make him an efficient and reliable computer. Insofar as he follows the spirit as well as the letter of these standards, he will be aided in developing habits that will have a genuine bread-and-butter value to him no matter what field he may ultimately enter. Because the ability to adhere to a set of standards is so important, all computations, diagrams, tables, and graphs are to be prepared in accordance with this chapter on Standards for the Computing Room. Failure to adhere to the specifications is sufficient reason to cause the checker to reject the work in whole or in part, even though the numerical results are correct. The quality of workmanship will be considered in evaluating each set of papers.

3.4 Equipment.

The practicing engineer is expected to provide himself with certain instruments and materials and be ready for any assignment that may be given. He is held responsible for maintaining this equipment in good working condition. He should have his own tools. The borrower is disliked, even by his fellow workers. Each man should see to it, therefore, that he has the following equipment, that it meets specifications, and that he has it at hand ready for work each day.

(*1*) **Paper.** Use standard 8.5- by 11-in. engineering computation paper, primrose color, having coordinate rulings on the back side, ruled margins on the plain front side, punched to fit loose-leaf binders.

(*2*) **Pencil.** Use automatic pencils. They are being adopted by large numbers of firms because of their economy and time-saving features. Have one that holds the lead rigidly. The propel-repel type of pencil seldom holds the lead firmly enough for good lettering or for drafting-room use. Absolute control of the point is essential to good lettering and line work. Sand the point now and then in order to obtain consistently clean-cut work.

(*3*) **Grade of lead.** Use a lead that will give a clear black mark without smudging. A hard lead will give lines that are too faint and gray. Soft leads will give fuzzy, smeary lines. A 3H lead is generally the best grade for engineering paper, but adapt the grade of lead to the surface of the paper used, as papers vary. For coordinate paper and all graphic work use a 5H lead.

(*4*) **Eraser.** Use an eraser that will clean thoroughly, but without roughing the paper or making a mess of dirty crumbs. Never use ink erasers on pencil work or the crumbly, cleaning erasers that are made for use in the drafting room.

(*5*) **Ruler.** Use a 12-in. ruler for making sketches and ruling lines. If you have a slide rule of the Mannheim type, it has a scale on one edge that serves as a ruler. The 6-in. pocket rulers are inefficient, as it is impossible to draw a line across the sheet in any direction without shifting the ruler.

(*6*) **Pencil pointer.** Use a fine file or sandpaper block to get the proper pencil point.

(*7*) **Loose-leaf binder.** Keep all papers in a ring binder. Sheets quickly become mussed and torn if kept loose in a folder. Reserve a section of the ring binder for back problems, notes, and any mimeographed data sheets that may be furnished from time to time. Keep all notes on standard engineering paper. Do not destroy corrected problems.

(*8*) **Books.** Keep this Manual, the Workbook,[1] and all other required books ready for use.

(*9*) **Compass.** So many diagrams require the drawing of circles that a small compass is desirable.

(*10*) **Protractor.** A protractor is needed in constructing to scale any diagrams that involve angles.

(*11*) **Irregular (French) curve.** This is an essential tool in the drawing of graphs and other irregular curves. A very useful

[1] DANA, F. C., and L. R. HILLYARD, "Engineering Problems Workbook," William C. Brown Company, Dubuque, Iowa, 1945.

shape is the K. & E. No. 1960-4. Small curves, 5 or 6 in. long, are much too small to be of value in the construction of most graphs.

(12) **Slide rule.** Be sure to get a slide rule that has at least the following scales: D, C, CI, CF, DF, A, K, and L.

3.5 Workmanship.

The group of specifications under this heading is one of the most important in this entire chapter. The man whose paper work shows scant evidence of intent to conform to the spirit of the various specifications and suggestions seldom realizes how much of his character and incompetence he is revealing. It is the seemingly little things that tell the story. The tone of the pencil work, the placement of the entries, the dimensioning of a diagram, the logical or haphazard arrangement of the steps in the solution—each one tells something of the attitudes, the habits, the skills, and reliability of the worker. Slovenly, scribbled papers are an almost unfailing indication of a lazy, careless, indifferent man. Neat, crisp, orderly, well-planned papers are the product of a thoughtful, dependable engineer. It is a mistake to assume that neat work means slow work. The competent man takes no longer than does the hasty, sloppy, slapdash worker and, moreover, has fewer blunders to locate and correct.

It should be noted that none of the specifications on workmanship depend in any way upon sheet rulings, the computation methods used, the nature of the problems, or any other details that may vary from office to office. Only one of them, Spec. *(20)*, refers to a man's skill in doing engineering lettering; all the rest can be followed by the engineer and the nonengineer alike. It is adherence to such standards as these that distinguishes the good craftsman from the poor one. The man who tries to conform to the spirit as well as to the letter of these suggestions will seldom need to apologize for the appearance of his papers. He will learn to adapt the quality of his work to the needs of the task in hand, but always following this rule:

Do the best and neatest work that time, money, and circumstances will justify.

(13) **Pencil work must be decisive.** Make clean-cut, reasonably black marks in all lettering and line work. Perfect legi-

bility should be the goal. The first definite impression made by a computation sheet is determined largely by the tone of the pencil work. Getting pencil work with a "snap" to it is an art perfected only by practice.

(*14*) **Find the right grade of lead.** Get the proper hardness for the paper to be used. Do not rely on the manufacturer's hardness marking. There is no uniform scale that is used by all makers of leads.

(*15*) **Do not make fine, faint, gray marks.** They are sufficient reason for complete rejection of a paper by the checker even though the sheet is otherwise acceptable. Hard lead or too sharp a point may be the cause, but generally lack of pressure accounts for faint marks.

(*16*) **Smeary pencil work is inexcusable.** It is ample reason by itself for the rejection of a paper. Too soft a lead, a dull point, and, usually, lack of pressure are the causes.

(*17*) **Bear down on the pencil.** Use enough pressure on the pencil so that the pencil mark will be embedded slightly in the paper. Too much pressure, however, will cause the paper to curl. Practice with various pressures, and then form the habit of using the pressure that gets the right effect.

(*18*) **Use backing paper.** Do not try to work on a single sheet of paper resting on a hard surface, because the lead cannot be embedded into the paper properly. Use a firm but resilient backing such as a sheet of smooth poster board or half a dozen sheets of unused problems paper. A yielding, spongy backing surface is even worse than one that is too hard.

(*19*) **Keep the pencil sharp.** Do not, however, try to work with a needle point. The result will be light, scratchy work that is hard to read and rubs off easily. If the point is too fine, it will break as soon as more pressure is applied in the effort to get clearer, permanent line work.

(*20*) **Use engineering lettering.** Figures and letters must be properly formed. They must be neither too large nor too small. Adhere to Type 6 as shown in the ASA "Standard for Drawings and Drafting Room Practice."

(*21*) **Do not scribble.** Computations may be rejected for this reason alone. There is a distinct difference between scribbled work where no thought has been given to appearance and lettering that is crude in form but clearly shows an effort to do neat

work. Work that is not worth doing legibly is not worth doing at all.

(*22*) **Do not use scratch paper.** There is a place on the computation sheet for every necessary figure; hence, there is no need or excuse for doing work on scraps of paper that are later thrown away. Use a pencil whenever you please, but do all your figuring on the sheet where it belongs. Read Topic 1.9, page 8.

(*23*) **Keep left column neat.** Computations in the left-hand column are to be made just as neatly as the work in the right-hand portion of the page. There is no proper place for scribbling [see Spec. (*21*)].

(*24*) **Never mark one figure over another.** Do not retrace a letter or figure. This is a sure sign of a careless man. Erase the faulty value, and put it in correctly.

(*25*) **Erasing must be clean and thorough.** Sheets may be rejected for slovenly erasing. It is often a good plan to rub the erased spots with the thumbnail before trying to mark over them again.

(*26*) **Repair any work damaged in erasing.** Repair any lines or entries that may have been touched accidentally with the eraser.

(*27*) **Remove eraser crumbs.** Clean the paper before turning in the sheet for checking. A paper covered with dirty eraser crumbs is a disgusting thing to handle and certainly is not of high quality.

(*28*) **Canceling incorrect work.** If a computer discovers that an entire step in a solution is incorrect, he should cancel it by ruling two diagonal, intersecting lines across the entire sheet through the discarded work. Separate the discarded material from the calculations to follow by ruling a horizontal line across the sheet.

(*29*) **Keep papers clean.** Never submit torn, dirty, or greasy papers. Such papers will be rejected by the checker even if results are correct.

(*30*) **Clearance is necessary.** A clearance must be maintained between any lettering and any lines. This applies to letters and figures themselves and to diagrams, rulings, underlinings, and the lines used in fractions, multiplications, etc. Lines must never be drawn through or in contact with figures or lettering Move one or the other.

(*31*) **Stay within rulings.** Never allow work to overrun the column space.

(*32*) **Avoid crowding.** A sheet may be technically correct as far as specifications are concerned, yet give the impression of being badly crowded. A crowded paper is difficult to read and check.

(*33*) **Use a straightedge.** A ruler should be used for drawing diagrams, tables, and all sheet rulings.

(*34*) **Make straight short lines.** Use a ruler for short lines if you cannot draw a reasonably straight line freehand.

(*35*) **Watch your spelling.** Be on guard against misspelled words. Carry and consult a pocket dictionary if you have trouble with spelling. Guard against errors in English.

3.6 Make-up of the Computation Sheet.

The following specifications for the make-up, or format, of the computation sheets may be used to advantage in engineering offices as well as in student work. Details may have to be varied, of course, to suit the needs of a particular company or department, but adherence to this system will produce a desirable uniformity in the preparation of records. It is through the adoption of such standards that time is saved in calculation and in checking. Mistakes will be reduced and be easier to locate if blunders do occur.

(*36*) **Use the plain side of the computation paper.** The quadrille rulings show through the paper plainly enough to serve as guide lines for diagrams and lettering.

(*37*) **Use only the printed side of any coordinate paper.**

(*38*) **Keep binding holes to the left.** The holes for the binders must always be at the left or at the top if the sheet is turned sideways. This will place the written work on the right-hand page when the book is opened.

(*39*) **Put no entries in left margin.** The left margin is reserved for binding purposes. No entries of any kind should be made in it other than the desk number. No figuring should be allowed to overrun the calculation column into this margin.

(*40*) **Top margin is to be complete.** The top margin of every page is to be ruled and subdivided as shown in Fig. 1, page 50. Start at the left corner of the sheet, and record the information listed below and in the order given.

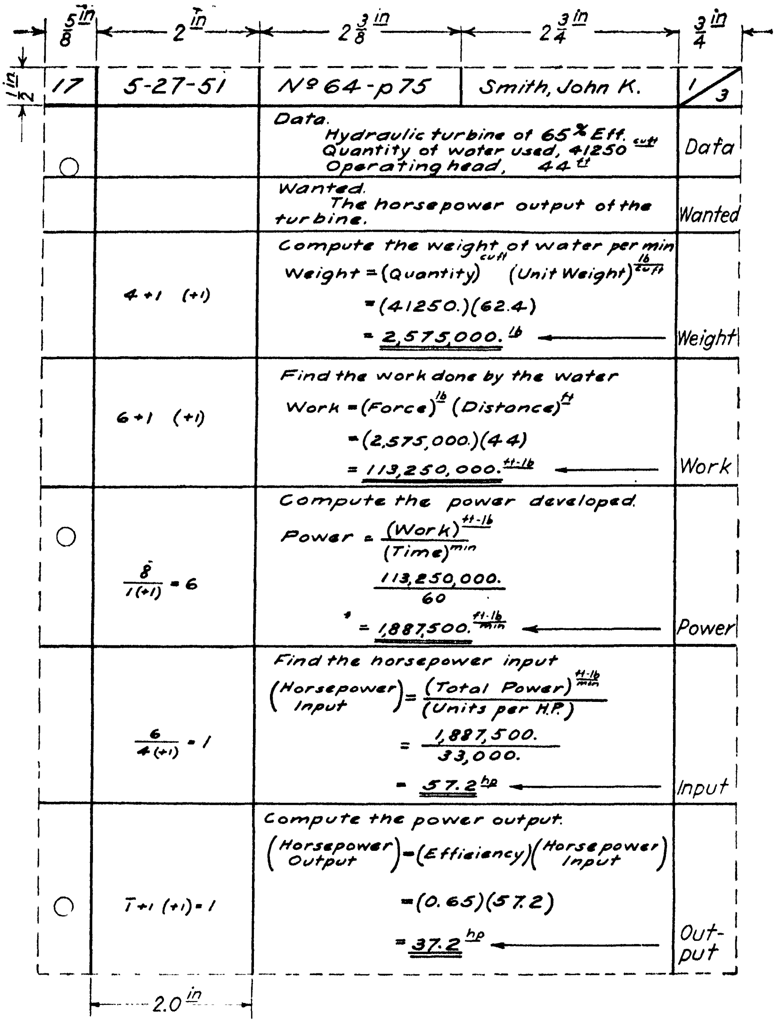


FIG. 1.—Slide-rule calculations.

(*41*) **Put desk or table number in left corner.**

(*42*) **Enter date in second space.** Record thus: 1-11-49. Do not use the form 1/11/49.

(*43*) **Record problem number in center space.** Show the problem number and also the page if it is from a textbook.

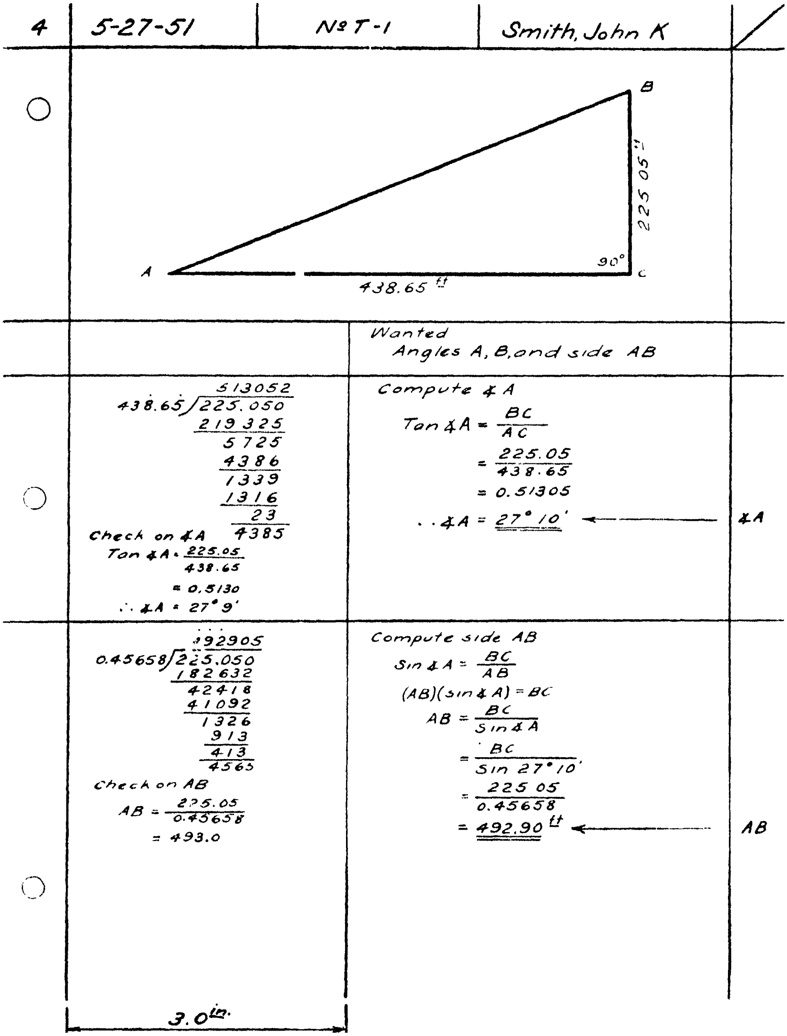


FIG. 2.—Right triangles. Typical page layout for cut-longhand calculations (see page 137).

(44) **Show name of computer in fourth space.** Record last name, then the initials. A neat rubber stamp may be used for this if desired.

(45) **Number pages.** Each page is to have its number recorded above the diagonal line in the upper right corner of the

page. The total number of pages in the assignment is to be entered on each sheet just below the diagonal line. Each day's work is paged as a unit starting with page 1 each period.

(*46*) **Center all top margin entries.** Keep them off the margin ruling.

(*47*) **Use right-hand margin for indexing.** The right-hand margin is reserved for indexing the answers to the various steps in the solution of the problem [see Spec. (*126*), page 62, and various sample pages].

(*48*) **Divide sheet vertically into two columns.** The widths will depend upon the method of calculation to be used.

(*49*) **When slide rule is used make left column 2 in. wide.** The characteristics of the terms may be entered here (see Fig. 1, page 50).

(*50*) **For longhand left column is to be 3 in. wide.** Refer to Fig. 2 which shows the use of cut-longhand. See Form 101 in the Workbook.

(*51*) **When logarithms are used make left column 3 in. wide** (see Form 35 in the Workbook and Fig. 3, page 53).

(*52*) **Tabulate logarithms.** For multiplication and division with logarithms rule a tabular form for the respective entries (see Fig. 3, page 53, and Form 118 in the Workbook).

(*53*) **Rule log forms first.** Rule the form for the logarithms before making any entries. Adhere exactly to the dimensions shown in Fig. 3, page 53.

(*54*) **Put ruling for logarithms at top of space.** Enter the figures for interpolation below it.

(*55*) **Do not break the vertical ruling.** The vertical column line specified in (49)–(51), should not be broken once it has been started.

(*56*) **Use right-hand column properly.** Reserve this column for all diagrams and the series of steps into which the problem is subdivided. The right-hand column should be used for all the entries needed in the analysis, reduction, and solution of the problem. No arithmetical or logarithmic calculations should be entered in this column; space has been provided in the left column for such work.

(*57*) **Put diagrams at top of sheet.** A diagram is often a vital part of the solution of a problem and hence should precede all other entries.

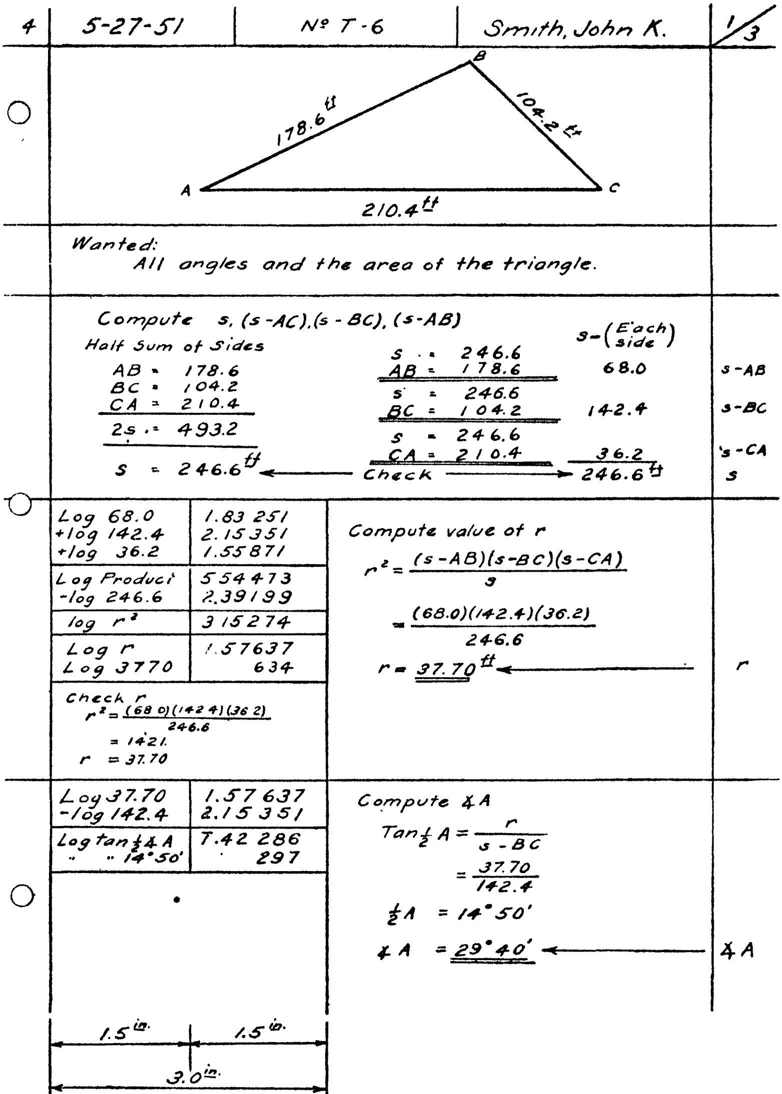


FIG. 3.—The radius-formula solution. Computations by five-place logarithms.

(*58*) **Make all diagrams of ample size.** The most common mistake of engineering students is the use of diagrams that are so small that they cannot be properly dimensioned, have data and results entered upon them, or be used to show necessary con-

struction lines. Use the entire width of the page or even a full page at any time that clearness will be gained (see Topic 3.12).

(*59*) **Show all given data.** Record the data on the diagram if one is used. If no diagram is drawn, be sure to list all given data in concise form.

(*60*) **State wanted items clearly.** A statement of the main or final objective of the problem should be given briefly and clearly immediately following the listing of given data and any diagrams used.

(*61*) **Keep operations simple.** Break the problem into a series of simple steps, each complete in itself as an intermediate problem and yielding an answer that becomes "data" for the following step.

(*62*) **Show five entries in each step,** as in Topic 3.9. Thus:

a. The item sought in that small step in the solution.
b. The word equation or formula applying.
c. Substitution of the data exactly as given.
d. Reduction of the mathematical equation.
e. The answer to that step in the solution including the units in which it is measured.

(*63*) **Emphasize the answer** [see Spec. (*124*), page 62].

(*64*) **Rule a line from edge to edge** of the paper to separate the steps. There is no reason for drawing the line if nothing more follows on that page.

(*65*) **Start each new problem at top of a new sheet.** If the assignment consists of several series of short, independent problems, then start each new series on a new sheet.

3.7 Mathematical Signs and Symbols.

Throughout the history of mathematics there have been many changes in the forms of the symbols used and a decided lack of uniformity. Only a few symbols can be said to be international. Various unsuccessful efforts have been made from time to time to standardize practice. Textbooks and handbooks, however, are usually lagging far behind the standards that have had general approval such as those of the ASA. Thus it happens that signs that were well known at one time but have passed out of general use are still being set by printers merely because they have the

characters and think them easier to set in type. Many of these characters are ambiguous in hand-written computations and hence are dangerous to use. The safest rule to follow is to abandon the use of any symbol or character or arrangement that can possibly be misread.

The following specifications are based upon the recommendations of various organizations and committees where any codes were available. In adopting each one, however, clearness and simplicity in handwork was the first consideration, not the convenience of the typesetter. His work, when it is completed, is done once for all, but the engineer makes new records daily, and each is an original record. So the rule should be "*Safety first in all ink or pencil work.*"

(*66*) **Use parentheses only to indicate multiplication.** In complicated equations use brackets and large parentheses to collect groups of terms.

(*67*) **Do not use the letter** x. Abandon both the letter x and the symbol $\times$ as a sign of multiplication. They are often confusing, especially in algebraic work.

(*68*) **Never use the dot as a sign of multiplication.** It is not a generally recognized symbol; moreover, it is readily confused with the decimal point in numerical work. The engineer works with decimal values constantly, and the danger of misreading the dot is so great that its use should not be tolerated in any engineering office. Symbols become very expensive when they invite mistakes.

(*69*) **Use horizontal-bar fractions to show division.** Do not use the inclined-bar (or shilling) fractions. They have caused numberless costly blunders. When one fraction is divided by another, overemphasize the horizontal line that tells this fact.

(*70*) **Do not use the $\div$ symbol.** It was discarded by a National Committee as early as 1923 and is becoming obsolete.

(*71*) **Proportion must be shown in fraction form.**

(*72*) **Put the unknown in the left-hand numerator.** Avoid one or more algebraic operations by using this form:

$$\frac{x}{b} = \frac{c}{d}$$

$$\therefore x = b\left(\frac{c}{d}\right)$$

(73) **Abandon the use of colons to indicate proportion.** This notation is obsolete. See page 113 of Marks' "Mechanical Engineers' Handbook," 1st ed., 1916.

(74) **Use decimal or fractional exponents.** Roots and powers should be indicated by this modern notation. Avoid the use of the radical (square-root symbol). It is often highly cumbersome, and the present trend in engineering is toward the exponential form.

(75) **Always show the decimal point.** Show and emphasize the decimal point regardless of its location. It is so important and mistakes from its misplacement or misreading are so costly that it is a good plan to exaggerate it somewhat.

(76) **Place a zero in front of numbers less than 1.** This notation has been spreading for years and was recommended by a national committee in 1927. Form the habit of using it. No zero is to be placed after the decimal point unless it is a significant figure needed to indicate the precision of the value.

(77) **Show characteristic of every logarithm.** It may be positive, negative, or zero, but a logarithm is incomplete, hence incorrect, without its characteristic.

(78) **Point off answers.** Point off numerical answers into groups of three digits each, starting at the decimal point and running each way. Thus: 0.004,62 or 26,320,000. This has nothing to do with reading values aloud but is for convenience in checking.

(79) **Write units as an exponent.** Units such as in., lb, etc., are much clearer and safer to use if written in the exponent (or superscript) position instead of on the same line with the numbers. Units are to be in approved symbol form (see Table 1, page 349).

(80) **Use capital letters for vertices of triangles.** Use these same letters for the sides of the triangles, thus: side AB, side BC, etc.

(81) **Do not use Greek letters for angles.** The use of Greek letters for angles, lower case letters for sides, and prime marks on lettered points is seldom, if ever, justified. Notations become complicated enough at best without using capital letters, small letters, Greek letters, and prime marks on one or two triangles.

(82) **Letters I, O, Q, X, Y, and Z should not be used on diagrams.** The I, O, and Q resemble figures too closely and are therefore likely to be misread in equations set up from the

diagram. Too many college students pounce onto X, Y, Z every time they need to letter a diagram, as if it would be a misdemeanor not to use X, Y, and Z somewhere in every problem. Start with the letter A; then run down the alphabet omitting the 6 letters mentioned. When the 20 safe letters have been used, it may then be time to consider the use of small letters or the Greek alphabet.

3.8 Graphs.

A graph is a formal bit of drafting whether or not it is prepared for publication, and as such it should be planned carefully and be neatly and accurately drawn. It should give its information to a user with the smallest amount of accompanying explanatory material. It should be in the form most likely to be understood by a reader and be free from needless pictorial effects or decorations. It should be prepared in accord with the recommendations of the best authorities and adhere to generally recognized standards wherever they apply. A computer or draftsman who violates such specifications as *(86)* and *(91)* has put himself on the defensive at once and must be able to give an airtight reason (not a mere excuse) for not following the specifications. Although it is true that there are plenty of cases where the maker of a graph has been too indifferent to learn the best practice, that fact does not mean that others should ignore the practices that have been approved by national organizations and industry in general. The following specifications are arranged in the order in which the various details should be decided upon before actually drawing the graph. Most of them are based upon or quoted directly from the ASA booklet "Engineering and Scientific Graphs for Publication," No. Z15.3, 1943. When the notation "(See 1.9, ASA)" follows a specification, it refers to that numbered recommendation in this ASA code. Also refer to Forms 30–32 in the Workbook.

(83) **Put independent variable on x axis** (horizontal axis, see 2.5, ASA). When the properties of a round piece of steel are studied, its diameter is the independent variable. The left-hand column of tabular data is the independent variable.

(84) **Put dependent variable on y axis** (vertical axis, see 2.5, ASA). A round steel bar might be studied for its weight per lineal foot, the power that it could transmit if used as a shaft, or its load-carrying ability if used as a beam or column. These are

all dependent variables. In tabular data the values in the body of the table constitute the dependent variables.

(*85*) **Keep graph paper in normal position.** Do not turn the sheet sideways unless there is an airtight reason for doing so.

(*86*) **Show zero point of both variables.** If the zero point of the independent variable is purely arbitrary, it may be omitted. The zero of the dependent variable must be shown. The best authorities condemn the amputation of this base line, saying that there are few, if any, cases justifying the practice. The excuse offered for doing so is that the changes in the dependent variable are exaggerated so they are easier to see. This is the lamest of excuses, because the resultant graph is a falsehood, giving the reader the impression of great variations which, in truth, are non-existent. Anyone drawing such a graph has put himself on the defensive, and his case is prejudiced from the start (see 2.3, ASA).

(*87*) **Use care in choice of scales for each axis.** The scale depends upon many factors such as size of the ruled surface, the lines per inch, the maximum values to be included, the precision desired. Compute the scale by the formula below; then round off to fit calibrations of the paper used (see 2.1 and 2.2 ASA).

$$\left[\begin{array}{c}\text{Scale in units}\\ \text{per inch, F.}\end{array}\right] = \frac{\left[\begin{array}{c}\text{Maximum value of the}\\ \text{variable in its units}\end{array}\right]}{\left[\begin{array}{c}\text{Available length of the}\\ \text{axis for it, in inches}\end{array}\right]}$$

(*88*) **Scale must fit paper used.** Choose scales that can be interpolated readily in both plotting and reading. Do not use any scale that will require awkward fractions in the smallest space on the paper. On 10-line-per-inch paper use only 1, 2, or 5 units per inch or these values multiplied or divided by 10, 100, 1000, etc., as is necessary to get into the data range (see 2.8 and 2.9, ASA).

(*89*) **Calibrate inch lines only.**

(*90*) **Choose scale consistent with precision of data.** This may at times require the use of larger sheets of graph paper.

(*91*) **Put origin at lower left corner of grid.** Do not move the axes into the ruled surface unless there are negative values to be plotted. Like the suppression of the zero line there is no legitimate reason for this rather frequent practice. Buy a sheet with a smaller grid if wider margins are needed. Putting calibration numbers and captions into the ruled grid merely obscures them,

making them harder to read and almost illegible if the graph is to be reproduced by blue printing or photography (see 2.6, ASA).

(*92*) **Record all calibration values and captions.** Place these in the white margins of the sheet (see 2.6 and 2.7, ASA). Center the captions along the axis.

(*93*) **The scale caption** or label should indicate both the variable measured and the unit of measurement (see 2.10, ASA). For example, "DIAMETER, in." or "POWER TRANSMITTED, hp." Use standard abbreviations in captions (see 1.10, ASA).

(*94*) **Place calibrations and captions properly.** All lettering and numbers on a graph should be placed so as to be easily read from the bottom and from the right-hand side of the graph, not the left-hand side (see 1.9, ASA).

(*95*) **Use a 5H pencil on graphs.** Unless a graph is to be inked or blueprinted from a pencil drawing, a 5H pencil should be used when plotting the data for calibrations, captions, and title plates.

(*96*) **Plotted points must be fine, clean, tiny dots.** A long, fine pencil point is needed. Set the point at the right spot; then spin the pencil half a turn, thus producing a small round dot.

(*97*) **Each dot should be circled.** Each point is to be indicated by a small, neat circle around it. Circles must not be over 0.1 in. or less than 0.05 in. in diameter (see 4.4 and 4.5, ASA).

(*98*) **Use a sharp, solid line for the graph.** It should be fine but black enough to be easy to follow (see 4.1, ASA).

(*99*) **Keep out of circles when drawing graph.** Draw the line up to but not across the circles. The reason for this rule is that the plotted point inside the circle is data and may be used repeatedly in studies or constructions based upon the graph (see 10.7, ASA).

(*100*) **Use a French curve for drawing the graph.** No credit is given for freehand curves.

(*101*) **Complete graph with a good title plate.** The title plate should be on the ruled grid for 8.5- by 11-in. graph paper but is put below the entire graph when it is reproduced in books or periodicals (see 5.1 and 5.2, ASA; also see Form 32 in the Workbook). The headings for Table 23, page 377, and Table 24, page 378, are worded in suitable title-plate form.

(*102*) **Balance the graph with the title plate.** The title plate should be put in such a position on the sheet that curves, captions,

and curve labels are balanced around horizontal and vertical axes. In order that the total effect may appear balanced to the eye, the center of gravity of the whole composition should be slightly above the mid-height of the sheet (see 9.3, ASA). No top margin or marginal entries are needed on graph sheets as the title plate replaces them. Show the page number, however.

(*103*) **Use vertical Gothic lettering in title plates.** Capital letters must not exceed 0.15 in., and lower case letters, such as e, m, n, etc., must not exceed 0.10 in. in height (see 12.1–12.8 inclusive, ASA).

(*104*) **Center each line in title plate.** Every line in the title plate must be balanced with respect to a vertical center line like the title page of a book, but do not have the title plate arranged in a rectangular block. No two lines should be the same length.

3.9 Recording the Analysis and Calculations.

The following series of specifications is intended to serve both as a check list of operations and as a detailed outline of a procedure applying the general method of attack on problems discussed in Topic 2.12 in the preceding chapter.

The most effective way to solve any problem, simple or long and involved, strange or familiar, is to break it down into a chain of short steps each of which involves only a few facts and well-understood principles. Each of these links in the chain should be shown in detail and be complete in itself. Each step should be clearly distinguished from the one before and the one to follow. Every such intermediate solution must answer each of the six questions "What for?" "How?" "What with?" "How was the answer obtained?" "What is the result?" "Is the answer correct?" Each question should be answered in the manner shown below (refer to Fig. 1, page 59).

(*105*) **"What for?"** is to be answered by a concise explanatory heading that definitely states the problem to be computed in that particular small step in the solution.

(*106*) **This heading is to be the first entry** in the step and is to be a complete sentence on a line by itself. It is not enough merely to mention the units in which the unknown is measured. Name it.

(*107*) **"How?"** is answered by a statement, in mathematical form, of the law or principle that is to be used.

(*108*) **Use a word-equation form for stating the principle** unless otherwise instructed (see Fig. 1, page 50, for typical word equations.) Arrange the word equation in such typographical form as to show clearly the mathematical relationship of the various factors.

(*109*) **Symbol formulas are not to be used** unless permission is given to do so.

(*110*) **"What with?"** is answered by now substituting the data to be used in this particular step. It may be new data used for the first time or a value computed in a previous step.

(*111*) **Enter the data,** with the proper units exactly as given. This is sometimes referred to as the "raw data."

(*112*) **Conversion constants for units must be included** in the problem setup the same as any other form of data.

(*113*) **Check all numerical values at this point** before proceeding further. Have they been copied correctly from the original source of the data or, if they are computed values, from the step where found? Are the units correct? Computations made with incorrect values are absolutely worthless; so do not gamble. It is disheartening to discover that hours of work are wasted because of a careless slip in copying values. Be sure all are right; then go ahead.

(*114*) **"How was the result obtained?"** is to be shown clearly and completely by a series of entries giving the reduction and solution of the data equation set up in Specs. (*108*) and (*110*). This part of the work must not be slighted. It should be remembered that many records will be consulted long after the computer has moved away, gone to another job, or died.

(*115*) **Do not make short cuts or omit operations.** The use of short cuts and tricks is permissible only in routine work by experienced computers, never for beginners or even "old heads" when they are starting into a new field. Many engineers of wide experience go further and say that "only the green or careless man attempts to short-cut; the experienced man knows better than to try it." Leave a plain trail for the checker. Perform one operation at a time, and leave a record of what you have done.

(*116*) **Keep all entries and steps in logical order.** Do not "backtrack." Backtracking is a sure sign that the computer was not thinking things through before he acted.

(*117*) **Do not use scratch paper.** See the quotation on page 10. Use a pencil whenever desired, but leave a record. Some of

the largest companies in the United States absolutely forbid the use of scratch paper. If anything needs to be written at all, it needs to be a part of the record. That is the purpose of the left-hand column. Record all interpolations and any other incidental figuring that is not done mentally.

(*118*) **Show conversions of units.** The conversion of units is a part of the reduction of the equation and generally should be shown with the first operation following the statement of numerical data. By doing this, one is assured that the answer to that part of the solution will be in the desired units.

(*119*) **Use given data in preference to calculated values.** When computed values must be used and there is a choice, take the ones that are closest to the original data. Once the logarithm of a number has been found by calculation or in tables, use it as it is in all following steps; do not recompute.

(*120*) **Be consistent in the number of significant figures carried.** Use the rules for calculation as given on page 134 to avoid the carrying of meaningless figures. Use calculation methods that reduce labor to a minimum.

(*121*) **Left-hand side of equation should not be repeated.** When a step has to be carried over to a following page, however, be sure to repeat the key word naming the thing sought, and proceed with the reduction of the equation.

(*122*) **Check each operation.** Be sure that it is correct. Do this, and the final answer to the entire problem will take care of itself.

(*123*) **What is the result?** This is, of course, the answer to the particular step being computed and is the final answer when that step is the last in the series.

(*124*) **Emphasize the answer.** Put it on a line by itself, and double underline it.

(*125*) **Show units of answer.** The units in which the result is measured must be shown. No answer is correct or complete unless its units are clearly indicated.

(*126*) **Index answer.** This is done by entering a key word or words in the right-hand (index) margin. A simple descriptive term of not over three words is to be used. Usually it is about the same as the explanatory heading.

(*127*) **Use words for indexing.** Avoid the use of letters, symbols, and unconventional abbreviations.

(*128*) **Place indexing correctly.** The key word is to be placed directly opposite the answer that it indexes, and a heavy arrow drawn from near the margin to within 0.5 in. of the answer. When the space is less than 1 in., omit the arrow.

(*129*) **Do not put answers in index margin.**

(*130*) **Record answers on any diagrams.**

(*131*) **"Is the answer correct?"** Check the result before starting the next step or another problem. Chief engineers in industry are constantly insisting that the young engineer must be trained to check his own work as he goes along. Know that the answer is correct before leaving it.

(*132*) **Do not rely upon someone else to find errors.** The computer should solve the problem in another way if need be, but he must find the error himself. He will have to stand on his own feet someday; so he should start doing it as a student.

3.10 Tabulating Data and Calculations.

The ability to design and construct well-planned tabular forms is of the utmost importance to the engineer. A high percentage of the facts and numerical values that he uses must be organized in such form that space is conserved, the relationships among values are clear, and calculation time and labor are reduced. It frequently takes as much or more time and thought to plan and rule a tabular form as it does to make a drawing of equal size. Preliminary sketches of column placing, headings, and widths should usually be made. Tables set in type frequently do not have side rules, but all forms made for reports, drawings, and office data books should be completely ruled—top, sides, and bottom. Refer to Table 22, page 376, and Table 23, page 377, for typical ruled forms. The "Engineering Problems Workbook" also is full of tabular forms.

The following numbered comments are as much a guide to help a beginner plan a table as they are specifications. Study the complete series before drawing a line, keeping in mind the particular problem at hand.

(*133*) **A descriptive heading must be put above the table.** Allow at least 1 in. for this heading, which tells what the table is about.

(*134*) **Do not draw any lines of random length.** The result is always a botched job even if the lines are erased or extended. Plan the table carefully before drawing any lines.

(*135*) **Determine the number of columns needed.** If there are only two to four variables and 12 or more readings for each, it is a good plan to break the table into two equal "banks" of values set side by side. Forms 4 and 14 in the Workbook show such arrangements.

(*136*) **Plan table width carefully.** Use good judgment in figuring column widths. Sometimes the columns must be wider than indicated in the table below in order to provide space for the column headings. In general, however, use these values as a basis for planning the table.

Number of digits	Column width	
	Fractional inch	Decimal inch
1 or 2	½	0.4
2 or 3	⅝	0.6
3 or 4	¾	0.8
4 or 5	1	1.0
5 or 6	1¼	1 2
6 or 7	1½	1.4

(*137*) **Turn sheet sideways for very wide tables.** Keep the binding holes at the top (see Form 233 in the Workbook).

(*138*) **Do not crowd column headings.** Use as many short lines as necessary for long headings, or turn the headings sideways as shown in several columns of Table 20, page 375, and Table 21, page 376.

(*139*) **Put units in headings.** Do not attach units to the column entries in the body of the table.

(*140*) **Keep column entries and headings inside the rulings.** Never allow any entries in a column to overrun the column or the limits of the table.

(*141*) **Do not use sheet margins for table rulings.** The table is to be an independant construction inside the margins and with a frame of clear space completely surrounding it.

(*142*) **Use 0.2-in. minimum side clearance.** Do not cut clearance below this value. Plan the table again if necessary to avoid this crowding.

(*143*) **Center the table.** Place the table so the clearance on each side is the same. Off-center placing indicates poor planning.

(*144*) **Do not crowd column entries.** Unless lettering size is reduced to about $\frac{1}{8}$ in. height, use 0.4 in. vertically for each entry. Avoid crowding at all times.

(*145*) **Allow space for totals.** If the values in one or more columns must be added, be sure to allow 0.4 in. vertically for the total.

(*146*) **Compute table width and length.** When Specs. (*138*)–(*145*) inclusive have been decided, compute the over-all size of the table, dot the four corners, and rule the outside frame with heavy black lines.

(*147*) **Use medium-weight lines for column rulings.** Complete the ruling of the table, and record all column headings before entering any numerical values.

(*148*) **Use double lines to separate unrelated columns.** Space not over $\frac{1}{16}$ in. apart.

(*149*) **Tables must not be broken.** Never break a table so it runs onto another page unless the complete table would be longer than a single sheet.

(*150*) **Keep all column entries in vertical lines.** Have units over units, tens over tens, and all decimal points in a vertical line. This helps to prevent blunders when totals must be obtained.

(*151*) **Do not draw horizontal lines between entries.** Such rulings are seldom necessary and are not used in standard tables. See any of the tables at end of this book. Do not rule such lines unless specific instructions are given to that effect.

3.11 Diagrams.

A diagram is, by definition, a drawing of the essential lines in an object, an area, a line figure, or any other construction intended to show the relation between lines, parts, and positions or to show the magnitudes of quantities, forces, and velocities or the ways in which such things act upon the shape being studied. A diagram may or may not be drawn to scale according to the needs of the particular situation, but in most cases the proportions used are relative and not to scale.

Diagrams are highly valuable tools for use in solving problems. They should be used to aid in making an intelligent analysis of the situation, not as mere records of what has already been done.

Both known and unknown quantities should be shown on the diagram before any computations are started, unknown values being indicated by dimension lines, letters, or other clear devices. If the computer completes his diagram before starting any computations, he will find that he has been led to reason his way through the entire analysis of the problem. This tends to eliminate false starts and mistakes due to the erroneous application of principles. The working diagram, therefore, should be a simple drawing upon which all of the facts, both known and unknown, are recorded in such manner that each item may be readily found, easily understood, and used with the least possible chance of mistake. The following rules will aid in obtaining satisfactory diagrams. For further suggestions on diagrams refer to a good text on engineering drawing and also consult the ASA Code, Z14.1, 1935, "American Standard Drawings and Drafting Room Practice."

(*152*) **Diagrams must precede all other work.** When several diagrams are needed, put each one just ahead of the work based upon it (see Figs. 2, page 51, and 3, page 53).

(*153*) **Diagrams must be large.** Any drawing less than 3 in. high or wide is too small. Redraw it. Small, crowded, poorly placed and dimensioned diagrams are a waste of time. They are really harder to draw neatly than large diagrams, difficult to check, and of little help in solving a problem.

(*154*) **Allow ample room for dimension lines.** As a general rule the minimum area needed for the whole diagram is two or three times that taken by the outline of the shape itself if clear work is to be obtained. Remember that the outline and principal lines showing the object itself form nothing more than a framework which is to carry the dimension lines. The outline shows the relation of the parts to one another, but the series of dimension lines gives the numerical values used in the calculations.

(*155*) **Do not put any diagram in left column.**

(*156*) **Full-page diagrams.** The entire width of the sheet, between margins, may be used for the diagram whenever necessary, omitting the vertical column line for that portion of the page. Use an entire page for diagrams having many details or a large number of dimension lines. Turn sheet sideways (clockwise) if the diagram fits better that way (see Fig. 24, page 207).

(*157*) **Avoid crowding.** A diagram that is otherwise very well drawn may have its appearance ruined by thoughtless crowding of even a few dimension lines, details, or numerical values.

(*158*) **Use proper tools for drawing diagrams.** Use ruler, compass, and irregular curve as needed. Adhere to the principles and conventions of good drafting.

(*159*) **Use plenty cf contrast.** Diagrams are colorless and ineffective where work is done in a monotone. Vary the thickness and blackness of lines according to their purpose (see below).

(*160*) **Scale is not essential.** Unless the answer is to be checked graphically as well as by calculation, nothing is gained by drawing the diagram to scale. We may, therefore, exaggerate proportions without hesitation when clearness in dimensioning is gained by so doing.

(*161*) **Use heavy, solid lines for outline of object.** In general adhere to the rules for line work as given in your drafting-room standards (see ASA code).

(*162*) **Do not draw pictures.** Needless picture effects are confusing rather than helpful. Omit all details that have no bearing on the solution of the problem.

(*163*) **Do not shade or crosshatch a diagram** unless absolutely necessary for a clear understanding of it. It takes too much time, and little or nothing is gained by doing it.

(*164*) **Dimension the diagram.** Remember that the sole purpose of the diagram on a computation sheet is to impart information to the users. Its primary value is to aid the computer in solving his problem. Its secondary use is a record of the complete solution, including construction lines and results. The diagram, therefore, must be dimensioned as clearly as possible, and the layout planned in advance. Use thought, care, and time in placing all dimension lines and their values. Time is not wasted, because well-placed dimension lines will speed the solution of the problem and help reduce mistakes. It may happen that a computer will spend more time drawing the diagram than he does in the actual figuring, but his total time on the problem is reduced simply because he has a well-planned diagram.

(*165*) **Put all dimension lines outside the extreme limits of the outline of the object.** In a few special cases this rule may be violated, but the man who does it is always on the defensive and

must be able to give excellent reasons for not following the rule (see ASA code; also see Fig. 61, page 283, for Prob. 195).

(*166*) **Put the smallest dimensions closest to the figure being dimensioned** and the progressively larger ones farther and farther away, with the over-all dimension being placed farthest of all from the figure. In this way unnecessary crossings of dimension and extension lines may be avoided.

(*167*) **Use thin, sharp, clear lines for all dimension lines.** They should be about a third as heavy as the line used for the outline of the object. One must have contrast here; otherwise the drawing will be hard to read.

(*168*) **Use thin limit lines.** The lines showing the limits of all dimensions must be the same weight of line as the dimension lines.

(*169*) **Limit lines must approach the points being dimensioned,** either just touching or with a little clearance.

(*170*) **Do not break dimension lines unless necessary.** To break them is generally a waste of time, because it takes at least a third longer to stop and start again in drawing the line, and frequently it develops later that the break was not in the right place for the clear recording of the numerical value.

(*171*) **Clearance between dimension lines is absolutely essential.** There should be a space of not less than 0.4 in. between any part of the outline and a dimension line, and a similar spacing is needed between parallel dimension lines.

(*172*) **Arrowheads must be properly drawn and spaced.** They must be neat, black, and tapering and actually touch the extension line to which they refer. Do not use seagull wings, triangles, loops, blacked-in arrowheads, or other scrawls as makeshifts for the standard arrowheads called for in the American Drafting Standards. Arrowheads are one of the important indicators of skill and ability.

(*173*) **Do not put dimension lines on trigonometry diagrams** unless it is impossible to show the values clearly otherwise. Write the numerical value parallel to the side concerned.

(*174*) **Record all numerical values on the diagram.** All numerical data that can be put on the diagram must be recorded there. Use care and thought in this job, as by so doing you may avoid miscopying when taking values from the diagram later on.

(*175*) **Recording numerical values.** Place the numerical or letter value parallel to and about 0.05 in. above the dimension

line. There will be no danger of misreading the values if a proper spacing has been maintained between dimension lines.

(*176*) **Use decimal fractions rather than common fractions** unless there is an excellent reason for doing otherwise. When data have both kinds, convert to decimal fractions as they are entered on the diagram. Because most computations are made on the slide rule, all dimensions should be ready to use.

(*177*) **Be consistent with the data when recording values.** Record the same number of significant figures, neither more nor less. If the data show a tolerance on any value, show it on the diagram in a similar manner.

(*178*) **Clearance is essential.** See that no values or descriptive lettering are in contact with any line.

(*179*) **All lettering must be clean-cut, black, and easily read** [see Specs. (*13*)–(*19*) inclusive, pages 46, 47].

(*180*) **Check the diagram.** Be sure that every value on it is correct before starting on a series of calculations. A blunder on the diagram may ruin a long series of calculations. No credit can be given to calculations made with incorrect data.

(*181*) **Record the results on the diagram** as soon as they have been checked. If this is done, the diagram frequently serves the purpose of a progress chart, and the computer thus can quickly determine the next logical step.

(*182*) **Different colors of lead may be used** to distinguish the given dimensions from the computed ones.

3.12 Derived Curves.

Sets of derived curves as discussed in Chap. 10 are not graphs but diagrams, since they are not plotted or drawn to scale. The computer should, therefore, follow the spirit of the specifications given for the construction of diagrams and also adhere to the following special instructions. The details are based upon many years of use and experimentation with derived curves. Curves drawn in accordance with the specifications will yield the maximum of information to the user with the least possible explanatory material.

(*183*) **Use standard top and side margins** as called for in Specs. (*40*)–(*47*) inclusive pages, 49, 52, including all entries (see Fig. 4, page 70).

(*184*) **Put initial ordinate 0.5 in. to the right of left margin line.**

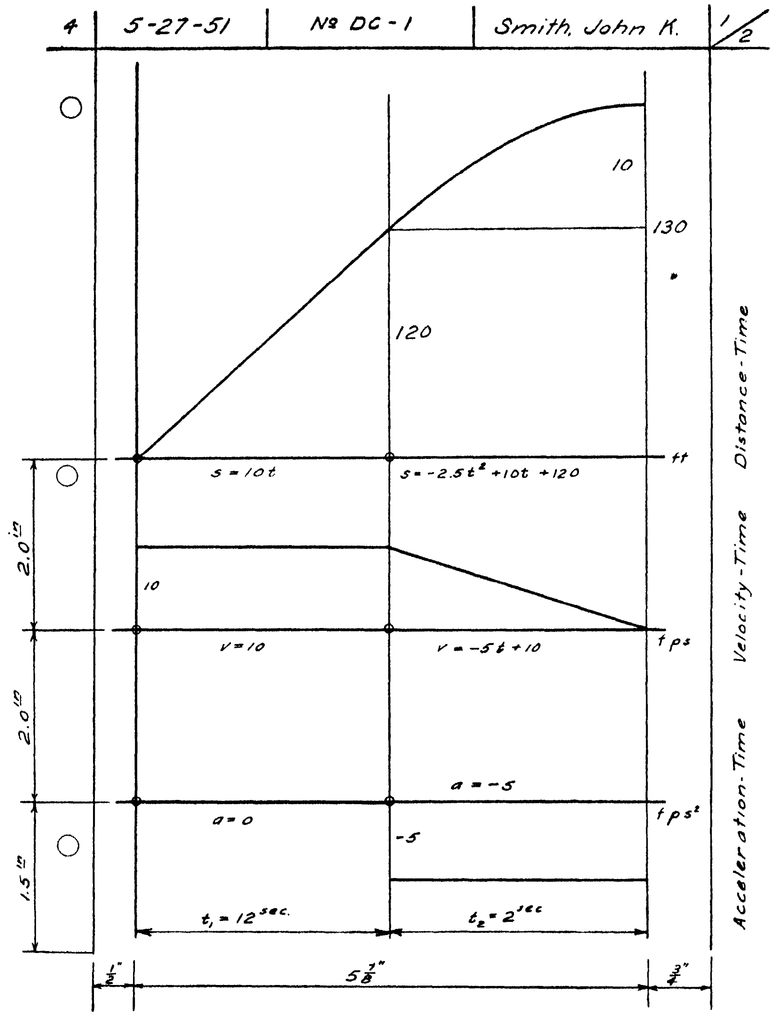


Fig. 4.—Derived Curves.

(*185*) **Use a very heavy black line for the initial ordinate.**

(*186*) **Put final ordinate 0.75 in. to the left of right margin,** regardless of the numerical value of the abscissa or the number of intervals.

(*187*) **Use a medium-weight line for the final and intermediate ordinates.**

(*188*) **Place the base lines as shown in Fig. 4.** In case only two curves are involved, omit the middle base line. If a fourth base line is needed, put it about 1.5 in. above the upper base line shown in Fig. 4.

(*189*) **Base lines are to be very heavy black lines** extending 0.25 in. beyond both the initial and final base lines.

(*190*) **Record the units of the ordinates** in the 0.75-in. space at the right-hand end of the base lines. Enter the unit just below the base line if the final ordinate is positive but just above it if the final ordinate is negative.

(*191*) **Index the curves in the customary manner** in the right-hand margin as shown in Fig. 4 and on Form 213 in the Workbook.

(*192*) **Keep units out of the index margin.**

(*193*) **Draw principal intermediate ordinates only.** These are the ones that indicate a special feature in one or another of the series of curves, such as a point of inflection, a maximum or minimum, or other change of shape.

(*194*) **Circle all origins of the curves.**

(*195*) **Do not attempt to draw curves to any scale.** Relative proportions only are needed and hence may be distorted any time that clearness is gained by so doing or the recording of values is easier.

(*196*) **Use heavy black lines for the curves.** Dotted or dashed lines should be used only when two or more curves must be shown in reference to a common base line.

(*197*) **Do not use dimension lines and arrows** to show any values except those parallel to the x axis.

(*198*) **When two sections of any curve are tangent to each other show them that way.** If they are not tangent, make that fact clear.

(*199*) **Check the shape of every section of each curve** before starting to compute values.

(*200*) **Record all calculations on a work sheet** (see Form 214 in the Workbook). Adhere to the standards of the specifications unless special instructions are given.

(*201*) **Record values of ordinates and abscissas as soon as found,** entering both partial ordinate and total value.

(*202*) **Place numerical values of ordinates beside the ordinate to which it refers.** Usually the partial, or difference, values are put on the left and total value on the right of the ordinate (see Fig. 4).

(*203*) **Check all values,** especially those to be used in later computations, before going ahead with the next step. Work done after the first mistake is usually of no value whatever.

(*204*) **Use either the second law or the calculus** to get the equations of the curves, whichever is the quicker.

(*205*) **Record the derivation of all equations** on computation sheets similar to those used for getting numerical results.

(*206*) **Record the equations on the sheet of derived curves,** writing them parallel and adjacent to the base line but outside the area under the curve.

(*207*) **Check the final value** by substituting the maximum limit of the abscissa in the equation of the highest curve, and see if it gives the same value for the final ordinate as was found by the second law.

3.13 Free-body Diagrams.

The equilibrium sketch, or free-body diagram, is a form of diagram that shows all of the forces that are acting upon an object. It is primarily a force diagram. It shows a body or a portion of a body isolated from all other bodies that were previously touching it or acting upon it in any manner. The actions of the removed bodies are to be shown by force arrows. The following points should be kept in mind when drawing free-body diagrams:

(*208*) **Force arrows** must indicate, by points of application and directions, the forces exerted by the bodies acting upon the one being studied (see Figs. 5 and 6, page 73).

(*209*) **A pull** (or tension) is shown by the force arrow pointing away from the body; **a push** (or compression) is shown by the force arrow pointing toward the body.

(*210*) **If the direction of the unknown force is not given,** assume its direction and then make the proper correction when it has been determined.

(*211*) **Never put in a force unless** a body or part has been taken away.

(*212*) **Never take away a body or part without** drawing a force arrow to represent its action.

(*213*) **All forces, whether known or unknown, should be shown before starting computations.** Mark an unknown force with a letter that will signify something of its nature.

(*214*) **If the body is in motion** or if motion is impending, draw a light arrow pointing in the direction of motion, placing this arrow an inch or more from the sketch.

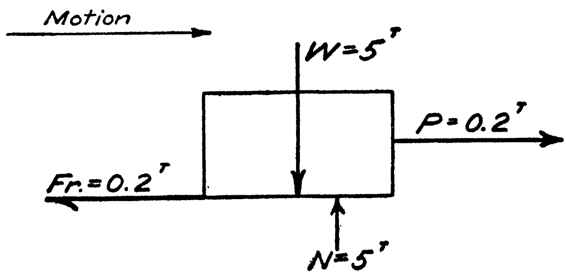


FIG. 5.—Body moving on level surface.

(*215*) **The inertia force should be shown** whenever the motion of the body is being changed. Its direction is always opposite that of the acceleration.

3.14 Integration.

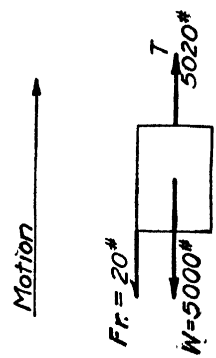


FIG. 6.—Body moving vertically upward.

Integrations involving properties of areas, volumes, and masses will be much easier to handle, especially when the computer is out of practice, if the technique shown below is followed.

(*216*) **Draw a good, clear diagram of ample size.** Show the curves, the limits, variables and elemental area, volume, etc.

(*217*) **Dimension this diagram** just as clearly and completely as it should be done if the object were to be made in the shop. Complete the diagram before setting down any part of the calculations. Adhere to the rules for good drafting.

(*218*) **The integral sign** is not to be written until nothing more can be done without it. The average student is in too great a hurry to get it down, but the bulk of the reasoning about the problem has to be done before, not after, this sign can be used.

(*219*) **Choose the element of area, volume, or mass with care.** When an elemental strip is used, take it in such direction as to require the simplest integrations. Show it on the diagram (see Fig. 7, page 74).

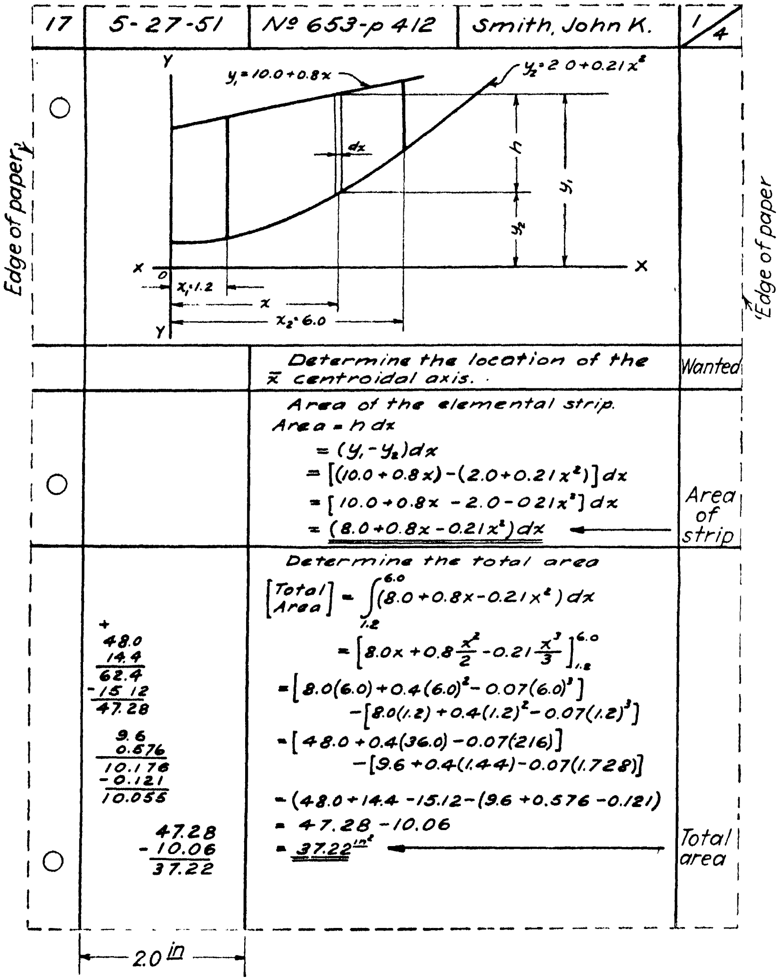


Fig. 7.—Setting up integration problems.

(*220*) **Do any preliminary algebraic work** that may be necessary at this point. If $x = f(y)$ is given and $y = \phi(x)$ is wanted, get the equations into proper form before going further.

(*221*) **Write down the simplest possible description of the element** of area, volume, or mass, using the dimensions shown on the diagram. Refer to the elemental strip in Fig. 7, where $dA = hdx$.

(*222*) **When this simple equation involves other variables,** bring in the equations showing the relations between them. The second and third equations $dA = (y_1 - y_2)dx$ and $dA = [(10 - 0.8x) - (2 + 0.2x^2)]dx$ of Fig. 7 illustrate this operation.

(*223*) **Simplify this new equation as far as possible.** See that the equation is in terms of the variable to be integrated and truly ready for integration, but do not record the integral sign yet.

(*224*) **Now indicate the required property** of the area, volume, or mass of the element, such as xdA.

(*225*) **Do not combine with any arithmetical operation.**

(*226*) **Do not record the sign of integration yet.**

(*227*) **Simplify as far as possible.** The smaller the number of symbols to drag through the integration process the smaller the chances of mistake.

(*228*) **Record the integral sign,** and show the limits.

(*229*) **Perform the indicated summation,** but do not substitute limits at the same time.

(*230*) **Substitute the given limits** exactly as given in the setup of the integral, making no arithmetical changes whatever.

(*231*) **Reduce the numerical equation to its final form.**

(*232*) **Do not short-cut or omit steps.** Mistakes are likely to creep in.

(*233*) **Check all operations, signs, and values** before indexing the final answer.

3.15 Specification Group Summary.

Topic No.	Subject	Specification numbers (inclusive)
3.4	Equipment	1 to 12
3.5	Workmanship	13 to 35
3.6	Makeup of the computation sheet	36 to 65
3.7	Mathematical signs and symbols	66 to 82
3.8	Graphs	83 to 104
3.9	Recording the analysis and calculations	105 to 132
3.10	Tabulating data and calculations	133 to 151
3.11	Diagrams	152 to 182
3.12	Derived Curves	183 to 207
3.13	Free-body diagrams	208 to 215
3.14	Integration	216 to 233

BIBLIOGRAPHY

Consult the following ASA codes for various standards and specifications. All are published by the American Standards Association.

Code Number	
Z-10*e*	"Aeronautical Symbols."
Z-10*f*	"Mathematical Symbols."
Z-10*gl*	"Letter Symbols for Electrical Quantities."
Z-10.1	"Abbreviations for Scientific and Electrical Terms."
Z-10.2	"Letter Symbols for Hydraulics."
Z-10.3	"Letter Symbols for Mechanics of Solid Bodies."
Z-10.4	"Letter Symbols for Heat and Thermodynamics."
Z-25 1	"Rules for Rounding off Numerical Values."
Z-32.2	"Graphical Symbols for Use on Drawings in Mechanical Engineering."
Z-14.1	"Drawings and Drafting Room Practice."

CHAPTER 4

EXPONENTS, LOGARITHMS, AND GRAPHS

4.1 Need for Special Notes on Algebra.

The topics to be discussed in the following pages concern only a few of the applications of greatest importance. The material is not intended as a substitute for a conventional algebra text. Its only purpose is to help remove some of the mystery that for some reason seems to attach to certain phases of fundamental algebra. Many students have an undue amount of trouble in getting a clear understanding of exponents and logarithms, and they use a table of logarithms mechanically, with no clear appreciation of why it works or its limitations. Freshmen have had very little experience with graphs, and as a result the undergraduate student frequently produces some rather awesome graphs in engineering work.

4.2 Scope of This Chapter.

This chapter will, therefore, contain a restatement of the simple laws of exponents, expand the idea of exponents to logarithms, and give suggestions on the use of these tools in calculations. The ideas developed in the discussion of logarithms will also serve as a foundation for the instructions on the slide rule. Much of the uncertainty in the use of logarithms and distrust of the slide rule exist because so many computers have never had a real understanding of the underlying algebraic principles. In other cases the worker is simply deficient in the common computing skills required.

Another section of this chapter will be a group of topics discussing graphs and other graphical tools. There are right and wrong ways of arranging a graph; and since there are definite, widely adopted standards, the young engineer should learn to use them. This discussion will, therefore, consider the application of the standards to various forms of graphic construction.

4.3 Setting up Formulas and Equations.

Time will be saved and frequently one or more entries in the reduction of an equation can be avoided if the computer will form the habit of making the unknown term the first entry and put it on the left side of the equals sign. For example, in simple equations of the types $x = ab$, $x = 3M + 1.4N - 0.2T$, etc., put the unknown, x, on the left side and the known values on the right-hand side. If a proportion is involved, start with the unknown in the first numerator and have it on the left side from the beginning. Reversing the above procedure makes a calculation appear as if the computer had thrown a series of values together on the basis of chance, then suddenly discovered that he had an answer.

4.4 Exponents as a Tool.

The work on exponents in an algebra text is, without much doubt, the most important part in the whole book. The engineer, the scientist, the research worker in any field soon discover that "Dame Nature" is undoubtedly a mathematician with a liking for exponential quantities. Again and again the mathematical statement of a "law of nature" uses exponents in one way or another. The problem may concern the deflection of a beam, the transfer of heat, or the flow of water, but in every case the mathematical formula requires the use of exponents. It follows that the engineer must have skill in the use of exponents in calculations as well as an understanding of the mathematical and engineering principles concerned.

4.5 The Basic Laws of Exponents.

There are four basic laws of exponents that must be known and used. Other expressions may be derived from them, but the four laws stated below are fundamental.

Case I. Multiplication.

Let $x = a^m$, $y = a^n$, and $z = a^s$, and the product $w = xyz$ is required.

Then

$$\begin{aligned} w &= xyz = (a^m)(a^n)(a^s) \\ &= a^{m+n+s} \end{aligned}$$

Rule: Add the exponents of the factors to get the exponent of the result.

Example:

Get w when $x = a^{2.4}$, $y = a^{-3.6}$ and $z = a^{2.9}$

$$\begin{aligned} w &= xyz = (a^{2.4})(a^{-3.6})(a^{2.9}) \\ &= a^{2.4+(-3.6)+2.9} \\ &= a^{5.3-3.6} \\ &= a^{1.7} \end{aligned}$$

Case II. Division.

Let $x = a^m$, $y = a^n$ and $z = a^s$ and the quotient $w = \dfrac{x}{yz}$ is desired.
Then

$$\begin{aligned} w &= \frac{x}{yz} = \frac{a^m}{(a^n)(a^s)} \\ &= a^{m-n-s} \end{aligned}$$

Rule: Subtract the exponents of the divisors from that of the dividend to get the exponent of the quotient.

Example:

Get w when $x = a^{4.8}$, $y = a^{1.2}$ and $z = a^{-2.1}$

$$\begin{aligned} w &= \frac{x}{yz} = \frac{a^{4.8}}{(a^{1.2})(a^{-2.1})} \\ &= a^{4.8-1.2-(-2.1)} \\ &= a^{4.8-1.2+2.1} \\ &= a^{6.9-1.2} \\ &= a^{5.7} \end{aligned}$$

Case III. Raising a Given Power to Another Power.

Let $w = (a^m)^n$
Then

$$w = a^{mn}$$

Rule: Multiply the exponents together to get the exponent of the result.

Example:

Get w when $x = a^{2.4}$ and $n = 3$
Then

$$\begin{aligned} w &= (a^{2.4})^3 \\ &= a^{2.4(3)} \\ &= a^{7.2} \end{aligned}$$

Case IV. Getting the Root of a Given Power of a Number.

Let $w = \sqrt[n]{a^m}$

Then

$$w = a^{\frac{m}{n}}$$

Rule: Divide the exponent that shows the power by the index showing the root to get the exponent of the result.

Example:

Get w when $x = a^{9.5}$ and $n = 5$

$$\begin{aligned} w &= \sqrt[5]{a^{9.5}} \\ &= a^{\frac{9.5}{5}} \\ &= a^{1.9} \end{aligned}$$

Note that Case IV can be converted into Case II and the calculations be simplified if the example is written thus:

$$\begin{aligned} w &= \sqrt[5]{a^{9.5}} \\ &= [a^{9.5}]^{\frac{1}{5}} \\ &= a^{9.5(0.2)} \\ &= a^{1.9} \end{aligned}$$

Several secondary relationships can be derived from the four basic cases, yielding certain forms that the engineer should recognize on sight. Three of these are as follows:

$$a^0 = 1 \qquad \text{because} \qquad \frac{a^m}{a^m} = 1 = a^{m-m} = a^0$$

$$a^{-m} = \frac{1}{a^m} \qquad \text{because} \qquad \frac{1}{a^m} = \frac{a^0}{a^m} = a^{0-m} = a^{-m}$$

$$\frac{1}{a^{-m}} = a^m \qquad \text{because} \qquad a^m = a^{0-(-m)} = \frac{a^0}{a^{-m}}$$

4.6 Numerical Calculation with the Aid of Exponents.

It should be noted in the four basic laws shown above that the degree of difficulty in the actual numerical work is lowered one step by the aid of exponents. Thus in Case I multiplication of numbers becomes one of adding exponents, an easier operation. Case II is changed from division to the easier operation of subtraction of exponents. In similar manner Cases III and IV are changed from power and root problems to the far simpler tasks of multiplication and division.

This simplifying of the numerical labor was recognized by several men around 1600–1615. In 1614 Baron John Napier

described a table of exponents that he had computed with 2.71828 (now known as the base e) for the value of the base a, which was used above in the statement of the laws of exponents. Other numbers have been suggested or used as the base for a system of exponents, but the table that has always been in most common use is the one devised by Henry Briggs about 1619. It has 10 as the base for the various values. Its general principles will be shown in the following topic.

4.7 Powers of 10.

The actual calculation of a table of exponents and powers for any base is a long, tedious process, but an idea of the principle and use of such a table can be obtained by actually computing a few values as follows:

$$
\begin{aligned}
10^{0} &= 1 \\
10^{0.25} &= (10)^{\frac{1}{4}} = \sqrt{\sqrt{10}} = \sqrt{3.1416} = 1.778 \\
10^{0.33} &= (10)^{\frac{1}{3}} = \sqrt[3]{10} = 2.1544 \\
10^{0.50} &= (10)^{\frac{1}{2}} = \sqrt{10} = 3.1623 \\
10^{0.67} &= (10)^{\frac{2}{3}} = \sqrt[3]{10^2} = \sqrt[3]{100} = 4.6416 \\
10^{0.75} &= (10)^{\frac{3}{4}} = \sqrt[4]{(10)^3} = \sqrt{\sqrt{1000.}} = \sqrt{31.623} = 5.623 \\
10^{1.0} &= 10 \\
10^{1.33} &= (10)^{\frac{4}{3}} = \sqrt[3]{10^4} = \sqrt[3]{10{,}000.} = 21.544 \\
10^{1.50} &= (10)^{\frac{3}{2}} = \sqrt{10^3} = \sqrt{1000} = 31.623 \\
10^{2} &= 100 \\
10^{2.33} &= (10)^{\frac{7}{3}} = \sqrt[3]{10^7} = \sqrt[3]{10{,}000{,}000.} = 215.44 \\
10^{2.50} &= (10)^{\frac{5}{2}} = \sqrt{10^5} = \sqrt{100{,}000} = 316.23 \\
10^{3} &= 1000 \\
10^{-1} &= \tfrac{1}{10} = 0.1 \\
10^{-2} &= \frac{1}{(10)^2} = \frac{1}{100} = 0.01 \\
10^{-3} &= \frac{1}{(10)^3} = \frac{1}{1000} = 0.001
\end{aligned}
$$

It will be noted that the exponent is made up of two parts: first, a decimal part which depends upon the sequence of the digits in the result obtained by raising 10 to the indicated power and, second, an integral part which changes only when the decimal point is shifted in the result. The values in the table above show that when the integral part of the exponent is other than zero, the magnitude of the integer indicates the number of places that the decimal point has been moved from the $(10)^0$ position and its sign + or − tells the direction of the shift. Thus, when the exponent is 2.50, the sequence of digits is 316.23, the same as

for $10^{0.50}$, but the decimal point must be moved two places to the right. If the integral part of the exponent is negative, $(10)^{\bar{2}.5}$, the decimal point is moved two places to the left and the result is $(10)^{\bar{2}.5} = 0.031{,}623$.

Calculations made by use of powers of 10 can be illustrated as follows:

$$\begin{aligned}
1.\ A &= (3.1623)(1.7778)\\
&= (10^{0.5})(10^{0.25})\\
&= 10^{0.5+0.25}\\
&= 10^{0.75}\\
&= 5.623\\
2.\ B &= \frac{464.16}{2.1544}\\
&= \frac{10^{2.67}}{10^{0.33}}\\
&= 10^{2.67-0.33}\\
&= 10^{2.33}\\
&= 215.44\\
3.\ C &= (17.778)^3\\
&= (10^{1.25})^3\\
&= 10^{1.25(3)}\\
&= 10^{3.75}\\
&= 5625
\end{aligned}$$

The foregoing small table of powers of 10 and the sample calculations will be recognized by students of algebra as an introduction to the study of logarithms. When Napier first introduced the idea, the tables of exponents were called *artificial numbers*, but in a few years, probably about the time when Briggs suggested using 10 as the base, the name was changed to *logarithm*. This is a word derived from the Greek, and the root meaning is "speaking number." The decimal part of the logarithm is known as the *mantissa*, and the integral part is called the *characteristic*.

4.8 A Logarithm Is an Exponent.

It should never be forgotten that the word *logarithm* is an arbitrary name given to one class of exponents; therefore, the same laws apply to logarithms, regardless of the base used, as apply to exponents in general.

$$\left[\begin{array}{c}\text{The }\textbf{logarithm}\\ \text{of }\textbf{any}\text{ number}\\ \text{to a given }\textbf{base}\end{array}\right] = \left[\begin{array}{c}\text{The }\textbf{exponent}\text{ of the power}\\ \text{to which the }\textbf{base}\text{ must be}\\ \text{raised to give the }\textbf{number}\end{array}\right]$$

The base should be positive and other than 1. The properties, sources, and relationships of the two parts of a logarithm are shown in the table below.

A LOGARITHM IS AN EXPONENT
It has *TWO* parts, both of which *MUST* be shown at all times.

An *INTEGRAL* part, called the CHARACTERISTIC, which shows the position of *THE DECIMAL POINT* in the number, nothing else.	A *DECIMAL* part, called the MANTISSA, which depends solely upon the *SEQUENCE OF THE DIGITS* in the number.
It never gives any indication of the sequence of the digits in the number.	It never gives any indication of the position of the decimal point in the number.
It may be positive, negative, or zero, but every logarithm has a characteristic.	It is always positive in all standard tables, either numerical or graphic.
It is determined by inspection and is not given in standard logarithm tables, either numerical or graphic.	It must always be found by means of tables, either numerical or graphic, and it cannot be determined by inspection.

4.9 Decimal Points by Characteristics.

The relationship between the magnitude and sign of the characteristic and the position of the decimal point in the power of 10 brought out in Topic 4.7 is of the highest importance. A clear understanding of this relationship is essential to the making of correct computations with Briggs, or "common," logarithms. Characteristics may be used to great advantage in other calculation methods as well as in logarithms. A person who has no knowledge of logarithms can quickly learn how to determine the characteristic of a number at a glance if he will study the table that follows. He can learn how to use them accurately and efficiently in computing if he will study the instructions given for cut-longhand in Chap. 6 or on the use of the slide rule in Chap. 5.

The characteristic method is simple, exact, speedy, and mathematically correct. It can be used equally well in longhand, contracted longhand, or work done by a mechanical calculator, also on slide rule or logarithmic computations.

4.10 Rules for Characteristics.

The following summary gives the important facts needed to determine the characteristic of any number correctly or to place the decimal point in an answer when the characteristic is known.

a. The characteristic of a number is a figure (or figures) with a definite relation to the position of the decimal point, in accordance with the following:

b. The normal position of the decimal point is immediately after the first, or left-hand, significant figure. The characteristic of the number is then zero. The number of significant figures has no relation to the characteristic.

c. The characteristic tells how many places the decimal point has been shifted from the normal position. If it has been moved to the right, the characteristic is positive; if it has been moved to the left, it is negative. The characteristic may be positive, negative, or zero. Negative characteristics should have the negative sign written above the characteristic. Every numerical value has a characteristic, but its magnitude and sign will depend solely upon the position of the decimal point in the value. The table below indicates the relation between the position of the decimal point and the characteristic.

DETERMINING CHARACTERISTICS

Number	Characteristic	
0.0005316	$\bar{4}$	The decimal point HAS BEEN moved to the left, hence these characteristics are negative.
0.005316	$\bar{3}$	
0.05316	$\bar{2}$	
0.5316	$\bar{1}$	
5.316	0	Characteristic for normal position.
53.16	+1	The decimal point HAS BEEN moved to the right, hence these characteristics are positive.
531.6	+2	
5316.	+3	
53160.	+4	
Normal position of decimal point is immediately after the first digit.		

4.11 Logarithms and the Laws of Exponents.

The basic laws of exponents must be kept in mind when logarithms are used; the laws will take the forms shown below. Although the statements apply equally well to either natural or common logarithms, the numerical examples are worked out using the table of common logarithms (see Table 32, page 401).

Case I. Multiplication by Logarithms.

If $z = xy$ with $x = a^m$ and $y = a^n$
then

$$\begin{aligned} z &= (a^m)(a^n) \\ &= a^{m+n} \end{aligned}$$

But by definition: m is the logarithm of x to the base a, and n is the logarithm of y to the base a; hence,

$$\begin{aligned} \log_a z &= \log_a x + \log_a y \\ &= m + n \end{aligned}$$

or, stated as a word equation,

$$\begin{bmatrix}\text{The logarithm} \\ \text{of a product}\end{bmatrix} = \begin{bmatrix}\text{The sum of the logarithms} \\ \text{of the factors}\end{bmatrix}$$

Example 1.

$$\begin{aligned} D &= (0.015)(882.) \\ \log_{10} D &= \log_{10} 0.015 + \log_{10} 882. \\ &= \bar{2}.17609 + 2.94547 \\ &= 1.12156 \\ \therefore D &= 13.23 \end{aligned}$$

Case II. Division by Logarithms.

If $z = \dfrac{x}{y}$ with $x = a^m$ and $y = a^n$
then

$$\begin{aligned} z &= \frac{a^m}{a^n} \\ &= a^{m-n} \end{aligned}$$

But by definition, m is the logarithm of x to the base a, and n is the logarithm of y to the base a; hence,

$$\begin{aligned} \log_a z &= \log_a x - \log_a y \\ &= m - n \end{aligned}$$

or, stated in word-equation form,

$$\begin{bmatrix}\text{The logarithm} \\ \text{of a quotient}\end{bmatrix} = \begin{bmatrix}\text{The logarithm} \\ \text{of the dividend}\end{bmatrix} - \begin{bmatrix}\text{The logarithm} \\ \text{of the divisor}\end{bmatrix}$$

Example 2.

$$
\begin{aligned}
E &= \frac{17.68}{707.2} \\
\log_{10} E &= \log_{10} 17.68 - \log_{10} 707.2 \\
&= 1.24748 - 2.84954 \\
&= \bar{2}.39748 \\
\therefore E &= 0.0250
\end{aligned}
$$

Case III. Raising a Value to a Power by Logarithms.

If $z = (x)^y$ with $x = a^m$

then

$$
\begin{aligned}
z &= (a^m)^y \\
&= a^{my}
\end{aligned}
$$

But by definition, m is the logarithm of x to the base a, hence

$$
\begin{aligned}
\log_a z &= \log_a(x^y) \\
&= y(\log_a x) \\
&= y(m)
\end{aligned}
$$

or, in words,

$$\left[\begin{array}{l}\text{The logarithm of the } y \\ \text{power of a number}\end{array}\right] = y\left[\begin{array}{l}\text{The logarithm} \\ \text{of the number}\end{array}\right]$$

Example:

$$
\begin{aligned}
F &= (54.3)^{1.8} \\
\log F &= 1.8(\log_{10} 54.3) \\
&= 1.8(1.73480) \\
&= 3.12264 \\
\therefore F &= 1326.
\end{aligned}
$$

Case IV. Finding a Root of a Value by Logarithms.

If $z = \sqrt[y]{x} = x^{\frac{1}{y}}$ with $x = a^m$

then

$$z = a^{\frac{m}{y}}$$

But by definition, m is the logarithm of x to the base a; hence,

$$
\begin{aligned}
\log_a z &= \log_a (x)^{\frac{m}{y}} \\
&= \frac{\log_a x}{y}. \\
&= \frac{m}{y}
\end{aligned}
$$

or, in words,

$$\left[\begin{array}{l}\text{The logarithm of the } y \\ \text{root of a number}\end{array}\right] = \frac{\left[\begin{array}{c}\text{The logarithm of the number} \\ \text{number}\end{array}\right]}{y}$$

Example:

$$G = \sqrt[5]{170{,}800.}$$
$$\log G = \frac{\log_{10} 170{,}800.}{5}$$
$$= \frac{5.23249}{5}$$
$$= 1.04650$$
$$\therefore G = 11.13$$

4.12 Working with Numbers Less than 1.

When a computer is using the table of common logarithms and has one or more values less than 1 (decimal values), such factors will, of course, have negative characteristics because the decimal point has been moved to the left of the "normal position," which is after the first digit in a number. Because some computers fear such negative characteristics and so do not like to work with them, they avoid the necessity of learning the basic principles by using an artificial device known as the *nine minus ten* system. That is, the whole logarithm, characteristic as well as mantissa, is made positive by adding 10 or a multiple of it to the negative characteristic and indicating that 10 or the same multiple of it should be subtracted from the logarithm. Thus: log 0.00432 = $\bar{3}.63548$ with the characteristic negative and the mantissa positive. It is changed into the 9–10 form thus:

$$\begin{array}{r} \bar{3}.63548 \phantom{{}-10} \\ +10.00000 - 10 \\ \hline 7.63548 - 10 \end{array}$$

The characteristic and mantissa are now both positive. To change the characteristic to the actual value, subtract the added 10's from each part of the logarithm.

Although this system is widely used and the engineer must be familiar with it because many handbook tables use it, it is, nevertheless, a derived system and requires more time, entries, and space. For problems involving decimal values and decimal powers, either positive or negative, it is actually more laborious than the method based upon the fundamental theory of exponents.

4.13 Using the Absolute Value of a Logarithm.

The logarithm of a number less than 1 always has a negative characteristic; and since the mantissa is always less than 1, the

true, or absolute, value of the logarithm is a negative value. For example, log 0.478 = $\bar{1}.67943$, but the absolute value of the logarithm is obtained by adding the characteristic and mantissa algebraically. Now $\bar{1}$ is the same as -1.00000; so $\bar{1}.67943$ really means $-1.00000 + 0.67943$ and $(-1.00000 + 0.67943) = -0.32057$, and the entire decimal part is now negative.

As just stated, the use of negative characteristics is usually faster and is closer to basic principles than the 9 − 10 system. In like manner the use of the absolute value of the logarithm will sometimes reduce the chances of error and also save time in calculations (see Example 2 below).

To illustrate the various methods and the use of each of the three systems, 9 − 10, negative characteristics, and absolute value of the logarithm, two examples will be solved by each of the three methods. Because many computers are at a loss when they meet such problems, each step in the solution will be shown.

Example 1. $A = (0.0674)^{0.4}$ (Positive exponent).

Negative characteristics	Absolute value of the logarithm	"Nine minus ten" system
log 0.0673 = $\bar{2}.82802$	log 0.0673 = $\bar{2}.82802$ Add the negative characteristic 2 and the positive mantissa 0.82802, giving the absolute value of log 0.0673 = −1.17198	log 0.0673 = 8.82802 − 10
log A = 0.4(0.82802 − 2)	log A = 0.4(−1.17198)	log A = 0.4(8.82802 − 10)
= 0.331208 − 0.8	= −0.468792	= 3.531208 − 4
Add and subtract 0.2	Add and subtract 1	Add and subtract 6
log A = 0.531208 − 1	log A = 0.531208 − 1	log A = 9.531208 − 10
= $\bar{1}.531208$	= $\bar{1}.531208$	= $\bar{1}.531208$
∴ A = 0.3398	∴ A = 0.3398	∴ A = 0.3398

In the example above there is relatively little to choose among the three methods; but since negative characteristics are the fundamental form, it is perhaps the best of the three. When negative powers are involved, the absolute value of the logarithm is much the shortest and most efficient, as will be seen in the example following.

Example 2. $B = (0.0673)^{-0.4}$ (Negative exponent).

Negative characteristics	Absolute value of the logarithm	Nine minus ten system
$\log 0.0673 = \bar{2}.82802$	$\log 0.0673 = \bar{2}.82802$ $= -1.17198$	$\log 0.0673 = 8.82802 - 10$
$\log A = -0.4(0.82802 - 2)$ $= -0.331208 + 0.8$ Add the -0.331208 to the $+0.8 = 0.468792$	$\log A = -0.4(-1.17198)$ $= +0.468792$	$\log A = -0.4(8.82802 - 10)$ $= -3.531208 + 4$ Add the -3.531208 to the $+4. = 0.468792$
$\therefore A = 2.943$	$\therefore A = 2.943$	$\therefore A = 2.943$

Since $(0.0673)^{-0.4}$ is the same as $\frac{1}{(0.0673)^{0.4}}$, the problem can be solved as in Example 1 and then the value of the reciprocal be computed. It is obvious, however, that such a method is very inefficient, since Example 1 is longer than Example 2 by any of the three methods and getting the reciprocal adds still another operation.

4.14 Natural Logarithms.

The base most commonly used in engineering work is 10, and logarithms computed from it are known as *Briggs'* or *common* logarithms. Any other positive number (other than 1) might be used as a base for a system of logarithms. A system that has many important uses in theoretical work is the natural or Napierian system based upon the numerical value of an algebraic series. This base is denoted by the letter e and is an irrational number having the value 2.71828+. Many problems in mechanics can be solved much more quickly through the use of natural logarithms than by common logarithms, and so a computer should be familiar with their use. Although some engineers shun them, there is nothing mysterious about them or their use in calculation. The only point to be remembered is that the characteristic cannot be determined by inspection. The easiest way to handle values outside the range from 1 to 10 is to rewrite the value as a number in this range multiplied (or divided) by the proper power of 10. Thus, 2,760. becomes $2.760(10)^3$, and $\log_e 2760.$ becomes $\log_e 2.760 + 3 \log_e 10$. If the value is 0.02760, we write $2.760\ (10)^{-2}$;

hence, $\log_e 0.02760$ becomes $\log_e 2.760 + (-2 \log_e 10) = \log_e 2.760 - 2 \log_e 10$.

The small table at the top of Table 34, page 412, gives a convenient set of the values of $\log_e 10^n$.

It is usually unnecessary as well as a waste of time to convert natural logarithms to common logarithms; but when it must be done, the conversion is simple and is expressed in word-equation form, thus:

$$\begin{bmatrix}\text{The logarithm of}\\ \text{a number } N \text{ to}\\ \text{the base } b\end{bmatrix} = \begin{bmatrix}\text{The logarithm of}\\ \text{the number } N \text{ to}\\ \text{the base } a\end{bmatrix}\begin{bmatrix}\text{The logarithm of}\\ \text{the base } a \text{ to}\\ \text{the base } b\end{bmatrix}$$

To convert from natural to common logarithms, or the reverse, we must use the values $\log_e 10 = 2.302585$ and $\log_{10} e = 0.434294$.

Example:

$$\log_{10} 4.32 = 0.63550$$
$$\log_e 4.32 = 1.46325$$

By the equation above

$$\begin{aligned}\log_{10} 4.32 &= 0.434394\ (\log_e 4.32)\\ &= 0.434294(1.46325)\\ &= 0.63550\end{aligned}$$

and

$$\begin{aligned}\log_e 4.32 &= 2.302585\ (\log_{10} 4.32)\\ &= 2.302585(0.63548)\\ &= 1.46325\end{aligned}$$

Since these multiplications are tedious, it is best to avoid them by making the calculation in natural logarithms if the quantity e is inherent in the problem.

4.15 Graphical Methods.

Graphical methods of presenting facts are of the utmost importance in engineering, because a properly constructed chart, graph, or diagram will transmit more information from mind to mind in a given time than will any other known means of communication. It has been said that a single picture tells more than can be said in many thousands of words and that to a very great extent the message is independent of language barriers. This is true because the eye has the ability to pick up a myriad of facts in a split second and to present them to the mind for interpretation with the least chance of being misunderstood. Although eye defects and inaccurate observation can be the cause of error in the

report to the mind of the things seen, the graphic and pictorial are nonetheless the most vivid and effective methods of presenting many classes of factual matter for study and use. Although words are the most common tools for transmitting ideas from mind to mind and will always be used, they do have many shortcomings. Most words have more than one meaning, and some have so many that only the context of the sentence or paragraph will reveal the meaning to be chosen. Time and long use affect words just as they do structures and machines; hence, they become timeworn and obsolete. The language of Chaucer's day, for example, is almost a foreign tongue to the users of modern English. The graphic language, however, suffers the least loss in value with the passing of time. The conventionalized pictures found on the walls of tombs in Egypt enable us to understand a great deal of the ways of life in that ancient civilization. In another part of the ancient East there once lived near Ur of the Chaldees an engineer-king named Gudea. About 4500 years ago he ruled one of the city-states, and instead of waging constant wars he was famed as a builder. He constructed canals, roads, irrigation systems, and transportation systems, even bringing in goods from ports a year's voyage away. We know that he was also a designer and builder, because in a Paris museum there is a headless statue of Gudea that tells the story. On his lap is a large tablet showing his plan for the reconstruction of a temple to the god Ningirsu. It is drawn to a scale that can be understood by the engineer or architect of the present time. Beside the plan lie the stylus for drawing and the scale used in laying out the building. The words that he had inscribed on the skirts of the statue are without meaning to all save a very few men, but the plan, the stylus, and the scale tell their story at a glance. Words may thus be ambiguous or in a forgotten tongue, but the graphic message need not be of dubious meaning. It is for this reason as well as the saving of time that the engineer and the architect are led to make increasing use of the graphic method for showing what is seen, measured, or wanted.

In our highly technical civilization the graphic language must be able to express many more ideas than were ever dreamed of by old King Gudea. It has become necessary, therefore, to adopt various conventions and standards that must be used if the graphic material is to be correctly interpreted by all users. The

ASA code No. Z 15.3, 1943, has been prepared and adopted by the leading industries and engineering technical societies of the United States. It should be followed unless the computer and designer have airtight reasons for ignoring it. The man who refuses to use such accepted codes is like the man who coins his own words—he talks only to himself.

The graphic presentation of facts ranges from the common blueprint to the exploded views used to teach beginners on assembly lines in factories. It includes the whole family of graphs plotted on one form or another of coordinate paper, pictorial and semipictorial representations used by magazines for popular consumption, the familiar bar charts, the pie charts, and the three-dimensional figures. It also includes the entire group of alignment charts, or nomographs, that are deservedly becoming increasingly popular with engineers. It requires a volume by itself to discuss all these varied forms of graphical presentations of facts.

4.16 Coordinate Papers.

A wide selection of well-printed coordinate papers is available to the engineer today. They vary in sheet size as well as in types of rulings. Sheet sizes vary from 3.75 by 6.75 in. through 8.5 by 11 in. and 11 by 16.5 in. to rolls 150 yd long and 12, 22, 32, and 33.5 in. wide. The most important rulings are the profile, rectangular, metric, logarithmic, semilogarithmic, isometric, polar, and triangular coordinates. These may be obtained in various colors of ruling and on tracing cloth, drawing paper, or tracing paper. In addition to these there are other rulings for specialized uses.

The rectangular coordinate rulings are by far the most important of any. The ones usually listed in catalogues are those with 4, 5, 6, 8, 10, 12, 16, and 20 lines per inch. Probably the one that is used more than all others combined is the one ruled 10 lines per inch. It is often called a 10-line paper. The advantage of this paper lies in the ease with which values can be plotted or readings be made from it.

4.17 Planning a Graph.

When it is necessary to plot a graph from a series of observed or computed values, the engineer should do some planning before he

starts his graph. He must consider and weigh each of the following items: range in the data, precision of the data, units involved, desired precision of the graph, size of probable graph, paper size, and kinds of rulings available. It should be recognized that changing one of the items can easily involve one or more of the others. Thus, a change in the range of the data may drastically affect the size of the graph and its precision. A restricted choice in coordinate papers may seriously limit the scales used, hence the precision of readings from the graph. For such reasons, therefore, good judgment is necessary in the planning and drawing of graphs.

One very important phase of this planning work is that of choosing the scales to be used. The scales must be chosen so that the paper is used to best advantage, the graph fills the sheet reasonably well, and no awkward fractions are encountered in estimating even the smallest readings on the sheet. The ASA specifications limit the choice of scales on 10-line-per-inch paper to 1, 2, and 5 units per inch. These may have the decimal point shifted either way to conform to the range in the data. Any other values such as 1.5, 3, 6, 7, or any other prime or fractional number are thus definitely ruled out. If, for some valid reason the worker must use 3, 6, or 12 units per inch, then he is compelled to get a 6- or 12-line paper. If the scale must be 4, 8, or 16 units per inch, then corresponding papers have to be used. Notice that in every such case, however, one can no longer decimalize the small spaces but must use common fractions. This is certainly a much slower process in plotting or reading the graph.

4.18 Avoid Some Common Blunders.

Without doubt the worst of several common blunders is that of omitting the zero of the dependent variable and using an arbitrary value for the base line. Of all the blunders made, the error of "suppressing the zero" is the one most severely condemned by every authority in the field of graphics. The excuse always offered is that the worker wants to magnify the minor variations "so they can be seen better." It should be remembered that one is entirely on the defensive when he omits the zero for either axis, especially the dependent axis, and must be able to give valid, airtight reasons to justify his violation of this fundamental rule.

Another common blunder is that of moving the coordinate axes

into the ruled grid and then putting calibrations and captions inside the coordinate rulings. This makes these entries very hard to read; and when the graph is to be reproduced by blueprinting or photography, they become almost illegible. The white margin is left on the sheet by the manufacturers for calibrations and captions; why not use it? Shifting the axes also limits the user on scales, because what was a 10-in. vertical axis on a 7 by 10 grid becomes only 8 or 9 in.

When the white margins are really too narrow by an inch or more, the worker probably should go to a larger sheet size. If this is not feasible, then a grid of the proper size should be drawn on a sheet of desired margins. Another way to get the same result is to cut a grid of proper size from a prepared paper, then mount it on a plain sheet of the desired size and margins. Whatever subterfuge may be used, however, the draftsman should keep his calibrations and captions out of the ruled grid.

When the magnitudes of the values plotted are so large that the margins become really crowded, it is often possible to increase the size of the units used and indicate this fact in the caption. If, for example, the stresses on a structural member are to be plotted and the data are in pounds per square inch ranging from zero to 100,000 lb, we have the choice of the scales, 1000 psi or 10 kips per sq in. per in. (The kip is 1000 lb.)

A third common blunder is that of extending the curve beyond the plotted data (extrapolation) without indicating in any way that the extended line is a guess. Any such extrapolated portions of a curve should be distinguished by using dotted or dashed lines contrasting with the known portion of the graph. Failure to do this produces a misleading graph, as it gives a semblance of validity to what is merely a guess.

Blunders in the use of papers having one or more logarithmic axes are usually in regard to the numerical values printed on the sheet. The 10-line paper has no numbers printed in the margins of the sheet, because several values can be assigned to the calibrations as well as decimal-point changes. On any paper with logarithmic scales, however, such as the semilogarithmic papers or those with logarithmic scales on both the x and y axes, one cannot change the value of the printed numbers. The only change possible is a shift of the decimal point, right or left, as necessary to get into the range of the data. The line marked 5, for example,

may be changed to 0.5, 50, 500, etc., but never to 1.5, 2.5, or any other number. On papers having two or more cycles of the logarithmic scale one must be careful to complete the calibration of each cycle by entering the necessary zeros and decimal points. This is of vital importance, as otherwise serious errors can occur in the plotting and reading of value.

CHAPTER 5

THE ENGINEER'S SLIDE RULE

5.1 Introduction.

The purpose of this chapter is to help the engineering student develop greater speed and accuracy in his slide-rule calculations. Engineers frequently lack confidence in this timesaving instrument because they have such a hazy knowledge of why it works at all. Unfortunately, many slide-rule instruction books seem to imply that the slide rule is just another and rather mysterious mechanical gadget. Such books give a series of arbitrary settings to be memorized and used blindly. These instructions sometimes dodge the explanation of the basic principles that underlie the design and operation of all slide rules and leave the user to trust in vague, machinelike, trick settings memorized by rote.

No one can develop a sure skill in any area of knowledge on such a poor foundation. Because the great majority of slide rules are used by engineers who have to be familiar with the use of exponents and logarithms, this chapter will endeavor to develop, step by step, a sound engineering method of using the rule. There will be no suggestions for fancy trick settings to be memorized blindly. A sure yet simple method of determining the location of the decimal point will be demonstrated, and proved usefulness is the basis for every suggestion.

Unfortunately, locating the decimal point correctly is brushed off as a matter of minor importance by some writers. As a matter of fact, a mistake of one place in locating the decimal point is a blunder five times as great as one in which the initial digit is half or double what it should be. When a writer tells a slide-rule operator to make a series of longhand or mental calculations in order to locate the decimal point, he is ignoring the fact that there is no gain whatever in using the slide rule if the computer has to work out the correct answer by some form of arithmetical figuring.

The "characteristic method" used in this book is the result of the pooling of the ideas of several independent workers in various

sections of the country. It has been thoroughly tested in the field, in the classroom, and in engineering offices for thirty years or more. No failures or exceptions to the method have been discovered in all this time. As a matter of demonstrable fact, none can occur, because the system is founded upon the mathematically true laws that govern all logarithmic operations. The computer who uses the characteristic method is independent of all the approximate solutions or rough calculation processes or calculating by tens. He who uses this system can place the decimal point swiftly and with unerring precision in 100 per cent of his slide-rule problems. It is a technique that can be applied with the same reliability whether the actual numerical figuring is done on a calculating machine, on the slide rule, by logarithms, or by cut- or full longhand. Thus with only one method to learn and, as will be seen later, only one thing to do, the mind of the user is not cluttered with other artificial or empirical rules.

The student will find that the approach to the entire subject of slide-rule calculation is simple and straightforward, each operation being based upon the fundamental laws of exponents. These laws of exponents as developed in the preceding chapter and carried on through the work in logarithms furnish all the foundation needed for a thorough understanding of the principles used in constructing the slide-rule scales and in the operation of the slide rule. Thus there need be no mystery about this invaluable engineering tool. Even the person who does not care to study the algebra of exponents and logarithms can use the rules for determining characteristics as given in the table on page 84 and learn to use the characteristic method with confidence and certainty.

5.2 On Choosing a Slide Rule.

For the great majority of engineers, and especially for those who have to carry a slide rule about with them on the job, a rule of the single-faced type such as the Polyphase Mannheim or the Mannheim Special is usually the wiser choice. It is to be preferred to the duplex type because it not only is much easier to operate but is lighter and quicker to adjust and holds its adjustment much better than any of the double-faced rules. It is smaller and will stand a lot of rough treatment yet is as accurate as any rule and has all the scales needed by the average engineer.

Only a few specialists in certain lines of work, where problems

with fractional exponents are encountered frequently, really are justified in buying a slide rule of the log-log type. These are double-faced rules and should be very carefully checked at the time of purchase. These slide rules have 15 or more scales on them, and any variation in scale length will introduce errors that will preclude the doing of precise work such as a fine slide rule should permit. It is a tedious job to adjust one of these rules so that all of the scales and the hairlines on the runner are in perfect alignment. They will not stand as rough treatment as will rules of the Mannheim type and should not be dragged around in the field or thrown around on a drafting table. They should be kept in the case when not in use, and the owner should form the habit of checking their adjustments at frequent intervals.

The prospective purchaser will discover that numerous names are given to their slide rules by the makers. Most of them are really descriptive of the scales that are the distinctive feature of each model. Regardless of make or model, there are certain basic scales that should be on an engineer's slide rule. There are at least six types of rules and three makes that have the following essential scales: D, C, CI, CF, DF, A, K, and L.

If a rule lacks the CI, CF, and DF scales, its owner can expect to make at least one more setting on many problems than will the owner of a rule that has these scales.

Since the quality and prices of slide rules vary considerably, there is a wide range of choice. Any slide rule should be inspected carefully before purchase. The D, C, CI, A, K, and L scales must be exactly the same length, and the π marks on the CF, and DF scales must line up exactly with the indexes of the C and D scales. Errors in scale length are permanent defects and cannot be corrected by any adjustments; hence, any rule having errors in scale length exceeding the thickness of a calibration line should be rejected.

5.3 Suggestions on Care and Adjustment of the Slide Rule.

Remember that a well-made slide rule is a precision instrument that will yield the owner a lifetime of excellent service if proper care is taken of it. If the face of the rule becomes soiled, it can be wiped with a rag that has been dampened slightly, but strong soaps, alcohol, or cleaning fluids should never be used. They may remove the markings or stain the celluloid facings. The rule

should be kept away from strong fumes of chemicals; otherwise the facings may be disfigured permanently. Do not leave the rule in a place where it will be subjected to great changes of temperature or humidity. If the etched line on the runner fills with dirt, tear a strip of paper and drag it under the runner of duplex rules. If this will not work, it may be necessary to remove one runner glass, clean each glass, and then readjust the hairlines. When the black pigment in the etched line wears out, either printer's ink or mimeograph ink can be rubbed into the line and allowed to dry. The excess ink on the glass can be scraped off with a razor blade.

For some reason a few men are "all thumbs" when it comes to making settings of the slide or the runner. When using a rule of the duplex type they put one hand on top of the rule, wrap their fingers around it, and take a firm grip. Then they complain that the slide is too tight. The stock or body of the rule should be held between the finger tips and preferably, if space permits, with the first finger and thumb holding only that part of the stock near the operator. The slide should be held by the edges, never by the calibrated faces, and with the fingers next to the body of the rule. Close, accurate settings are made by "pinching" the slide with the thumb and finger close against the end of the stock. Then with a slight rolling movement of the first finger the slide can be brought to the exact setting without sticking or jumping. In like manner the runner should be set by crowding it to position with a similar rolling movement of the fingers. The fingers should really grip the rule, not the runner at all.

5.4 Historical.[1]

The close connection between logarithms and the slide rule is seen when one is familiar with the story of its development. It was in 1614 that John Napier described his table of natural (Napierian) logarithms. Shortly afterward Henry Briggs suggested using 10 as a base for a system of logarithms. He journeyed to Scotland to see Napier, who liked the idea. Together they worked out the table of common, or Brigg's, logarithms, an account of which was published in 1619. A year later Edmund

[1] For a complete account of the development of the slide rule consult Cajori, Florian, "A History of the Logarithmic Slide Rule," Engineering News Publishing Company, 1909.

Gunter made an invention that was the direct ancestor of the slide rule. He plotted the mantissas of the common logarithms of the numbers from 1 to 10 along a straight line, then added or subtracted the logarithms of numbers as distances on the scale. He used a pair of dividers for adding or subtracting the distances that corresponded to the logarithms of the factor in his problem. The scale shown in Fig. 8, page 102, is a Gunter's line broken into four short sections. If matched properly into a single long scale this line could be used exactly as Gunter used his scale. This *logarithmic line of numbers*, as it was called, was used quite generally in London and by ship's navigators for many years. Gunter's line is far from being obsolete today, even though the slide rule has replaced it for computing work. Not only is this logarithmic scale the basic scale for the slide rule, but it appears in the spacings of the rulings of a series of logarithmic and semi-logarithmic coordinate papers. The construction of many of the nomographs seen so frequently today in engineering literature depends upon the reliability and simplicity of Gunter's line. The most efficient four- and six-place tables of logarithms available today are Gunter's lines some 60 and 360 ft long, respectively, cut into short sections and assembled in book form.[1] Logarithms and antilogarithms can be obtained from this table in less than a third of the time required by any known numerical table of equal capacity. The engineer owes a great debt to Edmund Gunter, the man who also devised the surveyor's chain (used for centuries in land measurements), a quadrant for measuring the sun's azimuth, and instruments for aiding navigators, and discovered the variation in the declination of the magnetic compass.

A few years after Gunter made his famous logarithmic scale an English clergyman, Rev. William Oughtred, invented the *logarithmic sliding scales*. He did not make his invention public until about 1630. He studied and taught mathematics as a hobby, and to him goes the credit for the first use of several symbols for mathematical operations as well as the invention of both the rectilinear and circular types of slide rule. The circular rules come and go, but the rectilinear form has continued to hold the favor of its users for over 300 yr. About 1657 a surveyor named Seth Partridge devised a rule of what is now known as the

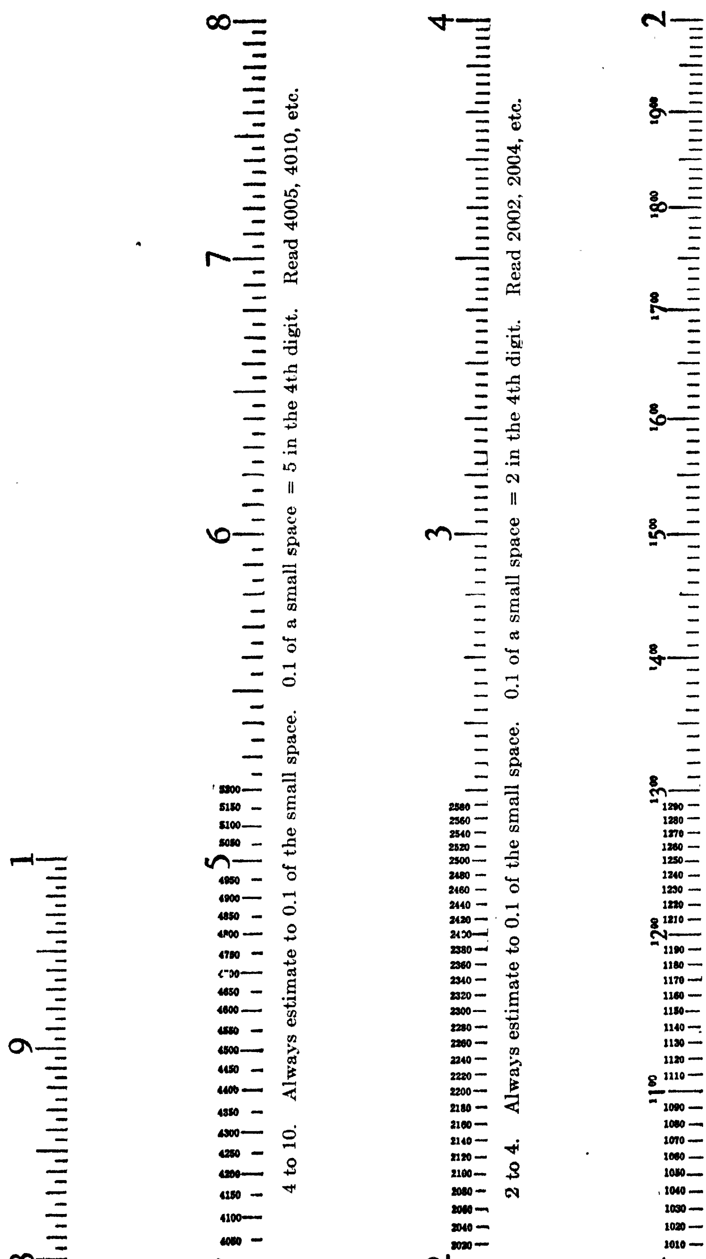
[1] LACROIX, ADRIEN, and CHARLES L. RAGOT, "A Graphic Table of Logarithms and Antilogarithms," Macmillan, 1927.

duplex design. All these slide rules were used without an indicator or runner. There is considerable doubt as to the inventor of the runner, but evidence exists that the credit should go to John Robertson who described his rule with a runner about 1775.

During the three centuries that the slide rule has been known many scales have been invented and scale arrangements suggested, but the basic C and D scales have kept their place as the most important scales on the instrument. That is why a competent understanding of these two scales is absolutely essential. About 1851 a French lad, 19-year-old Amédée Mannheim, a lieutenant in the Army, devised the shape of rule and combination of scales that bears his name. The Mannheim slide rule is probably the favorite of more experienced users than all the complicated rules on the market today. It is doubtful if any user realizes the full capacity of the Mannheim type of slide rule. Certainly few men use or need to know all the settings that are possible. The scale arrangement of the Mannheim special rule (devised in 1931 by F. C. Dana) carries the CF and DF scales in addition to the usual Mannheim scales. With this scale arrangement (whether on the Mannheim or duplex types of rule) there are over 150 different possible settings that require either the use of the runner only or not more than a single setting of the slide. These settings will handle problems with from one to four factors (the fourth factor being π). This is certainly enough capacity for most operators and solves problems that many men will never meet; hence, no one should attempt to memorize such a long list of trick settings. If one of them is encountered frequently in routine work, it is then time to think about memorizing it.

Unless the worker knows that he will have to solve frequent problems of the types $A = (4.36)^{1.23}$ or $B = (0.0142)^{0.2}$, there is no need to buy the more expensive and complex rules of the log-log duplex type. If, however, such problems are routine matters, as in electrical and thermodynamics problems, log-log rules are indispensable. Some engineers like to keep both types of rules handy, a rule of the Mannheim type for the common, "run-of-mine" problems and a log-log rule for roots and powers and for work with trigonometric functions.

These log-log rules are a collection of logarithmic scales based upon both the common logarithms (base 10) and the natural logarithms (base e). This rule has all of the basic scales found on



4 to 10. Always estimate to 0.1 of the small space. 0.1 of a small space = 5 in the 4th digit. Read 4005, 4010, etc.

2 to 4. Always estimate to 0.1 of the small space. 0.1 of a small space = 2 in the 4th digit. Read 2002, 2004, etc.

1 to 2. Always estimate to 0.1 of the small space. 0.1 of a small space = 1 in the 4th digit. Read 1001, 1002, etc.

FIG. 8—Illustrating the reading of the graduations on a slide rule. (Only the large, black numbers are printed on the rule.)

the Mannheim Special and, in addition, another set of log-log scales. These LL scales are really natural (Napierian) logarithms plotted to work in conjunction with the scales plotted to the base 10, so that a problem such as $A = (4.36)^{1.23}$ is changed to this form, $\log_e A = 1.23 (\log_e 4.36)$. The multiplication $1.23 (\log_e 4.36)$ is then carried out directly by means of the proper LL scale and the C scale on the slide. Such problems can, of course, be solved on the ordinary slide rule, but the making of two extra settings is required.

5.5 Precision of Slide-rule Readings.

Most of the errors made by slide-rules users are mistakes in the setting and reading of values on the various scales. The student should, therefore, learn to observe, to train his fingers in setting the slide or runner, and to use care in all settings and readings. A well-made slide rule is a precision instrument, justifying its owner's best efforts to use it as it should be used. Even though the given data may require only three-digit precision, it is not a bad idea to form the habit of reading four digits and then rounding off the answer to the required precision. Such procedure is in the spirit of Holman's rules for computation as given in Chap. 6.

The scales shown in Fig. 8, page 102, are a reproduction of the C and D scales but enlarged several times, then broken into four sections. A portion of each section is numbered down to the smallest calibration mark, so the beginner can see clearly the way in which the value of a setting is read.

The slide-rule student should remember that the slide-rule scales most often used (the C, D, CI, CF, DF scales) are calibrated on the decimal basis. That is, the full scale length from 1 to 10 is divided into 10 parts, then each tenth into 10 parts. These, in turn, are divided into smaller parts; but because the subdivisions are now becoming rather small, the number of them, hence their value, varies with the particular section of the rule in which they lie. Thus, from 1 to 2, the smallest set of calibrations gives single-unit changes in the third digit, such as 161, 162, 163, etc. From 2 to 4 the calibration marks give the even values only, as 214, 216, 218, etc. Lastly, from 4 to 10, the steps are five units each, thus, 510, 515, 520, 525, etc.

In all sections of the rule, however, the careful worker can

estimate the values to one-tenth of the space between successive calibrations; hence, the above values become 1610, 1620, 2140, 2160, 5100, 5150, etc., all being four-digit numbers. It is seen, then, that from 1 to 2 the calibrations indicate a 10-unit change in the fourth digit between successive marks: from 2 to 4, a 20-unit change; from 4 to 10, a 50-unit change. If the worker will estimate to a tenth of a space, he can read values such as the following: from 1 to 2, 1611, 1612, 1613, 1614 (each unit); from 2 to 4, 2140, 2142, 2144, 2146 (even values); from 4 to 10, 5100, 5105, 5110, 5115 (changing by 5's).

Values such as 2141, 2143 are impossible unless the reader can estimate to twentieths of a space, which is very doubtful. Values like 5112, 5113, 5114 can be rated as impossible because the human eye cannot divide the space between marks into fiftieths and thus detect the difference, for example, between 12 fiftieths and 13 fiftieths. It is, therefore, folly to record such improbable values in answers.

It is necessary for the beginner to practice, practice, practice. Do not be misled into thinking that a magnifier will increase the precision of the readings. It cannot do so, since the basis of the estimate is unchanged. All that the magnifier does is to reduce eyestrain and sometimes help in setting the hairline of the runner.

One excellent way of devising practice problems for home study is to practice the reading of the reciprocals of numbers. For example (since the CI scale gives the reciprocals of the values on the C scale), set the runner to 8150 on the C scale. You should read 1227 on the CI scale. Since the tolerance is always one-tenth of a calibration space, acceptable readings would be 1226, 1227, or 1228. Compare your reading with the digits in the reciprocal of 815 as given in a table of functions of numbers such as the one in the "Engineering Problems Manual." It is 122699; hence, 1227 is the best possible slide-rule reading.

Try setting the runner to 3800 on the C scale. It should read 2632 on the CI scale. Compare this with the value in the text, 2631579 or 2632 if rounded off to four digits. Since the allowed tolerance is one-tenth space, or 0.1 (20 units) = 2 units, the permissible readings would be 2630, 2632, 2634.

To get a reading in the third group, set the runner to 1650 on the C scale. The text gives the answer as 6060606 or 6061 to four

digits. The tolerance is one-tenth space, or 0.1 (50 units) = 5 units; hence, acceptable readings would be 6055, 6060, 6065.

5.6 The Slide Rule Is a Logarithm Table.

The slide rule is essentially a group of Gunter's logarithmic scales of various lengths conveniently arranged for the mechanical addition and subtraction of logarithms. Many users of the instrument do not realize that the rule is based upon logarithms, and as a result they are not able to make full use of its possibilities. They rely upon memorized settings, guess at decimal points, and frequently do not have a great amount of confidence in the results obtained with it. If the origin of the instrument is kept in mind, the mystery surrounding its manipulation vanishes, and there is no need of memorizing trick settings.

The following facts should be kept clearly in mind when working with the rule:

a. The slide rule is a **graphic logarithm table.**

b. **Logarithm tables,** numerical as well as graphic, **give the mantissas only.**

c. The mantissas determine only the sequence of digits.

d. Characteristics are used to locate the decimal points.

e. Characteristics are **not** shown in either form of logarithm table.

f. Characteristics of the various terms are determined by inspection.

g. The determination of the decimal point in logarithmic computation may be considered as an independent operation.

h. Since the slide rule is based upon logarithms, the same laws apply to its operation as to numerical logarithms and therefore:

The decimal point can be as definitely located with a slide rule as with numerical logarithms.

5.7 Graphic and Numerical Logarithms.

The principal difference between the two forms of logarithmic computation is this: In one case numbers must be looked up, recorded, then added or subtracted; in the other the complete solution of the problem is obtained by using distances instead of numbers. Thus:

For multiplication

a. With numerical logarithms,
add the logarithms.

b. With the slide rule,
add the distances that represent the logarithms.

For division

a. With numerical logarithms,
subtract the logarithms.

b. With the slide rule,
subtract the distances that represent the logarithms.

5.8 Multiplication on the Slide Rule.

This is a process of adding distances that represent the mantissas of the logarithms. The sequence of operations is as follows:

a. Move slide (C scale) so that its index comes to the first factor on the D scale.

b. Set runner to second factor on the slide (C scale).

c. Read answer on the D scale under the runner.

This operation has laid off the mantissa of the first factor on the D scale, the mantissa of the second factor on the C scale, and their sum is read on the D scale.

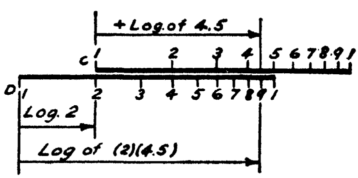


FIG. 9.—Multiplication.

Example 1 (see Fig. 9). Multiplication (2)(4.5).

a. Move the slide so the initial index of the C scale coincides with 2 on the D scale.

b. Set the hairline of the runner to 4.5 on the C scale.

c. Read the answer, 9.0 on the D scale under the runner.

These operations have really laid off a distance corresponding to the mantissa of 2 on the D scale, the length corresponding to the mantissa of 4.5 on the C scale, and the sum of the two lengths is the mantissa of the product.

5.9 Division on the Slide Rule.

This operation is performed by subtracting the distance corresponding to the mantissa of the divisor from that of the dividend. The process is as follows:

a. Set the runner to the numerator on the D scale.

b. Move the slide so the denominator comes on the C scale under the runner.

c. Read the answer on the D scale under either index of the slide (C scale) (See Fig. 10).

This operation has set the numerator on the D scale; the denominator over it on the C scale; the quotient is on the D scale.

Example 2 (see Fig. 10). Division, 9.6 divided by 4.0.

a. Set the runner to 9.6 on the D scale.

b. Move the slide to the right until 4.0 on the C scale lies under the runner.

c. Read the quotient 2.4 on the D scale under the initial index of the C scale.

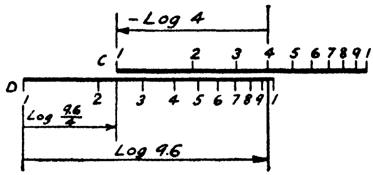


FIG. 10.—Division.

These operations have laid off a length on the D scale corresponding to the mantissa of 9.6, then subtracted the length on the C scale corresponding to the mantissa of 4.0. The remaining length on the D scale corresponded to the mantissa of the quotient, which is read as 2.4.

5.10 The Slide Does Most of the Work.

Beginners should remember that most of the work is done by means of the slide and the runner. The first factor and the answer are the only values on the D scale; all the rest, no matter how long the problem may be, are brought into the settings by use of the slide. The runner is merely a convenient pointer used to hold values on the D scale and to speed up the laying off of values on the slide. It should also be noted that settings of the slide and the runner alternate. To make two successive movements of either means that one of the terms has been thrown away. It is also worth remembering that in multiplication the answers are always read under the runner but in division the quotient is always under an index of the C scale. It may be under either the initial or the final index, depending on the magnitude of the respective factors, but the quotient is always available.

5.11 Characteristics and the Slide Rule.

When numerical logarithms are used, the characteristic is added to the mantissa and the two are taken together. This

cannot be done with the slide rule, however, and the characteristics must be treated separately. This is a simple matter, though, and takes less time than other methods of determining decimal points.

In problems involving the addition of numerical logarithms, it frequently happens that the mantissas add to more than 1.0, and so 1 must be carried over into the column of characteristics. Also in subtracting numerical logarithms, the mantissa

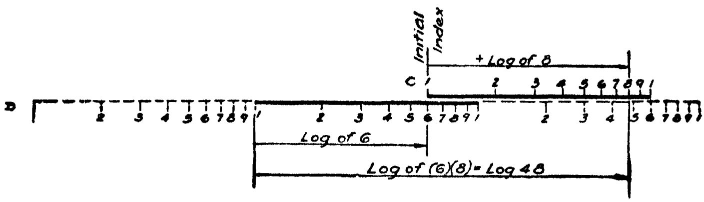


Fig. 11.—Multiplication.

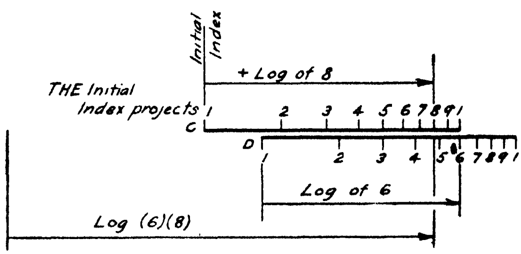


Fig. 12.—Multiplication.

of the divisor may be greater than that of the dividend, and it becomes necessary to borrow 1 from the characteristics of the dividend. For example,

a. Multiply (6.0)(8.0).

$$\begin{aligned} \log 6 &= 0.77815 \\ + \log 8 &= \underline{0.90309} \\ & 1.68124 = \log 48. \end{aligned}$$

b. Divide 30. by 9.

$$\begin{aligned} \log 30 &= 1.47712 \\ - \log 9 &= \underline{0.95424} \\ & 0.52288 = \log 3.3333 \end{aligned}$$

If these same problems are solved on the slide rule, it will be found that there is an exact parallel between the numerical and graphic methods. The distances representing the man-

tissas will add to one full scale length or more. This corresponds to carrying 1 into the characteristic column; hence, a note should be made each time this occurs. If the mantissas are being subtracted, it may be necessary to "borrow" another scale length just as 1 may be borrowed from the column of characteristics. A record should be made when this occurs.

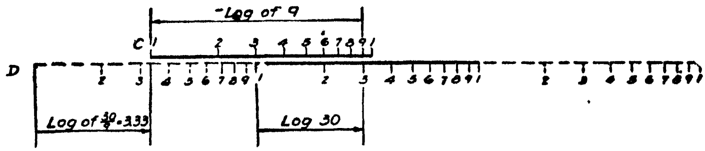


FIG. 13.—Division.

In both of these cases (if the full-length C and D scales are being used), the answer can be read only by shifting the slide so that its relation to the D scale is the same as it would have with respect to a second D scale were it available and the slide in the normal position (see Figs. 11 and 13).

Since there is but one section of the D scale, the C and D scales must be in the position shown in Figs. 12 and 14.

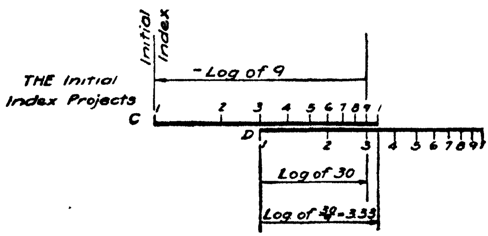


FIG. 14.—Division.

There is a simple, quickly applied, and absolutely accurate rule for decimal points based upon the foregoing facts. This method completely does away with the need of longhand checking, mental shifting of the decimal point, approximate calculations, or any other so-called "system" not based upon logarithms.

5.12 The Initial Index.

In the following rule for the location of the decimal point, reference is made to **THE Initial Index.** There are many indexes on a rule; those at the beginning end of any scale are "initial indexes," and those at the far end are "final indexes."

There is one initial index, however, that is so important that it is **THE Initial Index.**

THE Initial Index is the beginning end of the C scale for all operations. The graduations begin at the initial index and have increasing values as the final index is approached.

When the slide is in the position shown in Figs. 12 and 14, it is said that **THE Initial index projects,** and there must be a change made in the characteristics. **THE Initial Index** never projects unless such a change should be made.

5.13 Rule for Decimal Points.

*If **THE Initial Index** of the C scale projects after any factor has been set on the slide, add 1 to the characteristic of the factor that has just been set on the slide.*

5.14 Multiplication.

In multiplication this 1 is added to the characteristic of the multiplier. The characteristic of the answer is the algebraic sum of the characteristics of the factors plus any added 1's.

5.15 Division.

In division this 1 is always added to the characteristic of the divisor. Never add it to the characteristic of the dividend; the answer will be one hundred times too big. The characteristic of the answer is obtained by subtracting the algebraic sum of the characteristics of the divisors (including added 1's) from the algebraic sum of the characteristics of the factors in the dividend (including any 1's added to them.)

5.16 There Are No Exceptions to the Rule.

The following facts should be remembered, because the rule for locating decimal points is really extremely simple. All who have trouble with it do so because they are looking for exceptions and are inclined to create troubles where none exist.

a. There are no exceptions to the rule for locating decimal points.

b. It is the initial index of the C scale that is watched. When the C scale is inverted to make the CI scale, the initial index is, of course, at the right-hand end. It is watched, therefore,

for all factors set on the CI scale, but it is still **THE Initial Index** and there is no change in the rule for decimal points. When the folded, CF scale is used, watch the initial index of the CI scale for projections. There is again no change or exception to the rule for decimal points.

c. It should be remembered distinctly that movements of slide and runner have nothing whatever to do with above rule for decimal points. It is simply a case of noting the position of the initial index of the C scale after each and every factor has been brought into the solution by setting it on the slide.

d. Under no circumstances is 1 to be added to a factor set on the D scale, nor is 1 to be subtracted from any characteristic.

e. There is only one thing to do, namely, if **THE Initial Index** projects (and only then), *add* 1 to the characteristic of the term just set.

f. Projection of the final index is meaningless. Ignore it all together.

g. For continued operation involving three or more factors note the position of **THE Initial Index** as soon as each new value is set on the slide; and if the index projects, record the added 1 to its characteristic immediately.

5.17 Sequence of Operations.

While becoming familiar with this method, the beginner should form the habit of going through the following steps in his slide-rule work.

a. Set the work up in form suitable for slide-rule computation.

b. Indicate the logarithmic characteristic of each term somewhere close by. (Just above the multipliers and below the divisors is convenient, or in the computation space to the left opposite the operation. See Fig. 1, page 50.)

c. Note the position of **THE Initial Index** as soon as each new factor is set on the rule.

d. Record the *added characteristic* if the index projects.

Remember that this extra 1 is invariably *added* to the characteristic of the term just laid off on the slide, regardless of whether it is division or multiplication that is being performed.

e. Determine the characteristic of the answer.

f. Check the results by taking the factors in reverse order.

5.18 Typical Problems.

Example 1.

$$x = \overset{2}{(165.5)}\overset{\bar{3}}{(0.004750)}$$

a. Record the characteristics of the factors, 2 for the 165.5 and $\bar{3}$ for the 0.004750, just above the factors.

b. Move the slide so that the left index of the C scale comes to 165.5 on the D scale.

c. Set the runner to 4750 on the C scale.

d. Note the position of **THE Initial Index.** It does not project. Hence, the characteristic of the product is $2 + \bar{3} = \bar{1}$.

e. Read the digits in the answer, 7860, on the D scale, under the runner.

f. Record and point off the answer, 0.7860.

Example 2.

$$y = \overset{5}{(843000.)}\overset{\bar{3}\ +1}{(0.004970)}$$

a. Record the characteristics of the factors, 5 and $\bar{3}$ respectively.

b. It should be obvious that the product cannot be read with the slide projecting to the right. Move the slide so that the right-hand (final) index of the C scale comes to 8430 on the D scale.

c. Set the runner to 4970 on the C scale.

d. Note the position of **THE Initial Index.** It is projecting. Therefore, 1 should be added to the characteristic of the factor, 0.004970, just set on the C scale. Hence, the characteristic of the product is

$$5 + (\bar{3} + 1) = 3$$

e. Read the digits in the answer, 4190, on the D scale under the runner.

f. Record and point off the answer, thus, 4,190.

Example 3.

$$z = \frac{\overset{3}{6270.}}{\underset{\bar{2}}{0.03920}}$$

a. Note the characteristics of the factors as 3 and $\bar{2}$ respectively.

b. Set the runner to 6270 on the D scale.

c. Move the slide so that 3920 on the C scale lies under the runner.

d. Note the position of **THE Initial Index.** It does not project. Hence, the characteristic of the answer is $3 - \bar{2} = 3 + 2 = 5$.

e. Read the digits, 1599, of the quotient on the D scale under whichever index lies within the limits of the D scale. It is the left index for this problem.

f. Record and point off the answer, thus, 159,900.

Example 4.

$$w = \frac{\overset{\bar{2}}{0.03260}}{\underset{1\ +1}{92.50}}$$

a. Note the characteristics of the factors as $\overline{2}$ and 1 respectively.
b. Set the runner to 3260 on the D scale.
c. Move the slide so that 9250 on the C scale lies under the runner.
d. Note the position of **THE Initial Index.** It is projecting. Therefore, 1 should be added to the characteristic of the factor, 9250, that was just set on the C scale. Hence, the characteristic of the quotient is

$$\overline{2} - (1 + 1) = \overline{2} - 2 = \overline{4}$$

e. Read the digits, 3520, of the quotient on the D scale under whichever index lies within the limit of the D scale. It is the right index for this problem.
f. Record and point off the answer thus 0.000,352,0.

5.19 Using the C, D Scales for Three or More Factors.

Some slide rules have only the basic C and D scales for the full-length, working scales. It is, therefore, important for a computer to know how to solve his problem efficiently when it has three or more factors and only the C and D scales are available. Most of the three factor problems are of the types outlined below.

Case I. $w = \frac{x}{y}(z)$

a. Set the runner to x on the D scale.
b. Move the slide so that y on the C scale lies under the runner. Note the position of the initial index of the C scale.
c. Set the runner to z on the C scale. Note the position of the initial index of the C scale.
d. Read the answer on the D scale under the runner.
e. Compute characteristic of the answer.

Case II. $w = xyz$

a. Move the slide so that the proper index comes to x on the D scale.
b. Set the runner to y on the C scale. Note the position of the initial index of the C scale.
c. Move the slide so that the index is under the runner.
d. Set the runner to z on the C scale. Note the position of the initial index of the C scale.
e. Read the answer on the D scale under the runner.
f. Compute the characteristic of the answer.

Case III. $w = \frac{x}{yz}$

a. Set the runner to x on the D scale.
b. Move the slide so that y on the C scale lies under the runner. Note the position of the initial index of the C scale.
c. Set the runner to the index of the C scale that is inside the rule.
d. Move the slide so that z lies under the runner. Note the position of the initial index of the C scale.

e. Read the answer under the index of the C scale that lies inside the rule.
f. Compute the characteristic of the answer.

5.20 The Inverted or CI Scale.

Most of the better slide rules have a special scale on the slide which serves several useful purposes. Some manufacturers mark it CI, meaning C inverted; another maker may label it CR, for C reversed; a third may call it the R, or reciprocal, scale. In each case it is simply the familiar C scale turned end for end. By means of it the reciprocals of numbers may be read with the aid of the runner only. If the runner is set to a value on the C scale, its reciprocal will lie on the CI scale under the runner. The reciprocal scale is usually numbered in red ink so that it will not be confused with other scales.

Two of the three factor problems, Case II and Case III, will generally be converted to single-setting operations if the CI scale is used. Every time that a resetting of the slide or runner can be avoided, precision is gained and the chances for a mistake are reduced. Remember, when reading the instructions to follow, that **THE Initial Index** is not left behind when the C scale is turned end for end to make the CI scale and so it lies at the right-hand end of the slide for every factor that is laid off on the CI scale.

Case I. $w = \frac{x}{y}(z)$ The CI scale is not used for this case.

Case II. $w = xyz = \frac{x}{\left(\frac{1}{y}\right)}(z)$

Notice that dividing x by the reciprocal of y does not change the value of the equation in any way but that it does convert Case II into a form resembling Case I. In this way the movements of slide and runner become exactly the same as Case I, and in most problems Case II becomes a single setting job, thus:

a. Set the runner to x on the D scale.

b. Move the slide so that y on the CI scale lies under the runner. Note the position of the initial index of the CI scale because y was laid off on it.

c. Set the runner to z on the C scale. Note the position of the initial index of the C scale because z was laid off on C scale.

d. Read the answer on the D scale under the runner.

Case III. $w = \frac{x}{yz} = \frac{x}{y}\left(\frac{1}{z}\right)$

It is apparent at a glance that multiplying by the reciprocal of z does not change the equation in any manner but it does convert Case III into a form like Case I. Again the movements of slide and runner are the same as Case I and consequently a single-setting job for most problems, thus:

a. Set the runner to x on the D scale.

b. Move the slide so that y on the C scale lies under the runner. Note the position of the initial index of the C scale.

c. Set the runner to z on the CI scale. Note the position of the initial index of the CI scale because the value z was laid off on the CI scale.

d. Read the answer on the D scale under the runner.

5.21 The Folded Scales, CF, DF, and CIF.

The CF and DF scales are simply C and D scales that have been cut at the π mark, the sections transposed so that the two indexes coincide, and the resulting "folded scale" then engraved on the rule so that the π marks at the ends are in alignment with the initial and final indexes of the C and D scales.

If the diameter of a circle is set on the D scale with the runner, its circumference is read on the DF scale without the aid of the slide. If the square of the radius is laid off on the D scale with the runner, the area will be read on the DF scale.

The folded scales are of more frequent value, however, in enabling an operator to solve in one setting many multiple-factor problems that would require two or more settings if only the C and D and CI scales were available. When the third factor in either Case I or Case II cannot be set on the C scale because too much of the slide lies outside the rule, set factor z on the CF scale and read the answer on the DF scale. In Case III if the factor z is out of reach on the CI scale, set it on the CIF scale and read the answer on the DF scale. There will always be a few problems, however, where so much of the slide is outside the rule that the third factor z cannot be reached on any of the regular or folded scales. In such situations revert to the instruction given for the C and D scales in Topic 5.19.

There will be no trouble in locating the decimal point in the answers when using the folded scales if the computer will watch the initial index of the CI scale when a factor is set on the CF scale, for Case I and Case II.[1] If the factor z is set on the CIF

[1] For step-by-step instruction on three-factor solutions on the several scales refer to the "Engineering Problems Workbook."

scale for Case III, then watch the initial index of the C scale just as is done when only the C and D scales are used.

One word of caution should be given here: Never set a factor on a folded scale unless it is impossible to set it on the C or CI scales. Decimal points will be misplaced if this is done because one will be referring to the wrong scale when watching for index projections. There is a simple way of checking to see if the folded scale was necessary:

a. Does the answer *w* lie between the central index (the 1.0) of the DF scale and the central index of the CF scale? If it does, the use of the folded scale was proper.

b. Does the answer *w* lie to the right or left of both of these central indexes? If it does, the factor *z* can and should be set in the usual manner on the C or CI scale depending upon the form of the problem.

5.22 There Are Still No Exceptions to the Rule for Decimal Points.

Now that instructions for the use of the CI, CF, CIF, and DF scales have been added to those for the C and D scales it is advisable to say again: There are no exceptions to the rule for locating the decimal point in answers. It still remains as follows:

If **THE Initial Index** projects, add 1 to the characteristic of the factor just set on the slide.

All that must be remembered is that inverting the C scale takes **THE Initial Index** along with the other calibrations.

One can sum it all up thus:

a. Watch the initial index of the C scale for all values set on the C or CIF scales.

b. Watch the initial index of the CI scales for all values set on CI or CF scales.

5.23 Multiple-factor Problems.

For continued operations involving four or more factors the engineer should be on the alert to use all of the above scales in order to reduce the number of settings and also to do the work, if possible, so that the answer is the only reading made. If the continued operation involves both multiplication and division, follow a zigzag path through the problem. That is, divide the first term in the numerator by the first term in the divisor,

multiply by the second term in the numerator, as for any three-factor problem. Now with the runner on the partial result bring in the second divisor and then the third multiplier, etc. Some computers waste time by breaking such a problem into three problems; they get the numerator and divisor separately and then make the final division. Such a procedure will introduce into a five-factor problem an extra setting of both slide and runner and two needless readings, making ten operations instead of six.

The computer should not memorize special or trick settings unless they are used frequently. He should, however, endeavor to master the scales discussed in the foregoing topics so that he can devise special settings when useful.

5.24 Scales Used for Squares and Square Roots.

The A scale on the usual slide rule consists of two logarithmic scales, placed end to end, and each section is half as long as the D scale. The A scale is, therefore, plotted so that the mantissas on it increase twice as fast as they do on the D scale. Since this means the same thing as multiplying the logarithm by 2 and

$$2\,(\log x) = \log x^2,$$

it follows that the A scale carries the squares of the values on the D scale. A few rules of the log-log type do not have an A scale, but the B scale can be used with the C scale the same way that the A and D scales are used for squares and square roots.

5.25 Squares, $y = x^2$

When squaring a number be sure to notice whether the value of x^2 is read on the first (left-hand) or the second (right-hand) section of the A scale, as this fact determines the location of the decimal point in the square. The operations are as follows:

a. Set the runner to x on the D scale.
b. Read the value of x^2 on the A scale.
c. Note the section on which x^2 is found.
d. The characteristic of the power is computed thus:

1. When the first section of A carries the value of x^2, double the characteristic of x to get the characteristic of x^2.
2. If the second section of A carries the value of x^2, double the characteristic of x, then add 1, to get the characteristic of x^2.

5.26 Square Roots, $y = x^{0.5}$

To find the square root of a number one must be sure to set x on the correct section of the A scale. This is determined by first marking off the number whose square root is desired into blocks of two digits each, beginning at the decimal point just as in longhand extraction of the root. The number of digits left in the first (left-hand) block tells the section of the A scale to be used. If there is one digit in this left-hand block, set x on the first section of the A scale. If there are two digits in the left-hand block, set x on the second section of the A scale.

The operations are as follows:

a. Point off the numerical value of x into blocks of two digits, starting at the decimal point.

b. If there is one digit in the left-hand block, set the runner to x on the first section of the A scale. If two digits, set the runner to x on the second section of the A scale.

c. Read $x^{0.5}$ on the D scale under the runner.

d. The characteristic of the root is computed thus:

1. The root is found under the first section of the A scale: Divide the characteristic of x by 2 to get the characteristic of the square root.
2. The root is found under the second section of the A scale: Subtract 1 from the characteristic of x then divide by 2 to get the characteristic of the square root.
3. Note that if, at any time, a fractional value is obtained when dividing the characteristic by 2, this fact proves that instruction *d*2 has not been followed.

Example 1.

$y = (5{,}150{,}000.)^{0.5}$ The characteristic is 6.

a. Point off in blocks of two digits each, thus: 5′15′00′00.

b. There is one digit in left block; use the first section of the A scale. Set the runner to 515.

c. Read the digits in the answer 2270 on the D scale under the runner.

d. Compute the characteristic of the answer: $\frac{6}{2} = 3$.

e. Point off the answer, 2,270.

Example 2.

$y = (0.000{,}515)^{0.5}$ The characteristic is $\overline{4}$.

a. Point off into blocks of two digits each, thus: 0.00′05′15

b. There is one digit in the left block: Use the first section of the A scale. Set the runner to 515.

c. Read the digits in the answer 2270 on the D scale under the runner.

d. Compute the characteristic of the answer: $\frac{\bar{4}}{2} = \bar{2}$

e. Point off the answer, 0.022,70

Example 3.

$y = (45{,}430{,}000.)^{0.5}$ The characteristic is 7.

a. Point off into blocks of two digits each, thus: 45′43′00′00.

b. There are two digits in the left-hand block: Use the second section of the A scale. Set the runner to 4543.

c. Read the digits in the answer 6740 on the D scale under the runner.

d. Compute the characteristic of the answer: The second section of A was used; so subtract 1 from the characteristic of x; then divide by 2, thus:

$$\frac{7-1}{2} = \frac{6}{2} = 3$$

e. Point off the answer, 6,740.

Example 4.

$y = (0.000{,}045{,}943)^{0.5}$ The characteristic is $\bar{5}$.

a. Point off into blocks of two digits each, thus: 0.00′00′45′43

b. There are two digits in the left-hand block: Use the second section of the A scale. Set the runner to 4543.

c. Read the digits in the answer 6740 on the D scale under the runner.

d. Compute the characteristic of the answer: The second section of A was used; so subtract 1 from the characteristic of x; then divide by 2, thus:

$$\frac{\bar{5}-1}{2} = \frac{\bar{6}}{2} = \bar{3}$$

e. Point off the answer: 0.004,543

Note that in each of the examples above there is one digit in the root for each block or partial block in the value of x. If the value of x is less than 1, there will be one zero between the decimal point and the first digit in the root for every pair of digits in a similar position in the value of x.

5.27 Scales Used for Cubes and Cube Roots.

The K and D scales are used for cubes and cube roots. The K scale consists of three complete logarithmic scales placed end to end, and each section is one-third as long as the D scale. The

whole procedure is similar to that for squares and cube roots and hence will be given in condensed form.

5.28 Cubes, $y = x^3$.

When cubing a number be sure to notice whether y is read on the first (left-hand) section, second (center) section, or third (right-hand) section of the K scale. The characteristic of the cube is computed thus:

a. If first section of K carries the value of x^3, multiply characteristic of x by 3 to get the characteristic of x^3.

b. If the second section of K carries the value of x^3, multiply characteristic of x by 3, then add 1 to get the characteristic of x^3.

c. If the third section of K carries the value of x^3, multiply the characteristic of x by 3, then add 2 to get the characteristic of x^3.

5.29 Cube Roots, $y = \sqrt[3]{x}$.

To get cube roots the number should first be marked off into groups of three digits, beginning at the decimal point, as for longhand extraction. There will be one figure in the root for each group of three digits in the value of x.

As with square roots, the number of digits in the left-hand group will tell which section of the K scale should be used. If one digit, use the first (left-hand) section. If two digits, use the second (center) section. If three digits, refer to the third (right-hand) section. Thus:

		Answers
$\sqrt[3]{4'560'000.}$	One digit in left-hand group. Use the first section of K scale.	165.8
$\sqrt[3]{0.000'006'400}$		0.018,57
$\sqrt[3]{54'700'000.}$	Two digits in left-hand group. Use the second section of K scale.	380.
$\sqrt[3]{0.000'089'600}$		0.0447
$\sqrt[3]{245'000'000.}$	Three digits in left-hand group. Use the third section of K scale.	626.
$\sqrt[3]{0.000'000'975}$		0.009,92

5.30 Decimal Points for Roots and Powers.

As shown in the topics above the characteristic method of determining the location of the decimal point can also be used for roots and powers. The rules may be put in tabulated form as follows:

CHARACTERISTICS OF ROOTS AND POWERS

Section of scale	Operation to be performed on characteristic of number to give characteristic of result			
	For squares	For square root	For cubes	For cube root
First......	(2)(Char.)	$\frac{\text{Char.}}{2}$	(3)(Char.)	$\frac{\text{Char.}}{3}$
Second....	[(2)(Char.)] + 1	$\frac{(\text{Char.} - 1)}{2}$	[(3)(Char.)] + 1	$\frac{(\text{Char.} - 1)}{3}$
Third.....	....	....	[(3)(Char.)] + 2	$\frac{(\text{Char.} - 2)}{3}$

5.31 The L Scale.

The most popular size of slide rule is known as a 10-in. rule, but actually it is 25 cm in length. The L scale is simply this 25-cm length divided uniformly into 10 principal parts, and these in turn into still smaller units so that the mantissas of the values on the D scale can be read directly. This permits the computation of fractional and decimal roots and powers. It is truly a graphic logarithm table, and the logarithms may be used in exactly the same manner as the numerical logarithms read from printed tables.

There are two ways in which the L scale of Mannheim type rules may be used to get log x, as follows:

a. Pull the C scale to the right until the value of x lies just above the final index of the D scale. Read log x on the L scale under the hairline of the insert.

b. Invert the slide, and align the initial indexes of the L and D scales. Set the runner to x on the D scale, and read log x on the L scale under the runner.

On the log-log rules the L scale is sometimes put on the back of the rule and at the upper edge of the body of the rule. If the index of the L scale lines up correctly with the index of the D scale, set the runner to x on the D scale and read log x on the L scale under the runner. If the indexes do not line up, set the slide so the index of the C scale is aligned with the L scale, then set runner to x on the C scale instead of the D scale. A problem will be solved to illustrate the use of the L scale for powers.

Example:

y	$= (36.0)^{2.3}$	
$\log y$	$= 2.3(\log 36.0)$	
	$= 2.3(1.556)$	Read 0.556 on L opposite 36.0 on D scale.
	$= 3.580$	Transfer 0.580 to L scale,
		Read 3805 on D scale.
		Characteristic in log is 3.
$\therefore y$	$= 3805.$	

5.32 Trigonometric Scales.

The back of the slide of the "Mannheim Special" slide rule has on it scales marked S, B, L, and T. The S, B, and T scales are used in getting the primary trigonometric functions. Most rules of the Mannheim type have either a mark engraved on the side of a notch in the stock or a small transparent insert with a hairline on it set in the notch. This mark or hairline is in line with the index on the face of the rule. If it is in the correct position, the functions may be read with the slide in the normal position. If the insert is not accurately placed, however, it does not affect the reading of the sine but the tangent should be read on the D scale with the slide upside down, as suggested later.

5.33 Sines.

Pull the slide to the right until the angle on the S scale lies under the hairline on the insert. Read the natural sine on the B scale beside the S scale. The table below gives the rules in concise form:

Range in angle	Setting	Function	Section of B scale where sine* is found	Char.
0° 34′ to 90°	Set angle on the S scale	Read the Sine on the B scale	Left-hand Right-hand	$\bar{2}$ $\bar{1}$

* For cosines read the sine of the complementary angle, that is, (90° − given angle).

5.34 Tangents and Cotangents.

To get the tangent of an angle pull the slide to the right until the angle on the T scale lies under the insert. Turn the rule over, and read the desired function on the C scale or D scale according to the table below. For angles under 5° 43′ it is necessary to

read the sine and use it instead of the tangent. The error is about 5 in the fourth decimal place for this angle and decreases as the angle becomes smaller.

Range in the angle	Setting	Tangent		Cotangent	
		Read	Char.	Read	Char.
0° 34′ to 5° 43′	Angle on S scale	On B scale	$\bar{2}$	On A scale opposite index of C scale	1
5° 43′ to 45° 00′	Angle on T scale	On C scale opposite index of D scale	$\bar{1}$	On D scale opposite index of C scale	0
45° 00′ to 84° 17′	(90° − A) on T scale	On D Scale opposite index of C scale	0	On C scale opposite index of D scale	$\bar{1}$
84° 17′ to 89° 26′	(90° − A) on S scale	On A scale opposite index of C scale	1	On B scale	$\bar{2}$

When the hairline on the insert is not exactly opposite the index of the D scale, the tangent and cotangent may be read as follows:

Tangent. Turn the slide over, and line up with the D scale. Set the angle on T scale; read the tangent on D scale.

Cotangent. Pull the slide to either right or left until the angle on the T scale lies opposite the index of the D scale. Read the cotangent under the index of the slide. Characteristics are as shown in the table above.

CHAPTER 6

PRECISION AND ARITHMETICAL CALCULATIONS

6.1 Arithmetic: the Most Essential Tool.

The oldest of the mathematical sciences is still the one of greatest importance to most people. All through the ages it has been of the greatest usefulness and interest to the majority of men. Ancient Babylonian clay tablets and long-buried Egyptian records show that even in that distant age arithmetic was regarded as an essential branch of knowledge. Today it is more important than ever that everyone know this basic tool. The ordinary citizen is expected to keep his accounts for income tax records; the farmer has to make many varied reckonings; and the housewife, no less than the others, must know the basic operations of arithmetic if she is to be competent in handling the household finances.

It should be obvious that the engineer must assuredly be competent in the science of calculation if such skill is expected of the ordinary citizen. Without ready skill and dependability in figuring, an engineer is of scant value to his employer. It is a strange fact, however, that although the engineering student presumably has been taught arithmetic from the time when he was in the first grade, he is frequently completely unreliable in such fundamental operations as simple addition and multiplication. Many of these young men feel that when they are given a problem and make a ruinous arithmetical blunder on it, they should nonetheless be given credit on the assignment just because "they tried hard" or "they knew how." They fail to realize that it takes cold, impersonal, accurate results to build bridges, to make machinery, or to operate a public utility. It is said that Hades is paved with good intentions. Note that even there the inhabitants do not think much of good intentions, because they tramp on them. Results only, not effort, count.

6.2 The High Cost of Blunders.

In every engineering design, construction, or operation project, no matter how abstruse the theory may be, the facts must eventually be resolved into measured, numerical values before the project can assume reality. Failure to handle these numerical calculations correctly will naturally be a cause of expense and delay or, worse still, the failure of the project. The utility, safety, convenience, and useful life of any engineer's creations will depend upon the correctness of his calculations. And yet competent engineers in executive positions in industry testify that nearly all of the mistakes that bedevil engineering offices are due to carelessness in calculations, not to troubles in theory or higher mathematics. No, wrong answers due to blunders in arithmetic cannot be brushed aside lightly with an off-hand remark about misplaced decimal points or mistakes in arithmetic. Accuracy is a mark of competence.

Men who are in a position to know have stated repeatedly that fully 90 per cent of the mistakes in figuring are due to careless arithmetical blunders. They are the sources of constant delay and expense. Estimators have cost their employers many thousands of dollars because of their blunders in making extensions from unit price to total price in their cost estimates. The contractor must absorb such costs, not the owner. Stories of losses from misplaced or omitted decimal points are legion.

6.3 The Source of Blunders.

When questioned about the causes of such blunders several psychologists have said that there was only one reason: *carelessness.* They did not know of any psychological reason or excuse for blunders. Often these careless mistakes are due to an unvoiced feeling that this phase of the work is relatively unimportant. Sometimes they are due to a momentary lapse of attention A bit of mind wandering while the pencil moves mechanically may well introduce a costly blunder into a chain of calculations. Such inattention frequently results in a transposing of digits when copying values. It is a time-consuming blunder, because an internal transposition of digits is one of the most difficult mistakes to detect. Thus it is that the mind wanders, the pencil keeps moving, and sooner or later a "bonehead" blunder creeps in.

Only an alert checker then stands between the faulty figures and the men who must use them.

6.4 The Luxury of Inefficiency.

The failure of instructors to teach and of employers to insist upon the use of simple, direct, timesaving calculation procedures cause expense, due to wasted time, in any computing room. Computing machines are not so common in offices as most men seem to believe, and there are countless situations where the use of the machine would be time-wasting. The mechanical calculator is not a substitute for the human mind.

In many cases the engineer who uses long, roundabout techniques is merely following the childhood teaching of some inexperienced teacher who never knew or cared that she (or he) was teaching inefficiency. In other cases he continues to use poor methods because he does not have time to learn better ways. Some men waste time by carrying too many figures through their calculations. One engineer, for example, used six-place logarithms, interpolated, on a job that is regularly handled on the slide rule. Another used a seven-place logarithm table for all values, even three significant figures, in the belief that he could thus secure answers good to six or seven figures. Such blunders indicate a deep-seated lack of understanding of the principles of measurement and calculation.

Using formulas without regard to the labor that they require in numerical calculations is another way to waste time and energy. A good illustration of the differences among formulas is found in various methods of getting the three angles in a triangle when the lengths of the three sides are known. The shortest, neatest method requires only four logarithms to solve for all three angles independently, to get the radius of the inscribed circle and the area of the triangle. A second handbook formula for this same problem requires seven logarithms and two extra square-root determinations to get only the angles.

The habit that some men have of trying to get each problem into the form of a single equation is another source of expense. They put far too many things into one bundle, thinking that they are saving themselves labor. They are, however, merely carrying what the old pioneer always referred to as "a lazy man's load." Some workers seem to have almost a mania for trying to solve long,

complicated problems by means of equally long and involved formulas. These men will construct such a formula when none is found in their handbooks. They couple term to term much as freight trains are assembled, thinking that they are thereby proving that they know their subject. Such efforts do not mark the worker as a man of superior intellect—quite otherwise. It is a common occurrence for a freight train crossing the mountains to be cut into smaller sections to "get it over the hump." A similar procedure in the handling of tough problems will also help the computer to "get over the hump." More than one man can tell of hearing chief engineers and superintendents make scathing, not to say profane, comments about the men who will not cut their problems into a series of simple, concise operations. Failure to do this results in lost time in making the original calculations, many unnecessary blunders, more time lost by the checker in going over the work, and then still more time lost in trying to locate and correct the inevitable blunders.

6.5 Sources of Efficiency.

To be efficient in calculating merely means that the computer is, first of all, accurate in his work and, second, that he uses good judgment in his choice of computing methods. He must, of course, be competent in the basic skills of figuring. He must also be familiar with the use of the slide rule, cut-longhand, logarithms, and the mechanical calculator. The slide rule ranks very high in importance, as various large companies report that at least 90 per cent of their calculations are done on the ordinary engineer's slide rule. He must have a clear understanding of the meaning of the terms *precision of measurement* and *significant figures* and hence be able to choose the computing method best fitted to the data at hand. If he carries too few figures, his work is of as little value as if the result were wrong. If he carries too many figures, then his results are false and misleading, because they give an appearance of precision that, in fact, they do not have.

6.6 Precision of Measurement.

Most numerical values as used in engineering represent measurements of one kind or another. Either they are observed values obtained by instrumental measurements of some sort,

or else they are computed values that have to be laid off, weighed out, or otherwise measured as they are applied to the job in hand. In either case, both measurements and calculations are involved. Now nothing has ever yet been, or ever can be, measured exactly, that is, with zero error. In some cases the measurement may be only roughly approximate, as in pacing a distance. In others, however, every possible artifice is used to make the error as small as possible. In the case of a certain machinists' gauging system, for example, the master gauges are checked by the use of light of known wave length, and the error is guaranteed not to exceed 0.000002 in. in a gauge block 1 in. long.

The fact that every value expressed as a measured quantity has an error in it great or small does not excuse mistakes and blunders. Every precaution should be taken to avoid these and get the results to the degree of precision that the methods of measurement will justify. Ordinary measurements usually can be made so that the error is less than 1 in 1000, that is, plus or minus 1 or less in the fourth figure. Thus a distance measured to the nearest foot, such as 1283 ft, may be in error as much as 6 in. either way. For this reason we say the 3 is "doubtful" because the true value may be anywhere between 1282.5 and 1283.5, a range of 12 in. Yet any value in that range, when written to four "significant figures," would be recorded as 1283 ft.

The precision of measurement will depend upon three factors as follows:

a. **Precision of the measuring tool itself.** Compare the precision of a dressmaker's cloth measuring tape with that of a steel tape. Or compare the cheap spring scale with the chemist's balance as a means of weighing an object.

b. **Fineness of the calibrations.** A steel tape that was calibrated only at 5-ft intervals would be useless in surveying. A 12-in. ruler that was calibrated only to inches would not be of much value in constructing diagrams to scale where sixteenths of an inch would be a coarse reading.

c. **Care used in measuring.** If high-precision instruments are used carelessly, the measurements obtained are no better than those made with poorer grade tools. The skill and care exercised by the man doing the measuring thus is a factor determining the reliability of the results.

6.7 Significant Figures.

When the expression *significant figures* is used, it refers to the series of digits, including zeros, that indicate both the magnitude of the measured value and the degree of precision with which it was measured. If a length of, say, 36.2 ft is measured to three figures, it is recorded as 36.2 ft. If, however, more careful work is done, it might be 36.19 ft, thus using four significant figures. Suppose that still more careful measurements were made; then five significant figures might be justified, and the value would be recorded as 36.194 ft.

The zero may be either a significant figure or just a space filler to show the position of the decimal point in the value. If the value mentioned above was still read as 36.2 when methods justifying the use of four significant figures were used, that fact is shown by recording it as 36.20 ft. If correct to five figures, it should be recorded as 36.200 ft. If, however, the same digits occurred in a very long distance, it might have to be entered at 3,620,000. ft, and the user cannot tell whether it is correct to three, four, five, six, or seven significant figures. In this case the ciphers are merely fillers for decimal-point location. To clear up an ambiguous situation like this the number should be written as follows:

$3.62(10)^6$ for three significant-figure precision.

$3.620(10)^6$ for four-figure precision.

$3.6200(10)^6$ for five-figure precision.

This notation has many advantages, especially for extremely large or very small values, such as $3.620(10)^{-7}$, because space is saved and calculations can be simplified by so doing.

The position of the decimal point has no connection with the number of significant figures. For example, the following values all have the same number of significant figures although the decimal point is shifted over a wide range.

627,500,000.	176,928,000.
6275.	1769.28
0.6275	17.6928
0.000,062,75	0.000,176,928

All the values in the first column have four significant figures, while those in the second column have six.

A clear distinction should be made between errors and mistakes.

Errors have to do with the precision of measurement, but a mistake is a mathematical blunder due to carelessness or ignorance. Accuracy is essential in all calculations, and mistakes or blunders cannot be tolerated no matter how few significant figures are carried.

To drop significant figures without good reason is a blunder that reduces the precision of the result. To record a string of doubtful figures beyond those justified is downright dishonesty. One writer said that doing this is telling a falsehood because it is saying to all readers that the data were measured with a higher precision than was actually the case. The results, therefore, are misleading, unreliable, and dangerous because they give an unjustified appearance of high precision. The use of calculation methods that are not in keeping with the precision of the data is an economic waste of time and equipment.

6.8 Error and Tolerances.

Error is defined as the difference between the true value and the one measured or computed. The error is positive or negative depending on whether the computed value is larger or smaller than the standard. The magnitude and sign of the error is determined from this equation.

$$\text{Error} = \begin{bmatrix}\text{The measured} \\ \text{or computed} \\ \text{value}\end{bmatrix} - \begin{bmatrix}\text{The true} \\ \text{value}\end{bmatrix}$$

Thus if a line is computed as being 173.5 ft long and the true value is 173.7 ft, the error is $173.5 - 173.7 = -0.2$ ft, or 0.2 ft too short. If the length is computed as 174.0 ft, then the error is $174.0 - 173.7 = +0.3$ ft. That is, it is 0.3 ft too long.

The word tolerance is used to tell the worker the magnitude and sign of any permissible error. Suppose that a class in trigonometry is given a problem and the instructor says that the tolerance on results is ± 0.2 ft. This means that when a certain length should be 176.4 ft, the checker will accept any answer lying between 176.2 ft and 176.6 ft as correct. Answers of 176.1 or 176.7 ft will have to be marked as incorrect, because they have errors exceeding the allowed tolerance. Many blueprints going to a machine shop will have dimensions recorded thus:

$$\text{Diam.} = 1.6500^{+0.0002}_{-0.0000} \qquad \text{or} \qquad \text{Diam.} = \begin{matrix}1.6502 \\ 1.6500\end{matrix}$$

This says that the material cannot be undersize but can be 0.0002 in. oversize. As stated in an earlier topic in this chapter it is impossible to make or measure anything with zero error; hence, tolerances are highly important, and the engineer who sets them unnecessarily "tight" is merely running up the cost with no tangible gain.

There is still another term with which the engineer must be familiar. It is *probable error* and is closely allied to the idea of tolerances. Thus the maker of steel blocks used in a high-precision gauging system guarantees one grade of gauges to have a probable error not exceeding ± 0.000008 in. per inch of length, the next grade to have a probable error of ± 0.000004 in. per inch of length, whereas the best set has a probable error of only ± 0.000002 in. per inch of length. With such information at hand a buyer can tell which set best fits his needs.

6.9 Percentage Error.

A clear distinction must be made between error and percentage of error. The percentage error is the ratio of the error to the true value, expressed as a percentage, thus:

$$\left[\begin{matrix}\text{Percentage}\\ \text{of error}\end{matrix}\right] = \left[\frac{\text{The error}}{\text{True value}}\right](100)$$

In certain fields it is customary to express tolerances in per cent of error rather than in the absolute value of the error itself. A clearer idea of the precision obtained is often had by this notation. For example, in the following figures the two computed results have identical errors, but the percentage errors differ widely.

		Case A	Case B
Computed	=	1642.8 ft	4.7 ft
True value	=	1642.5 ft	4.4 ft
Error	=	0.3 ft	0.3 ft
Per cent of error	=	$\frac{0.3(100)}{1642.5}$	$\frac{0.3(100)}{4.4}$
	=	0.0183%	6.825%

The percentage values show that in Case A there is a high degree of precision in the computed value whereas Case B has very low precision. The absolute value of the error (0.3 ft), however, gave no indication of this fact.

The tables given on pages 132–133 give helpful information regarding customary tolerances in several fields of work and the suitable calculation methods for various degrees of precision. Note that the values for errors in angles are only approximate, as they vary widely depending on the size of the angle and the function being used.

COMMERCIAL TOLERANCES

	Tolerances Plus or Minus
Surveys:	
Farm and open land	
Ordinary work	1 in 500
Good work	1 in 2,000
Best work	1 in 5,000
Federal triangulation	1 in 1,000,000
City surveys	
Ordinary commercial work	1 in 5,000
Good commercial work	1 in 10,000
Best commercial work	1 in 50,000
Railroads and bridges	
Ordinary work	1 in 25,000
Best work	1 in 100,000
U. S. Coast and Geodetic Survey	
1st grade	1 in 25,000
Error in angles	1″
2d grade	1 in 10,000
Error in angles	3″
3d grade	1 in 5,000
Error in angles	5″
Machine Shop Practice:	
Ordinary bearings, allowable error	0.005 to 0.010 in.
Good work, allowable error	0.002 in.
Best work, allowable error	0.0002 in.
Fine gauge and tool work, allowable error	0.001 to 0.000001 in
Rough work, clearances	$\frac{1}{16}$ in.
Rough work, bolt holes, clearances	$\frac{1}{32}$ to $\frac{1}{8}$ in.
Chemical Analysis:	
Ordinary	0.3 of 1 per cent
Good	0.2 to 0.1 of 1 per cent
Best, metal traces, etc	0.03 of 1 per cent
Structural and Architectural Work:	
Ordinary buildings, main dimensions	$\frac{1}{2}$ to 1 in.
Steel bridges	
Member lengths	$\frac{1}{16}$ in.
Over all	$\frac{1}{4}$ in.

CALCULATION METHODS SUITABLE FOR VARIOUS TOLERANCES
SINGLE OPERATIONS

Significant figures in data	Suitable calculation methods	Tolerances: Maximum error	Tolerances: Per cent of error	Tolerances: Errors in angles*
3	Slide rule or three-place logarithms	1 in 100	1.0	30′
4	Slide rule or four-place logarithms	1 in 1,000	$\frac{1}{10}$ of 1	2′
5	Five-place logarithms or cut-longhand	1 in 10,000	$\frac{1}{100}$ of 1	10″
6	Six-place logarithms or cut-longhand	1 in 100,000	$\frac{1}{1000}$ of 1	1″ to 2″

CONTINUED OPERATIONS

Significant figures in data	Suitable calculation methods	Tolerances: Maximum error	Tolerances: Per cent of error	Tolerances: Errors in angles*
3	Slide rule or three-place logarithms	2 in 100	2.0	1°
4	20 in. slide rule or four-place logarithms	2 in 1,000	$\frac{1}{5}$ of 1	5′
5	Five or six-place logarithms or cut-longhand	2 in 10,000	$\frac{1}{50}$ of 1	20″
6	Six-place logarithms or cut-longhand	2 in 100,000	$\frac{1}{500}$ of 1	2″

* Davis, Foote, and Raynor, "Surveying," McGraw-Hill Book Company, Inc., 1940, pp. 35–37, gives a thorough study of errors in connection with angles.

The list of tolerances in current commerical practice in various fields of engineering can be used safely and will aid the inexperienced man to avoid the twin mistakes of using too few or too many figures. As better measuring devices come into use, some of the high-precision tolerances will undoubtedly be reduced and more precise calculations will then be necessary. Here and there firms may depart from the tolerances indicated, but they will be found satisfactory in the majority of cases.

6.10 Holman's Rules for Numerical Calculation.

Since it is misleading as well as time-wasting to carry needless figures through a calculation, the engineer should know how to round off values properly and also to know what the true results will be when values of varied precision are brought together in a series of calculations. The precision of a result depends upon the precision of both the various factors and the mathematical operations performed. It is necessary, therefore, for one to become familiar with the facts that are summed up in Holman's rules of calculation. In all of the following instructions and examples the doubtful figures are printed in small italics.

a. **In addition or subtraction.** The result cannot be accurate beyond the column having the first doubtful value.

Addition	Subtraction
128*3*.	**645.3*6***
+23.8*7*	**−219.*4***
+ 1.94*2*	**425.*96***
130*8.812*	

b. **In multiplication or division.** The result will be accurate only to the same number of figures as the least precise factor involved. This will be true regardless of the number of factors involved. Note the loss of precision in the examples below.

Multiplication	Division
	18.*5153*
1473.*6*	**37.*4*)692.47*3***
2.5*3*	**37*4***
44208	**31*84***
7368*0*	**299*2***
2947*2*	**1*927***
37*28.208*	**18*70***
	573
	37*4*
	1990
	1870
	1200
	1122
	780

c. **Carrying doubtful figures.** In general, carry through all the intermediate calculations one more figure than is wanted in the end result of a chain of calculations. That is, carry two, but only

two, doubtful figures through the intermediate steps; then cut back to one doubtful digit in the final answer. Remember, however, that one doubtful figure will always be left when any rounding off is done. Dropping it merely makes the one to its left doubtful. Refer to Topic 6.11 for the rules for rounding off numerical values.

When measured values are to be used in connection with constants such as π or conversion constants that are known to many places, time will be saved if the constants are cut back to the same number of significant figures as the measured values.

6.11 Rules for Rounding off Numerical Values.

The following instructions, including the numbering of the rules, are taken from the ASA Code No. Z 25.1, 1940, "Rules for Rounding Off Numerical Values." They should be followed whenever it is necessary to round off values that have more figures than can be used in the final answers.

In setting up rules for rounding off decimals there are three general cases that should be considered. They may be stated as follows:

When the figure next beyond the last figure or place to be retained is less than 5, the figure in the last place retained should be kept unchanged.

Example

1.2342	1.234	1.23	1.2

When the figure next beyond the last figure or place to be retained is more than 5, the figure in the last place retained should be increased by 1.

Example

1.6789	1.679	1.68	1.7

When the figure next beyond the last figure or place to be retained is 5, and

(*a*) there are no figures, or only zeros, beyond this 5, if the figure in the last place to be retained is odd, it should be increased by 1; if even, it should be kept unchanged;

Example

1.35	1.4
1.3500	1.4
1.45	1.4
1.4500	1.4

(*b*) If the 5 next beyond the figure in the last place to be retained is followed by any figures other than zero, the figure in the last place retained should be increased by 1, whether odd or even.

Example

1.3501	1.4
1.3599	1.4
1.4501	1.5
1.4599	1.5

The above rules for rounding off decimals may be restated as follows:

The figure in the last place to be retained should be kept unchanged

(*a*) when the figure in the next place is less than 5;

(*b*) when the figure in the next place is 5 followed by no other figures or only by zeros, and the figure in the last place retained is even.

The figure in the last place to be retained should be increased by 1

(*a*) when the figure in the next place is more than 5;

(*b*) when the figure in the next place is 5 followed by no other figures or only by zeros, and the figure in the last place retained is odd;

(*c*) when the figure in the next place is 5, followed by any figure or figures other than zero.

The final rounded value should be obtained from the most precise value available and not from a series of successive roundings. For example, 0.5499 should be rounded off successively to 0.550, 0.55 and 0.5 (not 0.6), since the most precise value available is less than 0.55. Similarly, 0.5501 should be rounded off as 0.550, 0.55 and 0.6, since the most precise value available is more than 0.55.

6.12 Time Economy in Longhand Calculations.

The subject of precision of measurement is seldom discussed or even given passing recognition in grade-school arithmetic books when the topics of long division and multiplication are discussed. As a result few students realize that ordinary longhand calculations as they use them really waste a lot of time and require much more writing than do equally good shortened methods. Computing machines carry a lot of useless figures and save time merely because of their mechanical speed and the fact that they are not subject to mind wandering and carelessness in the meshing of gears as is the human brain. So, whether the work is done by hand or machine, the man who does the work will have to cut off the useless figures. He has to get the result in line with the known precision and given tolerances.

There are numerous ways in which to economize on time with-

out loss of accuracy, and the good engineer should know and use a number of them. There are good books in print that suggest other methods not mentioned here. Contracted longhand, or cut-longhand as it will be called in this text, is a highly valuable tool that all engineers should know. Surveyors have appreciated its usefulness for many years, as it can be used to great advantage in the field, where no computing machines are available.

The computer who chooses his computation methods to fit the precision of his data will appreciate the time- and labor-saving possibilities of the following contracted or "cut-longhand" methods. Surprising economies are possible when the slide rule is used in conjunction with cut-longhand, especially in division and square root. A timed test with 150 students who were equally inexperienced in the use of logarithms and cut-longhand indicated a 2 to 1 advantage for the cut-longhand on multiplication or division involving two numbers of six significant figures each. A 35 to 50 per cent reduction in the number of digits to be recorded is no small saving, especially since the omitted digits are all worthless. Decimal points are just as easy to place as in the older, extended calculations if the "characteristic method" is used. When the data require the multiplying or dividing of numbers with four or more digits, there is a gross waste of time and absolutely no increase in accuracy through the use of full longhand figuring. Logarithms may be used, of course, but there will be no saving of time until four or more long factors have to be handled.

6.13 Division by Cut-longhand.

Contrary to the idea held by a considerable number of people division is actually easier than multiplication. This is true for slide-rule work, for most computing machines, and for cut-longhand. If longhand and slide-rule work are combined, the engineer can get results in far less time than he can by the use of logarithms. The quotient will have the same precision as the data, and there is no lost motion.

Stated concisely the method is the following:

a. Use contracted long division for all but the last three digits of the quotient, dropping useless doubtful figures.

b. Set the last remainder found in *a* on the slide rule, and divide as usual to get the last three figures in the quotient.

The sample problem below shows the savings obtainable by the use of this system. The doubtful figures are shown in small italics in each example. Since there is no gain whatever in carrying more than two doubtful digits, the contracted form is definitely a timesaver.

Example:

Divide 694.72*8* by 47.362*9*

Cut-longhand

	14.668*18*
47.362*9*)	**694.72***8*
	473 62*9*
	221 09*9*
	189 45*2*
	31 64*7*
	28 41*7*
	3 23*0*
	2 84*2*
	38*8*

Ordinary Longhand

	14.668*18*	
47.362*9*)	**694.72***8*	*00000*
	473 62*9*	
	221 09*9*	*0*
	189 451	*6*
	31 64*7*	*40*
	28 417	**7***4*
	3 22*9*	*660*
	2 841	**77***4*
	38*7*	*8860*
	378	**903***2*
	8	*98280*
	4	*73629*
	4	*24651*

The cut-longhand and slide-rule combination requires 39 entries of digits in addition to those for the dividend, divisor, and quotient. The conventional long division requires 80 digits with no gain whatever in precision. The net saving, then, is 41 characters, or 51 per cent.

6.14 Step-by-step Example of Cut Division.

a. Set up the work in the conventional manner and note the characteristics of each term as below.

b. Divide on the slide rule to get the first three digits of the quotient and to avoid trial divisions. Read 680.

$$A = \frac{\overset{2}{501.493}}{\underset{\bar{1}+1}{0.736984}} = \overset{2}{680}. \quad \text{Approx.}$$

c. Note the characteristic of the answer. With the decimal point now definitely located no more thought need be given to it.

d. Arrange the work for long division, and get the first remainder, using the 680 as a guide. Since the first digit in the divisor

is larger than the first digit in the dividend, add one zero to the 501.493 but no more, thus:

```
              680
0.736984)501.4930
         442 1904
         --------
          59 3026
```

e. Divide this remainder on the slide rule to verify the 0 obtained when the 680 was read on the slide rule. Now read $\frac{5930}{7370} = 804+$.

f. In order to avoid annexing useless zeros to the dividend, drop the last figure (the 4) in the divisor and put a dot above it to indicate that it will not be used again. Put a similar dot above the 6 in the quotient to show it has been used as a multiplier.

g. Now use the second digit (the 8) in the quotient to get the second remainder, thus:

```
             6̇804
0.736984̇)501.4930
         442 1904     Right-hand figure is a 7 from
         --------     (8)(8) = 64 to which 3 must
          59 3026     be added from the (8)(4) = 32
          58 9587←    that would have been found if
          -------     the 4 had not been dropped.
             3439
```

h. Next the 8's in the divisor and quotient are dotted to show that they have been used. Note that the figures are always dotted in pairs, one in the divisor to shorten it and one in the quotient to show that it has been used as a multiplier in getting the next remainder.

i. Since the third figure in the quotient has been found to be a zero, the 9 is dotted but the third remainder is the same as the second, and normally the following entry is not recorded but is shown here to make this statement clear.

```
             6̇8̇0̇4
0.7369̇8̇4̇)501.4930
         442 1904
         --------
          59 3026
          58 9587
          -------
             3439
             0000
             ----
             3439
```

j. Lastly, divide this third remainder 3439 by the rounded-off divisor 7370, using the slide rule. Read 4670. This confirms

the 4 found in Step *e* and adds the last two figures, the 67+ to the quotient which can now be recorded in full, thus: 680.467. By computing machine the answer is found to be 680.4666; but since the original data had only six significant figures, both longhand and computing-machine results would them be rounded off to 680.467.

k. If it seems desirable, however, to get a seventh digit in the quotient, thus making two doubtful digits in the answer, the long division is carried one step further. Drop the 9 in the divisor, and dot it. Dot the 0 in the quotient. Multiply by the 4 in the quotient, and get the remainder 491, thus:

$$
\begin{array}{r r l}
 & \dot{6}\dot{8}\dot{0}4 & \\
\underline{0.7369\dot{9}\dot{8}\dot{4}}) & \overline{501.4930} & \\
 & \underline{442\ 1904} & \\
 & 59\ 3206 & \\
 & \underline{58\ 9587} & \\
 & 3439 & \\
 & \underline{2948} \leftarrow & \left\{ \begin{array}{l} (4)(6) = 24. \text{ Add 4 to be} \\ \text{carried over from rounding} \\ \text{off the product, } 4\ (9) = 36. \end{array} \right. \\
 & 491 &
\end{array}
$$

l. Divide the 491 by the original divisor on the slide rule, reading 677. This gives a final quotient of 680.4677; hence, the six-figure reading appears to be 1 too low. This, however, is within permitted tolerances, hence, little was gained by getting the seventh figure.

When the divisor has more significant figures than the dividend, either round it off to the same number of figures or add zeros to the dividend to give it the same number of figures.

6.15 Decimal Point Rules for Cut-division.

The slide rule is the best way of locating the decimal point in the quotient; but if no slide rule is at hand, the following rules will locate the decimal point just as reliably.

a. If the first significant left-hand figure of the dividend is greater than the corresponding figure of the divisor, the characteristic of the quotient is the characteristic of the dividend minus the characteristic of the divisor.

b. If the first significant left-hand figure of the dividend is less than the corresponding figure of the divisor, add 1 to the characteristic of the divisor before subtracting from the characteristic of the dividend.

c. If the first significant left-hand figures in divisor and dividend are equal, take the first from the left that are not equal and treat as above. If they are all equal, treat as though the dividend was greater.

6.16 Multiplication by Cut-longhand.

In contracted multiplication the two numbers are arranged in the usual order, preferably with the number having the smaller digits as the multiplier. If the numbers do not have the same number of significant figures, round off the multiplicand (figure being multiplied) so that it has not over one more significant figure than the multiplier. The work of multiplication is begun with the left-hand figure in the multiplier. This first product determines the number of digits in the answer. Any figures falling to the right of this first product are worthless; so there is no point to getting them at all. The example below will give the step-by-step process. Pay no attention to decimal points while multiplying, but run the problem on the slide rule as an over-all check and to locate the decimal point.

Example:

Multiply 463.86 by 0.9748578

a. Round off the larger number to six figures, thus: 0.9748578 to 0.974858. See Holman's rules, Topic 6.10.

b. Set the work up in the usual fashion, multiplying by the number having the smallest digits, thus:

```
0.974858
  463 86
--------
```

c. Start multiplying with the 4. This will determine the digits in the answer, thus:

```
   0.974858
     463.86
   --------
3899432
```

d. Now multiply by the 6, but first dot the 4 in the multiplier to show that it has been used; then drop the final 8 in the multiplicand, and put a dot over it to indicate that it is dropped. Note the amount to be carried, however, if the 8 had not been dropped, thus:

$$\begin{array}{lll} \quad\;\; 0.97485\dot{8} & & \\ \quad\;\;\;\; \underline{\dot{4}63.86} & & \\ 3899432 & & \\ \;\; 584915 \leftarrow 6(5) = 30 & & \text{Add 5 from rounding off} \\ & & (6)(8) = 48 \text{ to a } 5. \end{array}$$

e. Next multiply by the 3, but now drop and dot the 5 in the multiplicand and the 6 in the multiplier, thus:

$$\begin{array}{rl}
0.9748\dot{5}\dot{8} & \\
\underline{\dot{4}\dot{6}3.86} & \\
3899432 & \\
584915 & \\
29246 & \leftarrow(3)(8) = 24 \quad \text{Add 2 from rounding off} \\
 & \qquad\qquad\qquad\;\; (3)(5) = 15 = 2.
\end{array}$$

f. Then multiply by the 8, first dropping the 8 in the multiplicand and putting the dot over it and a dot over the 3 in the multiplier, thus:

$$\begin{array}{rl}
0.974\dot{8}\dot{5}\dot{8} & \\
\underline{\dot{4}\dot{6}\dot{3}.86} & \\
3899432 & \\
584915 & \\
29246 & \\
7798 & \leftarrow(8)(4) = 32 \quad \text{Add 6 from } (8)(8) = 64 = 6
\end{array}$$

g. The last multiplication is by the final 6 in the multiplier. Drop and dot the 4 in the multiplicand and the 8 in the multiplier, thus:

$$\begin{array}{rl}
0.97\dot{4}\dot{8}\dot{5}\dot{8} & \\
\underline{\dot{4}\dot{6}\dot{3}.\dot{8}6} & \\
3899432 & \\
584915 & \\
29246 & \\
7798 & \\
584 & \leftarrow(6)(7) = 42 \quad \text{Add 2 from } (6)(4) = 24 = 2
\end{array}$$

h. The multiplications are completed so that the partial products are added, the problem solved on the slide rule, and the characteristics noted. The answer is pointed off and cut back to one doubtful figure, and the job is done, thus:

$$\begin{array}{r}
0.97\dot{4}\dot{8}\dot{5}\dot{8} \\
\underline{\dot{4}\dot{6}\dot{3}.\dot{8}6} \\
3899432 \\
584915 \\
29246 \\
7798 \\
\underline{584} \\
4521975
\end{array}$$

Round this off to 5 significant figures, thus: 452.20. To check this and place the decimal point, round off the factors to slide-rule precision and solve the problem on the rule.

$$\overset{\bar{1}}{(0.9750)}\overset{2\ +1}{(464.0)} = \overset{2}{452.0}$$

When this problem is solved on the computing machine, the product is read as 452.197539, the first seven digits being the same as those found by cut-longhand. The machine reading, too, must be rounded off to 452.20, the same as above.

6.17 Economy in Cut-multiplication.

The examples below show a comparison of cut-longhand and the older form. The numbers that are called *doubtful figures* because they are at the limits of precision of measurement for the data used are printed in *italics;* the certain values are in bold-faced type.

Note the large number of doubtful, hence useless, figures that are recorded in ordinary longhand.

Cut-longhand

97.38*6*	
46.92*8*	
38954*4*	
5843*2*	
876*4*	
19*5*	
78	
4570*13*	

Ordinary Longhand

97.38*6*	
46.92*8*	
77	*9088*
194	**77***2*
8764	**7***4*
58431	*6*
38954*4*	
4570*13*	*0208*

All digits to the right of this line are of no value.

The total number of digits recorded after the statement of the problem is 26 for cut-longhand and 40 for ordinary longhand, a saving of 14 entries, or 35 per cent. In the problem below, the saving is nearly 38 per cent.

847.62*6*	
269 43*2*	
169525	
50857*6*	
7628*6*	
339*0*	
25*4*	
1*7*	
22837*75*	

847.62*6*	
26.943*2*	
16	*95252*
254	**287***8*
3390	**50***4*
76286	**3***4*
508575	*6*
169525*2*	
22837*75*	*68432*

All digits to the right of this line are of no value.

6.18 Decimal Point in Cut-multiplication.

a. If the first significant left-hand figure of the result equals or is greater than the largest significant left-hand figure of either factor, the characteristic of the result equals the algebraic sum of the characteristics of the factors.

b. If the first significant left-hand figure of the result equals or is less than the smallest left-hand figure of either factor, the characteristic of the result is 1 plus the algebraic sum of the characteristics of the factors.

c. If the left-hand figures are alike, take the first from the left that differ and apply the rule as in *a* or *b*.

6.19 Square Root by Cut-longhand.

Sometimes square roots are needed to a higher degree of precision than can be obtained on the slide rule, that is, to five, six, or more figures. Time and labor can be saved by using a combination of conventional square-root calculation plus the slide rule. There will be no loss of precision by this method.

a. Run the problem on the slide rule using the A and D scales to get the first few digits in the answer and the decimal-point location. This will avoid false trials.

b. Decide on the number of figures wanted in the square root.

c. Compute all but the last three digits in the root by the usual method (see the following topic).

d. Divide the last remainder by the last trial divisor on the slide rule by ordinary division. This will give the last three figures in the desired root, with a probable error of 1 or 2 in the last digit.

Example:

Get the square root of 119.768

```
      119.7680)10.9
      1
   209)19 76
        18 81
    2184)9580
```

Now divide 9580 by 2184 on the slide rule in the usual manner, reading 438. Annex 438 to the first three figures in the square root 10.9, obtained above. This gives 10.9438 as the complete root.

When conventional methods of longhand are used, or seven-place logarithms, exactly the same value is obtained as was obtained in less time by the contracted form.

6.20 Usual Longhand Method of Getting Square Root.

The following step-by-step instructions give the usual long method of getting square roots. It is advisable to check the first three digits on the slide rule using the A and D scales (see Topic 5.26, page 118) and then verify the last few figurès as in the example above by ordinary division.

a. Divide the number into groups of two figures each, beginning at the decimal point and pointing off both left and right, if there are figures on both sides of the decimal point (see example).

b. Find the greatest figure whose square is contained in the left-hand group. This figure is the first in the root.

c. Subtract the square of this figure from the left-hand group, and annex the second group to the remainder for a dividend.

d. Double the root already found for a trial divisor, and set it down to the left of the dividend, leaving a space at its right for one additional figure for a completed divisor.

e. Divide the dividend by ten times this divisor. The quotient will be the next figure in the root.

f. Annex this quotient to the trial divisor in the space left for the additional figure. This completes the divisor.

g. Multiply the completed divisor by this last root figure, and subtract the product from the dividend. If the product is greater than the dividend, the root figure is too large and must be reduced until the product is contained in the dividend.

h. Annex the next group to the remainder thus found, and proceed as before, always doubling all the root already found for a trial divisor.

One figure will appear in the root for each group in the original number. Locate the decimal point in compliance with this fact.

CHAPTER 7

BASIC TRIGONOMETRY

7.1 Introduction.

This chapter does not give the proofs for any of the formulas; hence, it will not serve as a textbook in the theory of trigonometry. The practicing engineer does not need nearly all the formulas for solving triangles that the average trigonometry text implies, nor does he need to memorize a long series of identities. Speed in calculation and relative freedom from chances of error are the important factors. When a short, simple, accurate method of solving a problem is available, it should always be used in preference to a more involved formula. A long, complicated equation with combined operations may be true and a "beautiful demonstration of theory" or of the computer's skill, but neither is a reason for using such a method when shorter, simpler tools are known. The closer the computer sticks to fundamental principles—to direct, well-known methods—the fewer time-consuming blunders will be made. There are relatively few mathematical tools needed by the average engineer. It is only the man in highly technical design who needs more than the basic principles. However, the computer must know these few basic methods thoroughly, have confidence in them, and use them with speed and accuracy.

A working knowledge of the right triangle is of vital importance to the engineer not only because it enters into so many of his design problems but also because it may properly be regarded as the fundamental basis of the solutions of all plane triangles. For his uses the definitions of the functions should be based upon the right triangle rather than the coordinate system, because it is then unnecessary to place the triangle in a particular position before solving it. It is also true that the solutions of triangles will generally be shorter and less liable to error if right triangle methods are used. Texts and handbooks usually list a number of more or less complex formulas for the solving of oblique

triangles. They are supposed to enable the computer to get his answers readily, but as a matter of fact some of the formulas that apparently solve a problem in one equation are really longer and offer greater chances of mistake than do methods that, sticking close to basic principles, call for breaking the problem up into short, simple steps. In the following notes, therefore, only the shortest and most efficient methods have been stated.

Stick to these methods if time is to be saved and mistakes reduced in solving the trigonometry problems in Chap. 11.

7.2 Fundamental Tools for Solving Plane Triangles.

Experienced men who make use of trigonometric calculations in their daily work, such as surveyors, insist that only a few, simple tools are necessary or even desirable. Here is a list of what they consider to be the essential principles that must be known and thoroughly understood.

a. Right triangles, which include the following:
 1. Primary functions.
  (*a*) Sine.
  (*b*) Cosine.
  (*c*) Tangent (and cotangent).
 2. Area formula.

b. Oblique triangles.
 1. Sine law.
 2. Three-sides laws.
  (*a*) Radius formula.
  (*b*) Whole-angle formula.
 3. Area formulas.
  (*a*) Base-altitude formula (same as for right triangle).
  (*b*) Two-sides and included-angle formula.
  (*c*) Three-sides formulas (two).

Note that this list leaves out the secondary functions such as secant and cosecant, the law of cosines, the "law of tangents," and a long series of three-sides formulas. They are omitted because they are longer in numerical work, mistakes are more likely to occur, or they can result in a loss of precision in the answers. The law of tangents is discarded completely in this book because it suffers from all three defects. The right-triangle solution is much

the best for handling the problem in which two sides and the included angle are given.

7.3 The Right Triangle.

In either of the right triangles shown below, the angle BAC is formed by the hypotenuse and one of the legs of the triangle. The angle BAC will be called *the angle* A. The leg AC is the *side adjacent* to this angle, and CB is the *side opposite* the angle A. The *hypotenuse* is the side AB.

If the figure is to remain a right triangle, it is impossible to change the angle A without also altering the length of two

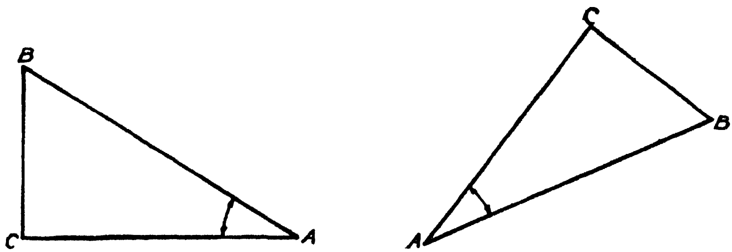


FIG. 15.—Right triangles.

sides. It is also impossible to change the length of only one side and still have a right triangle. If two sides are changed, the size of the angle A will also be changed. It is possible to change the lengths of all three sides, however, without affecting the angle A, but it will be seen that the new triangle is similar to the original one. In other words, the sides will still have the same ratios to each other. Thus the size of the angle is determined, not by the length of any one side, but by the ratios of the sides to each other. With three sides it is possible to set up six ratios, but three of the six will be merely the reciprocals of the others. Thus:

$$\frac{BC}{AB}, \frac{AC}{AB}, \frac{BC}{AC}, \frac{AB}{BC}, \frac{AB}{AC}, \frac{AC}{BC}.$$

The first three of these are the ratios most often used and will, therefore, be called **the primary functions** of the angle. They have been named *sine, cosine,* and *tangent.*

7.4 The Primary Functions.

$$\text{Sine of angle } A = \frac{\text{Opposite side}}{\text{Hypotenuse}} \qquad \sin A = \frac{BC}{AB} \tag{7.4a}$$

$$\text{Cosine of angle } A = \frac{\text{Adjacent side}}{\text{Hypotenuse}} \qquad \cos A = \frac{AC}{AB} \tag{7.4b}$$

$$\text{Tangent of angle } A = \frac{\text{Opposite side}}{\text{Adjacent side}} \qquad \tan A = \frac{BC}{AC} \tag{7.4c}$$

The above definitions apply to either of the acute angles of a right triangle in whatever position it may be placed, and it is quite unnecessary to turn the triangle around to any particular position before it can be solved. Many other functions are derived from these three primary ratios, and many formulas called *identities* are given in trigonometry texts, but all are of secondary importance, being useful only in special problems. Because the primary functions involve both the sides and the angles in a right triangle, it is possible to solve for any of the unknown sides or angles provided at least two of the terms are given, one of them a side.

The engineer meets many problems involving the solution of oblique triangles; but since such triangles may be divided into a series of right triangles, they are readily solved by using the right-triangle relationships. Broadly speaking, the primary functions are the only tools really needed for finding the unknown sides or angles in any plane triangle.

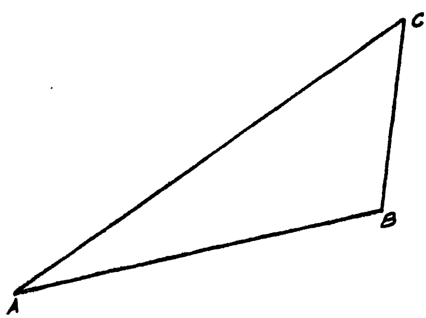


FIG. 16.—Oblique triangle.

7.5 Oblique Triangles.

As stated above, the right-triangle laws are enough to enable a computer to solve oblique triangles, but there are a few instances where it is more convenient to use special formulas which give the

relationships between sides and angles. A whole series of oblique-triangle formulas are given in trigonometry texts, but several of them are less efficient than the right-triangle method. Only those which are efficient or have some other special advantage will be given in this chapter.

7.6 Sine Law.

The most valuable of all the oblique-triangle laws is the one called the *sine law*. It is so useful that the engineer considers it as important as the primary functions. It is well adapted to use with either logarithms or computing machines. The sine law says:

In any oblique triangle the sines of the angles are proportional to the sides opposite the angles, thus:[1]

$$\frac{\text{One side}}{\text{Sine of the angle opposite}} = \frac{\text{Either of the other sides}}{\text{Sine of the angle opposite that side}} \tag{7.6a}$$

or, in symbols,

$$\frac{\text{Side } BC}{\sin A} = \frac{\text{Side } AC}{\sin B} = \frac{\text{Side } AB}{\sin C} \tag{7.6b}$$

The sine law furnishes a simple, legitimate short cut to the solution of all problems in which three of the four terms in the proportion are known.

7.7 Three-sides Laws.

There are numerous formulas for determining the angles in an oblique triangle when only the three sides are known. There is considerable difference in their efficiency, however, especially in logarithmic computation. For this reason, only three of the shorter methods are given here, but they will serve the needs of most computers. In order to have a check upon the accuracy of the work it is advisable to solve for all three angles, even though only one is required for the rest of the solution. The first two laws make use of the perimeter of the triangle, as follows:

a. **The half-angle solution**[2] (see Fig. 3 and Form 118 in the Workbook). Let $2s$ equal the perimeter of the triangle.

Then $s = 0.5$ (sum of the sides).

[1] See Form 110 in the Workbook for a proof of Eqs. (7.6*a*), (7.6*b*).
[2] See Form 117 in the Workbook for a proof of Eqs. (7.7*a*), (7.7*b*).

Let r denote a constant; then

$$r^2 = \frac{(s - BC)(s - AC)(s - AB)}{s} \tag{7.7a}$$

$$\tan \frac{1}{2} A = \frac{r}{s - BC} \tag{7.7b}$$

This law is much the shortest to apply, as it uses the least number of logarithms, only four being required to obtain all three angles, to check them, and to find the area of the triangle. Since this law gives the half angle, it is especially convenient for cases where the whole angle approaches 90°. The value r is the radius of the inscribed circle; hence, this formula has various special uses.

b. **The whole-angle solution**[1] (see Form 116 in the Workbook). Let K denote a constant and s equal to 0.5 (the sum of the sides) as above; then

$$K^2 = s(s - BC)(s - AC)(s - AB) \tag{7.7c}$$

$$\sin A = \frac{2K}{(AB)(AC)} \tag{7.7d}$$

This solution is a trifle longer than the preceding but has the advantage that K equals the area of the triangle and also that the whole angle is given directly. When the area or the sines of the angles are wanted, this solution will give these results as quickly as the half-angle formula.

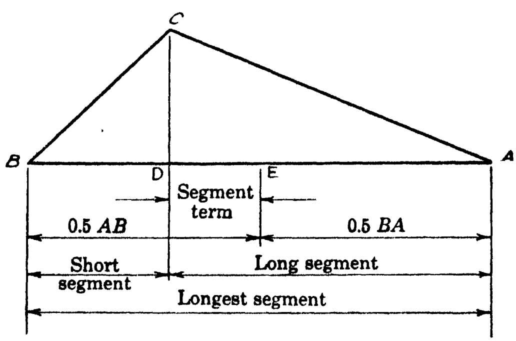


FIG. 17.—Construction used in the segment solution.

c. **The segment solution**[2] (see Forms 120, 121 in the Workbook). There is a third formula which is based upon the right-triangle method of analysis. It does not involve the quantity s

[1] See Forms 114, 115 in the Workbook for a proof of Eqs. (7.7c), (7.7d).

[2] See Form 119 in the Workbook for a proof of Eqs. (7.7e), (7.7f).

and hence is an excellent independent method of verifying the results obtained by either of the first two methods.

In the triangle ABC (Fig. 17) the vertex C is opposite the longest side AB. Point E is midway between A and B. If a perpendicular is dropped from C to AB, thus locating point D, the side AB will be divided into two segments AD and DB. When these two segments are computed, we have two right triangles each with two sides known, and hence the angles at A and B are readily found. Angle C is determined by subtracting $(A + B)$ from 180°. The "segment term" is the length DE which tells the amount that the segments AD and DB vary from being equal to half of AB.

$$\begin{bmatrix}\text{The segment}\\ \text{term}\\ DE\end{bmatrix} = \frac{\begin{bmatrix}\text{Sum of the}\\ \text{shorter sides}\end{bmatrix}\begin{bmatrix}\text{Difference between}\\ \text{the shorter sides}\end{bmatrix}}{(\text{Twice the longest side})} \tag{7.7e}$$

$$= \frac{(AC + BC)(AC - BC)}{2(AB)} \tag{7.7f}$$

$$\begin{bmatrix}\text{The long}\\ \text{segment}\\ AD\end{bmatrix} = \begin{bmatrix}\dfrac{\text{Longest side}}{2}\end{bmatrix} + \begin{bmatrix}\text{The segment}\\ \text{term}\end{bmatrix} \tag{7.7g}$$

$$= \frac{AB}{2} + DE \tag{7.7h}$$

$$\begin{bmatrix}\text{The short}\\ \text{segment}\\ DB\end{bmatrix} = \begin{bmatrix}\dfrac{\text{Longest side}}{2}\end{bmatrix} - \begin{bmatrix}\text{The}\\ \text{segment}\\ \text{term}\end{bmatrix} \tag{7.7i}$$

$$= \frac{AB}{2} - DE \tag{7.7j}$$

This method is about as short as the first two given above and is well adapted to machine calculation. The work of getting the segments should generally be broken into the three steps suggested by the word equations. Refer to Forms 120 and 121 in the Workbook for arrangement of the calculations.

This formula does have two rather serious weaknesses. First, it is not self-checking, as is the case with the two methods above. Second, when the sides BC and CA are roughly the same length, there is a great loss of precision, because the number of significant figures may drop from, say, five or six for BC and AC to two or three for the value $(AC - BC)$.

7.8 Cosine Law.[1]

A problem that frequently arises in the solution of triangles is the one in which two sides and the angle included between

[1] See Form 122 in the Workbook for a proof of Eqs. (7.8c), (7.8d).

them are given. If the data consist of four or more significant figures, the right-triangle solution should be used, as it is much the quickest method. For approximate results and for three-figure data the formula known as the *cosine law* is sometimes used. It is of considerable value in the study of vector problems in physics, mechanics, etc. The statement of the law is as follows:

$$\begin{bmatrix}\text{The square}\\ \text{of the un-}\\ \text{known side}\end{bmatrix} = \begin{bmatrix}\text{The sum of}\\ \text{the squares}\\ \text{of the given}\\ \text{sides}\end{bmatrix} - \begin{bmatrix}\text{The}\\ \text{product}\\ \text{term}\end{bmatrix} \tag{7.8a}$$

$$\begin{bmatrix}\text{The product}\\ \text{term}\end{bmatrix} = \begin{bmatrix}\text{Twice the}\\ \text{product of}\\ \text{the given}\\ \text{sides}\end{bmatrix} \begin{bmatrix}\text{The cosine}\\ \text{of the angle}\\ \text{included between}\\ \text{the given sides}\end{bmatrix} \tag{7.8b}$$

Thus, in letters,

$$(BC^2) = [(AC)^2 + (AB)^2] - 2[(AC)(AB)(\cos A)] \tag{7.8c}$$

Break the work into three steps: (*a*) The squares term, (*b*) the product term, and (*c*) the unknown side.

When angle A is wanted, solve Eq. (7.8*c*) for $\cos A$.

$$\cos A = \frac{(AB)^2 + (AC)^2 - (BC)^2}{2(AB)(AC)} \tag{7.8d}$$

The labor can be reduced considerably if tables of squares and roots are used and the calculations are made on the slide rule. This law should not be used in logarithmic computation because it requires several more logarithms and operations than are needed if the triangle is divided into two right triangles. Do not forget that the product term will be negative if the included angle is over 90°. Refer to Form 123 in the Workbook for arrangement of the calculations.

7.9 Areas of Triangles.

There are several ways of obtaining the area of a triangle, and the computer should choose the one that is closest to the given data or that requires the least computation in addition to that which he has already done in solving the triangle.

$$\text{The area of a triangle} = 0.5\,(\text{Base})(\text{Altitude}) \tag{7.9a}$$

$$= \frac{1}{2}\begin{bmatrix}\text{The product}\\ \text{of}\\ \text{two sides}\end{bmatrix}\begin{bmatrix}\text{The sine of the}\\ \text{angle included}\\ \text{between them}\end{bmatrix} \tag{7.9b}$$

$$= \begin{bmatrix}\text{The radius of the}\\ \text{inscribed circle}\end{bmatrix}\left[\left(\frac{1}{2}\right)\left(\begin{matrix}\text{The sum of}\\ \text{the sides}\end{matrix}\right)\right] \tag{7.9c}$$

$$= rs \tag{7.9d}$$

$$= \sqrt{s(s - BC)(s - AC)(s - AB)} \tag{7.9e}$$

7.10 Problem-analysis Outline.[1]

The following outline can be used to advantage by the inexperienced computer when he analyzes his trigonometry problems. No outline can serve as a substitute for initiative, imagination, perseverance, and courage. Only the computer himself can break his problem into its component parts, put in the necessary construction lines, and decide on the principles that apply to the problem in hand.

a. **Right triangles.**

1. Primary functions only are needed. Do not square any sides.
   (*a*) Sine function.
   Use when an angle (other than 90°), the hypotenuse, and a side opposite the given angle are involved.
   (*b*) Cosine function.
   Use when an angle (other than 90°), the hypotenuse, and a side adjacent to the given angle are involved.
   (*c*) Tangent or cotangent.
   Use when an angle and the legs of the triangle are involved.
2. To solve any right triangle one must know:
   (*a*) Two sides or
   (*b*) One side and one angle other than the 90° angle.
3. Area formula.
   Area = 0.5 (base) (altitude).

b. **Oblique triangles.**

1. All oblique triangles can be divided into two or more right triangles, and each right triangle should be solved with the use of the tools listed above. No construction line should be drawn so that it cuts the only known angle.
2. There are four cases based upon the data that are given. Each case can be solved in two or more ways but with varied efficiency of calculation. Only the preferred methods are mentioned below, arranged in the order of their speed in calculation and their relative freedom from possible mistakes.
3. In order to solve any oblique triangle, one must know three parts, one of which must be a side.

[1] Refer to Form 100 in the Workbook.

(*a*) Case I. Given: One side and two angles.
  (1) Get the third angle by subtraction.
  (2) Use the sine law to get the sides.
  (3) Right-triangle method. Draw one or two construction lines to form right triangles, then solve by using the primary functions as above.
  (4) Use Eqs. (7.9*a*) or (7.9*b*) to get the area.

(*b*) Case II. Given: Two sides and an angle opposite one of these sides (see Forms 111 and 112 in the Workbook).
  (1) Use the since law twice. First to get the angles, then again to get the third side.
  (2) Right-triangle method as in Case I.
  (3) Area. Same as Case I.

(*c*) Case III. Given: Two sides and the included angle.
  (1) The right-triangle method is the most efficient. Draw a construction line from a vertex (but not through the given angle), perpendicular to a side or a side extended, thus forming right triangles. Then solve using the principles of Part (*a*) above.
  (2) For three or less significant figures, the cosine law may be used. Make all calculations on the slide rule. This method is too slow for use with logarithms or longhand (see Form 123 in the Workbook).
  (3) Areas. Same as Case I.

(*d*) Case IV. Given: Three sides.
  (1) The half-angle, or radius, formula is by far the fastest method. It is excellent for use with logarithms (see Form 118 in the Workbook).
  (2) The whole-angle, or K, formula. It is somewhat slower than the radius formula, but a good tool. It is excellent with logarithms (see Form 116 in the Workbook).
  (3) Area. Use Eq. (7.9*d*), page 153, when the radius formula was used for angles. Use Eq. (7.9*e*) when the whole-angle formula was used.

CHAPTER 8

CURVE FITTING AND DERIVED CURVES

8.1 Introduction.

In analytic geometry students learn that it is possible to construct various lines, curves, and geometrical shapes when the equations are given. Unless students do more than read the assignments and go through the motions of solving a few problems, they will fail to realize that the usefulness of this branch of mathematics lies not merely in certain facts about "conic sections" but also in the relationships between experimental graphs and some of the standard types of equations. The engineer should recognize the straight line and its various forms of equation. He should be able to visualize the parabolic or hyperbolic types of curves and to write the equation of such a curve, even though he does not have any data regarding focus, directrix, etc. He may need to use considerable patience and ingenuity in finding the probable equation fitting some graphs.

8.2 The Straight Line.

The most useful form of the several straight-line equations is the one known as the "slope, y-intercept" form:

$$y = mx + b \tag{8.2a}$$

In this equation m is the slope of the line and b is its y intercept.

The "parallel-intercept" form is also of considerable use but is essentially the same as the slope form. This equation is

$$\frac{y - b}{y_1 - b} = \frac{x}{x_1} \tag{8.2b}$$

In this form, b is the y intercept, y_1 is the y ordinate when the abscissa is x_1. This form converts into the slope form by solving for y, thus:

$$y = b + \left(\frac{y_1 - b}{x_1}\right) x \tag{8.2c}$$

The general form is sometimes of value, and it likewise reduces to any of the other forms. It reads

$$Ax + By + C = 0 \tag{8.2d}$$

In this equation the y intercept b equals $\left(-\frac{C}{B}\right)$ and the slope m equals $\left(-\frac{A}{B}\right)$.

Now and then the engineer needs the angle between two straight lines whose slopes are known. If E is the desired angle, and m_1 and m_2 are the slopes of the lines, the angle is obtained by this formula:

$$\tan E = \frac{m_2 - m_1}{1 + m_1 m_2}$$

For other properties and equations of straight lines and normals consult a text on analytic geometry.

8.3 Fitting Equations to Straight-line Graphs.

When experimental data are plotted, it sometimes approximates a straight line. It then becomes necessary to determine the equation of the line that is the best approximation to the truth. The equation obtained in this manner is empirical until enough facts are at hand to enable one to develop and prove the equation by rational analysis. Three methods of determining the approximate equation that best fits the given data are outlined in this chapter. They differ in speed, precision, and convenience, so the method used should be chosen with due regard to the precision of the data.

a. **Method of selected points.** The easiest and quickest method to use is one based directly upon the graph obtained when the data are plotted. The worker places a transparent straightedge on the points and moves it about until it seems to him to best fit the group of points; then a straight line is drawn through the entire field of plotted values. This "best line" is one that passes through the maximum number of points, yet has about the same number of missed points to one side of the line as the other. The missed points should not be bunched with, say, three points below the line at one end and three above it at the opposite end.

When the best line has been drawn, it slope is determined, the y

intercept is read, and the slope form of the equation, $y = mx + b$ is written.

It is obvious that the precision of this method depends greatly upon the skill and judgment of the man who makes the constructions.

b. **The method of averages.** This method derives the equation of the line from the original data by calculations involving the "raw" data. No graph is plotted. Since it uses calculated values instead of drawing an average curve by eye as in the previous method, it usually gives results of higher precision than can be obtained by the method of selected points. The method is briefly as follows:

Step 1. Divide the data into two equal or nearly equal groups; then get the Σx and Σy for each group.

Step 2. Substitute these sums in the standard slope equation $y = mx + b$, thus:

$$\Sigma y = m\Sigma x + nb \tag{8.3a}$$

In this equation n is the number of observations or readings in the group. Two equations are thus obtained which are solved to get the value of m or b.

Step 3. Substitute this value in one of the equations, and solve for b or m.

Step 4. Write the equation of the line.

c. **Method of least squares.** Although the proof of this method depends upon an understanding of calculus, and hence is outside the field of the subject matter here, the actual operations are simple enough to justify inclusion of the method in this discussion of curve-fitting methods. This method also uses the slope form of the equation of a straight line. It is the most precise of the three methods and should be used on careful work. The operations are as follows:

Step 1. Get the totals, Σx and Σy.

Step 2. Square the x values, and get Σx^2.

Step 3. Compute the product of each x and its corresponding y value; then get Σxy.

Step 4. Substitute these sums in the slope equation $y = mx + b$ as follows:

$$\Sigma y = m\Sigma x + nb \tag{8.3b}$$
$$\Sigma xy = m\Sigma x^2 + b\Sigma x \tag{8.3c}$$

Step 5. Solve this set of equations for b or m.

Step 6. Substitute b or m in Eq. (8.3b), and solve for the other unknown.

Step 7. Write the equation of the line.

8.4 Curves of the Parabolic Type.

The family of curves classed as parabolic is probably of more general usefulness than any other. Research in many branches of science and engineering has indicated that an equation in this large family will best describe countless facts that have been discovered and organize them for practical application or further study. Typical formulas of this type are found in the study of motion problems, kinetic energy, hydraulics, beam analysis, certain heat problems, and numerous other branches of knowledge. The simple parabola studied in analytic geometry texts is but one member of this family. The most important facts, because they are useful, are those which enable the worker to identify curves and write equations for them. The average engineer probably will never construct a parabola, as such, in all his professional career. He will, however, frequently use the equations of curves that fall in the parabolic family.

This family of curves, otherwise known as the *power function*, $y = f(x)^n$, where n is positive, includes not only the standard equations of the parabola as listed below but such curves as the cubic and higher degrees and power polynomials. The exponent n must be positive; otherwise the curve falls into the hyperbolic family.

In the standard equations listed below, $0.5p$ is always the distance from the focus of the parabola to its vertex. The line that is perpendicular to the principal axis of the parabola at distance $0.5p$ from the vertex (or distance p from the focus) is called the *directrix*.

Three important cases are to be considered when discussing the second-degree parabola, each having two subcases.

Case I. The vertex is at the origin, and the principal axis of the curve is one of the coordinate axes.

a. The x axis is the principal axis of the curve.

$$y^2 = 2px \tag{8.4a}$$

b. The y axis is the principal axis of the curve.

$$x^2 = 2py \tag{8.4b}$$

Case II. The vertex is not at the origin, but the principal axis of the curve is on a coordinate axis.

a. The principal axis coincides with the y axis, but the vertex is at distance k from the x axis.

$$x^2 = 2p(y - k) \tag{8.4c}$$

b. The principal axis coincides with the x axis, but the vertex is at distance n from the y axis.

$$y^2 = 2p(x - h) \tag{8.4d}$$

Case III. The principal axis is parallel to one of the coordinate axes, and the vertex is at a distance from the other coordinate axis.

a. The principal axis is parallel to the y axis at distance h from it, and the vertex is at distance k from the x axis.

$$(x - h)^2 = 2p(y - k) \tag{8.4e}$$

b. The principal axis is parallel to the x axis at distance k from it, and the vertex is at distance h from the y axis.

$$(y - k)^2 = 2p(x - h) \tag{8.4f}$$

The two general forms of the equation for second-degree parabolas, corresponding to Eqs. (8.4e) and (8.4f), respectively are

$$x^2 + Dx + Ey + F = 0 \tag{8.4g}$$
$$y^2 + Dx + Ey + F = 0 \tag{8.4h}$$

8.5 Areas under Parabolic Curves.

Information about the areas under parabolic curves is of considerable importance to engineers. The various properties of the areas adjacent to a parabolic curve are of use in studies of beam or arch design, of motion problems and others. In many of the situations only the degree of the curve is known and nothing is known about its equation, focus, directrix, or principal axis. Under such conditions one needs to know how to get the areas when the information at hand is limited to the degree of the curve and a few ordinates in the curve.

The most general case is illustrated in Fig. 18, which shows a second-degree parabola cutting through a parallelogram. In the diagram, line AD is tangent to the curve at point A. AB may be at any angle with AD. Line $CD = h$ and is parallel to AB,

cutting the curve at any point C. Line BC is parallel to the tangent AD.

The second-degree parabola has the property of cutting the parallelogram into two areas, such that Area 1, known as the *parabolic spandrel*, equals half of Area 2, known as the *parabolic*

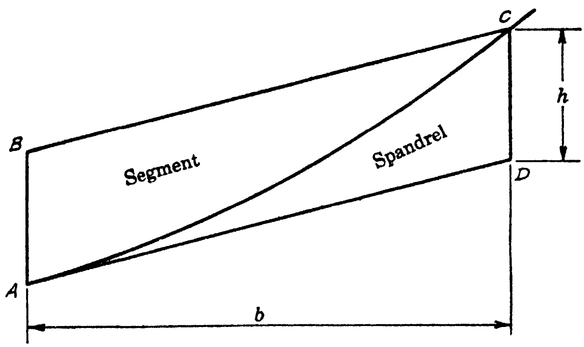


FIG. 18.

segment. If the perpendicular distance between the lines AB and CD is b, then the areas are as follows:

$$\text{Parabolic spandrel area} = \tfrac{1}{3}bh \tag{8.5a}$$
$$\text{Parabolic segment area} = \tfrac{2}{3}bh \tag{8.5b}$$

These formulas hold true for all values of angle BAD greater than 0° and less than 180°.

When the curve is a cubic parabola, the formulas become, repectively,

$$\text{Spandrel area} = \tfrac{1}{4}bh \tag{8.5c}$$
$$\text{Segment area} = \tfrac{3}{4}bh \tag{8.5d}$$

Generalized formulas can be written based upon the degree of the curve n as follows:

$$\text{Spandrel area} = \left[\frac{1}{n+1}\right] bh \tag{8.5e}$$
$$\text{Segment area} = \left[\frac{n}{n+1}\right] bh \tag{8.5f}$$

Certain other properties of the spandrel and segment will be found in Tables 3 and 4 in Chap. 12. They will prove useful in the work on derived curves discussed in Topic 8.13 and centroids and second moments discussed in Chap. 10, Topics 10.7–10.15 inclusive.

8.6 Fitting Equations to Parabolic Curves.

When the worker knows that his curve is not a straight line and he suspects that it belongs to the parabolic family, he has three approaches to his problem as follows:

a. **The method of selected points.** The quickest method of getting an approximate equation of a curve of the parabolic type is to plot x and y on logarithmic paper. If such paper is not at hand, then plot $\log x$ and $\log y$ on rectangular coordinate paper. If the plotted points are approximately on a straight line, the best line is drawn with a straightedge. The slope triangle is measured in any convenient unit, using the same scale for altitude and base; then the slope m is computed. The y intercept is read where $x = 1$ or $\log x = 0$. The equation of the curve can now be written $y = bx^m$, where b is the y intercept and m the slope of the straight line on the logarithmic paper.

When the points yield a curve on logarithmic paper but one feels certain that the curve is parabolic in form, there are still ways to rectify the curve. If the equation is of the form $y = bx^m + c$, the value of c may be found by successive trials or it may be computed. If the curve on logarithmic paper is concave upward, it can be straightened by subtracting a constant c from the values of y. Successive trials will give a close approximation to the true value. If the curve is concave downward, the value c is added to the respective values of y. The value of c can also be computed by reading, from the graph, the value of y_3 at the point where $x_3 = \sqrt{x_1x_2}$, then substituting the values of the ordinates in the following formula:

$$c = \frac{y_1y_2 - y_3^2}{y_1 + y_2 - 2y_3} \qquad (8.6a)$$

In the rather infrequent case where one is quite certain that the vertex of the curve lies on the y axis, the value of c is the y intercept of the curve as plotted on rectangular coordinate paper. It is usually advisable, however, to verify the value of c by calculation, using Eq. (8.6*a*) above.

b. **The method of averages.** In the method of selected points the best or mean line is determined by eye, and even the computed value of c depends upon a reading of y_3 from the curve; hence the final equation depends greatly upon the skill of the computer and the care that he used in doing the graphic work.

The method of averages, however, avoids the personal factor for an equation of the form $y = bx^m$ by using calculated values entirely. There are several steps in the calculations as follows:

Step 1. Tabulate the value of x and y.

Step 2. Tabulate the values of $\log x$ and $\log y$. Use the absolute values of the logarithms for any data values less than 1.

Step 3. Divide $\log x$ and $\log y$ into two equal or nearly equal groups, and get the subtotals.

Step 4. Substitute these subtotals in the standard equation

$$\log y = m\Sigma \log x + n \log b \tag{8.6b}$$

Step 5. Solve the simultaneous equations for m and b, and write the equation of the curve.

If the curve is of the form $y = bx^m + c$, one must determine the value of c as in the method of selected points, then proceed as indicated above, using $(y - c)$ instead of y.

c. **The method of least squares.** Just as in the case of the straight line, the most accurate way of determining the approximate equation of a parabolic curve is the method of least squares. The various operations for a curve of the form $y = bx^m$ is as follows:

Step 1. Tabulate the values of x and y.

Step 2. Tabulate the values of $\log x$ and $\log y$. Then get $\Sigma \log x$ and $\Sigma \log y$.

Step 3. Compute and tabulate the values of $(\log x)^2$ and the product $(\log x)(\log y)$.

Step 4. Get the totals $\Sigma(\log x)^2$ and $\Sigma[(\log x)(\log y)]$.

Step 5. Substitute the proper sums in the two standard equations

$$\Sigma \log y = m\Sigma \log x + n \log b \tag{8.6c}$$
$$\Sigma[(\log x)(\log y)] = m\Sigma(\log x)^2 + \log b\Sigma \log x \tag{8.6d}$$

Step 6. Solve these simultaneous equations for m and b, and write the equation of the curve.

If the curve is of the type $y = bx^m + c$, it will be necessary to determine c as in the method of selected points, then proceed as above. Remember that one uses $(y - c)$ in place of y in above tabular values and summations.

8.7 Exponential and Logarithmic Curves.

The third important family of curves is also of considerable interest to the engineer, because some of the common laws and

formulas assume either the exponential form $y = e^x$ or the logarithmic form $x = \log_e y$. This form of equation is sometimes called the *law of organic growth* or the *compound-interest law*. A large number of the formulas that describe the action of living organisms take on the exponential form when the relationships are put in mathematical equations. Usually the base for the logarithms will be found to be the value e, the base of the natural or hyperbolic logarithms. Sometimes labor will be saved on such calculations if Table 34 is used instead of changing all logarithms over to the base 10.

8.8 Fitting Equations to Curves of the Exponential Type.

When the data yield a curve on either rectangular or logarithmic coordinate paper and it cannot be straightened by introducing a constant c, as above, the equation may be of the exponential type $y = be^{dx}$. If the curve is of this form, the constants can be calculated and the equation be determined by three methods.

a. **Method of selected points.** This is again a graphical solution, and the resulting equation depends upon the skill and judgment of the worker for its reliability. The data are plotted on semilogarithmic paper. If they yield a straight line, the curve is of the exponential type, and the equation determined as follows:

Step 1. Throw the type equation $y = be^{dx}$ into the logarithmic form

$$\log y = (dx)(\log e) + \log b \tag{8.8a}$$

This equation can be put into the slope-intercept form, thus:

$$\log y = (d \log e)x + \log b \tag{8.8b}$$

corresponding to

$$y = mx + b \tag{8.8c}$$

for the straight line except that logarithms are involved.

Step 2. Read two ordinates on this line, y_1 and y, preferably the initial and final values. The value of b is read directly from the sheet, because it is on the logarithmic ruling.

Step 3. Get $\log y_1$ and $\log y$ and solve for the slope of the line

$$\begin{aligned} m &= d \log e \\ &= \frac{\log y - \log y_1}{x - x_1} \end{aligned} \tag{8.8d}$$

Since $e = 2.7183$ and $\log_{10} e = 0.4343$, the value of d can be computed:

$$d = \frac{m}{\log_{10} e} = \frac{m}{0.4343} \qquad (8.8e)$$

Step 4. The values of b and d are now known; so their values are substituted in the type equation, and the solution is complete. The equation should be checked by substituting some other value of x in it and solving for the corresponding value of y. See if this y value falls on the line and on the curve plotted on rectangular coordinate paper.

When the points yield a curve on semilogarithmic paper but one feels certain that the curve is exponential in form, it is possible to verify this assumption. If the equation is of the form $y = be^{dx} + c$, the value of c may be found by successive trials or it may be computed. If c is positive in sign, the curve is concave upward, it will be straightened when the proper value of c has been subtracted from the given values of y. If c is negative in sign, then the curve is straightened by adding the numerical value of c to each given value of y. The magnitude of c can be obtained by a few successive trials, or it can be calculated as follows: Read the value of y_3 from the graph at the point where $x_3 = 0.5(x_1)(x_2)$ Compute c:

$$c = \frac{y_1y_2 - y_3^2}{y_1 + y_2 - 2y_3} \qquad (8.8f)$$

b. **Method of averages.**

Step 1. Tabulate the values of x, y, and log y as for previous equation types.

Step 2. Divide x and log y into two equal or nearly equal groups, and get the subtotals.

Step 3. Substitute these subtotals in the standard equation, thus getting simultaneous equations, and solve for log b or d log e.

$$\Sigma \log y = (d \log e)\Sigma x + n \log b \qquad (8.8g)$$

Step 4. Write the equation of the curve, and test as above.

If the curve is of the form $y = be^{dx} + c$, calculate the value of c as in the method of selected points, then proceed as indicated above, using $(y - c)$ instead of y in all summations. Note that

the result depends upon the accuracy of the original graph because the value of y_3 is found only from the graph, not by calculation.

c. **Method of least squares.**

Step 1. Tabulate the values of x, y, x^2, $\log y$, and $x \log y$.

Step 2. Get the totals, Σx, $\Sigma \log y$, Σx^2, and $\Sigma(x \log y)$.

Step 3. Substitute these totals in the appropriate standard equation, and solve for $\log b$ and $d \log e$

$$\Sigma \log y = (d \log e)\Sigma x + n \log b \tag{8.8h}$$
$$\Sigma(x \log y) = (d \log e)\Sigma x^2 + \log b \Sigma x \tag{8.8i}$$

Step 4. Substitute these values in the type equation; then test as suggested above.

If the curve is of the form $y = be^{dx} + c$, calculate the value of c as in the method of selected points, then proceed as indicated above, using $(y - c)$ instead of y in all summations. Note that the result depends upon the accuracy of the original graph because the value of y_3 is found only from the graph, not by calculation.

8.9 The Harmonic or Periodic Type of Curve.

If the data is plotted on rectangular coordinate paper and yields a regular, wavy line resembling a sine curve, then one should suspect an equation of the form

$$y = a \sin bx \qquad \text{or} \qquad y = a \cos bx.$$

When a sine curve is plotted, it will be noted that it is symmetrical with respect to a horizontal line passed through the y intercept. The maximum value of y with respect to this line is called the *amplitude*, and the value of x from crest to crest of the waves is the *period*. The cosine curve is of the same shape as the sine curve but shifted one-quarter period to the right, because the cosine function is 0 when the sine is 1.

If the heights of the waves seem to be decreasing in such a manner that a smooth curve, resembling the logarithmic curve, is formed when the crests of the waves are connected, then one should suspect an equation of the type $y = (be^{-ax}) \sin (cx + g)$. This curve is known as the *damped harmonic* and is frequently encountered in electrical and mechanical problems.

A unique coordinate paper has been devised and can sometimes be purchased from the importers that has its rulings so arranged

that data plotted on it will yield a straight line for each quarter period if the equation is of the form $\sin y = ax^m$.

The slope of this line is the value m, and the displacement of the line right or left of a parallel line through sin 90° gives the value of a in the equation.

8.10 Curves of the Polynomial Type.

Sometimes one encounters a situation where none of the foregoing types of curves seem to apply. Perhaps the curve is best fitted by an equation of the polynomial type such as

$$y = a + bx + cx^2 + dx^3 \tag{8.10a}$$

In this type of equation any of the coefficients a, b, c, etc., may be positive, negative, or zero. Such an equation really represents the algebraic sum of several curves in which $y_1 = a =$ a constant, $y_2 = bx =$ a straight line, $y_3 = cx^2 =$ a parabolic curve, $y_4 = dx^3 =$ a cubic parabola, etc. Such an equation is, in fact, a concise plotting or mapping instruction for getting from the x axis to the curve. The equation $y = 2.4 - 1.2x + 2.4x^2 + 0.1x^3$, for example, is really saying

"To get from the x axis to the curve move upward 2.4 units, downward a length of $1.2x$ units, upward $2.4x^2$ units, and upward again $0.1x^3$ units to the curve."

A neat and highly practical approach to problems of this type is opened up in the topics on derived curves in this chapter. The polynomial equations are frequently solved swiftly and accurately by this method. In the following chapter a graphical method is described that will enable the worker to get the equations of many polynomial curves.

8.11 Miscellaneous Curves.

The preceding topics have discussed the curves that will be of greatest importance to the engineer. There are a few other curves whose typical equations will be given as found in texts on analytic geometry. Since they are of minor importance to the average worker, however, no suggestions on curve fitting will be given. These equations are as follows:

a. **Ellipse.** If the origin is at the center, long axis horizontal, short axis vertical,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{or} \qquad Ax^2 + By^2 = C \tag{8.11a}$$

a is the long radius, and b is the short radius.

If the origin is at the center, long axis vertical, short axis horizontal,

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \tag{8.11b}$$

If the origin is not at the center, long axis horizontal, short axis vertical,

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \tag{8.11c}$$

b. **The circle.** Since the circle is a special case of the ellipse in which the major and minor axes are equal or $a = b$, it will not be discussed here. The two most useful equations are shown below, but the circle is seldom encountered in curve-fitting problems.

If the origin is at the center and r = the radius,

$$x^2 + y^2 = r^2 \tag{8.11d}$$

If the origin is not at the center,

$$(x - a)^2 + (y - b)^2 = r^2 \tag{8.11e}$$

a and b are coordinates of the center.

c. **Hyperbola.** This curve is closely related to the parabola because the only difference in the equations is that m is positive for the parabola and negative for the hyperbola. The same methods of attack may be used in starting the determination of the equation. Thus, in the method of selected points, negative slope to any straight line obtained on logarithmic paper is a sign that the curve is hyperbolic in form. The four most useful equations are as listed below:

$$y = bx^{-m} \tag{8.11f}$$

If the origin is at the center, transverse axis horizontal, conjugate axis vertical,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \tag{8.11g}$$

a is the distance on the transverse axis from the center to the vertex, and b is the distance on the conjugate axis to the vertex.

If the origin is not at the center, transverse axis horizontal,

conjugate axis vertical,

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \tag{8.11h}$$

h and k are coordinates of the center.

In the above, if the transverse axis is vertical and the conjugate axis horizontal, y and x should change places.

If the asymptotes are the axes,

$$xy = \text{a constant} = \frac{a^2 + b^2}{4} \tag{8.11i}$$

8.12 Summary of Suggestions on Curve Fitting.

a. Plot the data on rectangular coordinate paper. If the best curve is a straight line, the equation is of the form

$$y = mx + b \tag{8.12a}$$

b. If this graph is a curve, examine it carefully and see if it resembles some known type.

c. If the curve goes through (0,0) or if it appears likely that the vertex is at the origin, plot x and y on logarithmic paper. If the curve is a straight line, then the equation is parabolic or hyperbolic, depending on the sign of m, and of the form

$$y = bx^m \tag{8.12b}$$

d. If the vertex of the curve, as plotted on rectangular coordinate paper, appears to be on the x or y axis and not at the origin, determine the y intercept c and plot x, $(y - c)$ on logarithmic paper. If this curve is a straight line, the curve is parabolic or hyperbolic, depending on the sign of m, and of the form

$$y = bx^m + c \tag{8.12c}$$

e. If none of the above suggestions work, then plot x and y on semilogarithmic paper. If this curve is a straight line, the curve is an exponential of the form

$$y = be^{dx} \tag{8.12d}$$

f. If this curve is not a straight line, solve for c and plot x and $(y - c)$ on semilogarithmic paper. If this curve is a straight line, then the equation is exponential of the form

$$y = be^{dx} + c \tag{8.12e}$$

g. Next try plotting x and $\left(\frac{y - y_1}{x - x_1}\right)$ on rectangular coordinate

paper. If this curve is a straight line, it is a second-degree parabola of the form

$$y = a + cx + dx^2 \tag{8.12f}$$

Also try the suggestions given in the next and following topics under the heading Derived Curves. This special approach may aid in solving the problem.

h. Check another possibility by plotting $\left(x, \frac{x}{y}\right)$ on rectangular coordinate paper. If the curve is a straight line, the curve is hyperbolic of the form

$$y = \frac{x}{a + cx} \tag{8.12g}$$

i. If the graph on rectangular coordinate paper is a curve and does not straighten out by any of the above devices, it may be of form

$$y = a + bx + cx^2 + dx^3 + cx^4 + \cdots kx^m \tag{8.12h}$$

Successive applications of the laws of derived curves may help to break this curve down to manageable form.

j. If the equation is of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{8.12i}$$

use the coordinates of six points and solve for A, B, C, D, E, and F in the equation. Check the results against several other points, and also plot the curve to see how it fits the original data.

8.13 Derived Curves.

In analytic geometry the student discovers that each of the geometrical shapes has its own distinctive algebraic equation. In the foregoing topics in this text he finds suggestions for getting the equations when the curves with which he is working do not have any easy, obvious solution, neatly fitted to textbook situations.

It does not seem to be recognized and appreciated by some students that various shapes of curves are related to each other and that given one curve, another can be derived from it easily and quickly by applying simple geometrical relationships. Even when this fact is mentioned in textbooks, it often "fails to register" in the student's mind and understanding. The straight line, the parabola, the third-degree parabola, and similar curves of higher degree are all mathematically and geometrically related,

one to the other, in what we can call the *parabolic family*. Thus, if given one of the curves or its equation, the others in the series can be derived from it. A knowledge of these relationships not only will aid in getting a clearer understanding of problems involving these curves but will also help to lay a sound foundation for the work in engineering and mathematics beyond the freshman year. It will then be found not only that the laws explained in this chapter apply to the parabolic family of curves but that they also can be extended to cover all sorts of equations and families of curves.

The parabolic family of curves is one of the most important to the engineer. The majority of engineering laws, formulas, phenomena, designs, and constructions make use of straight lines or parabolic curves. One study indicated that over three-fourths of engineering calculations involved equations in this family of curves. Since a clear visual picture of the relationships among some of the curves will help one to perceive the geometrical significance of any operations performed upon the equations, the following topics will develop the idea of derived curves.

8.14 Slope and Area Relationships.

In any given family of curves the successive curves are related to each other through two laws. The first law concerns an ordinate in one curve and the slope of a line tangent to the related curve at the corresponding ordinate. The second law states the relationship between the ordinates in one curve and the area between corresponding ordinates in the related curve. The derived curves must be considered in a definite sequence and should be arranged in the same sequence when the graphs are drawn. There are no exceptions to the laws that state these relationships, and the user does not need to fear that an exception to them must be considered. In certain cases the complete curve may consist of several sections such as inclined straight lines, horizontal lines, or parabolas. Each section has its own set of derived curves which, when completed, will lead to the solution of the whole series. There may, therefore, be many curves in the vertical bank for each section as well as several sections or banks.

If a few ordinates are known and it is also known that the curve is a straight line, or second-degree parabola, or some other curve in a series, it is possible to sketch the curves and eventually to

write the equations of them. The curves are not plotted but sketched only relatively to scale. Thus, if one knows that a final ordinate is larger than the initial ordinate, it is shown that way, but it is immaterial if exact proportions are not maintained. A curve that is concave upward is drawn that way, but it need not

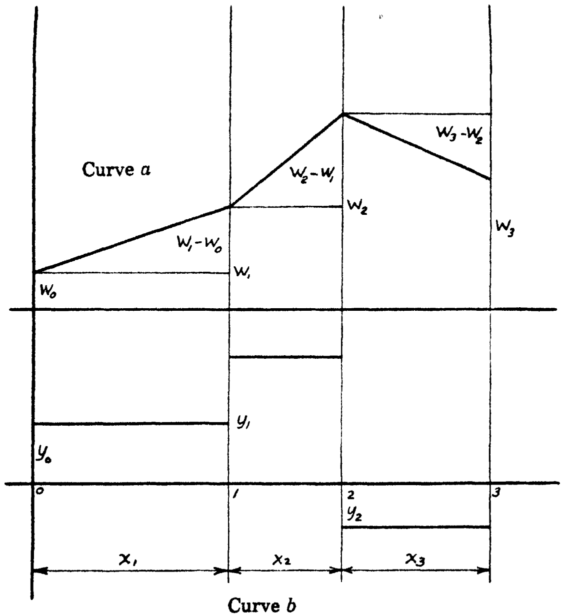


FIG. 19.

be constructed to scale. Since exact scaling is not essential, exaggeration is permissible and often desirable, especially when details have to be studied and dimensioned.

8.15 The First Law of Derived Curves.

The first relationship mentioned above is that which connects the slope of one curve to ordinates in its derived curve. This law enables one to sketch the shape of many curves before any of the numerical values have been computed. The first law states that

$$\left[\begin{matrix}\text{The } \textbf{slope} \text{ at } \textbf{any} \text{ Point in} \\ \textbf{any} \text{ continuous Curve}\end{matrix}\right] = \left[\begin{matrix}\text{The } \textbf{length} \text{ of the Ordinate} \\ \text{at the corresponding point} \\ \text{in the } \textbf{next lower} \text{ curve}\end{matrix}\right] \qquad (8.15a)$$

By "higher" and "lower" curves is meant both the degree of the curve and its position of the diagram sheet. In Fig. 19a,

page 172, is shown a curve consisting of three straight-line segments starting with an initial positive value. Each of these segments describes the situation that exists throughout the indicated x interval.

The next lower derived curve is constructed in Fig. 19*b* in accordance with this law. In the interval x, the slope of the first section of "Curve a" is $\frac{w_1 - w_0}{x_1} = y_0$. Since this slope is constant and positive, the value of y_0 is positive and constant. The only line that can indicate these two facts is a horizontal straight line drawn above the axis, as in Fig. 19*b*.

In similar fashion the slope of the second section of "Curve a" is seen to be $\frac{w_2 - w_1}{x_2} = y_1$; and since the slope is obviously steeper than in the first section, the value of y_1 is shown as another horizontal straight line in Fig. 19*b*, but higher than that drawn for y_0.

Figure 19*a* shows that the third section of "Curve a" has negative slope; that is, it points down to the right. The slope calculation would yield the same information. Since w_3 is less than w_2, the slope is $\frac{w_3 - w_2}{x_3} = y_2$ and is negative. This is shown in Fig. 19*b* by a straight line again, but now it must be drawn below the x axis, since y_2 is negative.

8.16 The Second Law of Derived Curves.

The relationship between the ordinates in "Curve a" Fig. 19 and the areas under "Curve b" in the same figure are inherent in the first law of derived curves but are more convenient to use if expressed as a second law, thus:

$$\left[\begin{array}{c}\text{The } \textbf{difference} \text{ in the}\\ \textbf{length} \text{ of } \textbf{any} \text{ two ordi-}\\ \text{nates in } \textbf{any} \text{ continuous}\\ \text{curve}\end{array}\right] = \left[\begin{array}{c}\text{The } \textbf{total net area} \text{ between}\\ \text{corresponding ordinates in}\\ \text{the } \textbf{next lower} \text{ curve}\end{array}\right] \tag{8.16a}$$

If in "Curve a" the slope is $\frac{w_1 - w_0}{x_1} = y_0$, then

$$(w_1 - w_0) = x_1 y_0$$

But if y_0 in "Curve b" is multiplied by x_1, the product is, of course, the area of the rectangle formed by the horizontal line

graph of y_0, the x axis, and the initial and final ordinates of the first section of "Curve b." Now $(w_1 - w_0)$ is only the change or difference in the length of the ordinates of "Curve a" for the first section; hence, the area in "Curve b" represents only this change and not the total ordinate w_1 at the end of interval x_1. To get w_1 one must include the initial condition w_0; hence,

$$w_1 = w_0 + \left(\begin{matrix}\text{Change in} \\ \text{the ordinate}\end{matrix}\right) = w_0 + (w_1 - w_0)$$

The second law can be used when going in either direction, up or down, in the bank of curves. For example, suppose that w_2 in "Curve a" is unknown but w_1 is known, also x_2 and y_1 in "Curve b." By the second law, the area x_2y_1 equals the change in ordinates $(w_2 - w_1)$ in "Curve a." Hence,

$$w_2 = w_1 + \left[\begin{matrix}\text{Area under the} \\ \text{next lower curve} \\ \text{from ordinate 1 to 2}\end{matrix}\right] = w_1 + (x_2y_1) \qquad (8.16b)$$

This second law should be applied after the shape of the bank of curves has been sketched, not before. It is the tool used for getting the numerical values of the various ordinates and is not used for determining the shape of the curves. That is the duty of the first or slope law. It should be emphasized that the second law gives the difference in the ordinates in the higher curve and not the total ordinate. In other words, it concerns the change in "Curve a" that occurs during the interval and in no way concerns the initial condition existing at the beginning of the interval.

There is a third law of derived curves which depends upon certain principles discussed in the following chapter. It is based upon the idea of moments of areas, but the mathematics of calculation is still in the realm of simple arithmetic. It is an excellent check method to use in verifying the results obtained with the second law. It also enables a computer to hurdle a curve now and then, going to the second higher curve without solving the intermediate curve.

8.17 Discussion of the Laws.

The two laws of derived curves are general and apply to any two or more continuous curves that are derived one from the other on the basis of the first law. The agreement between slope

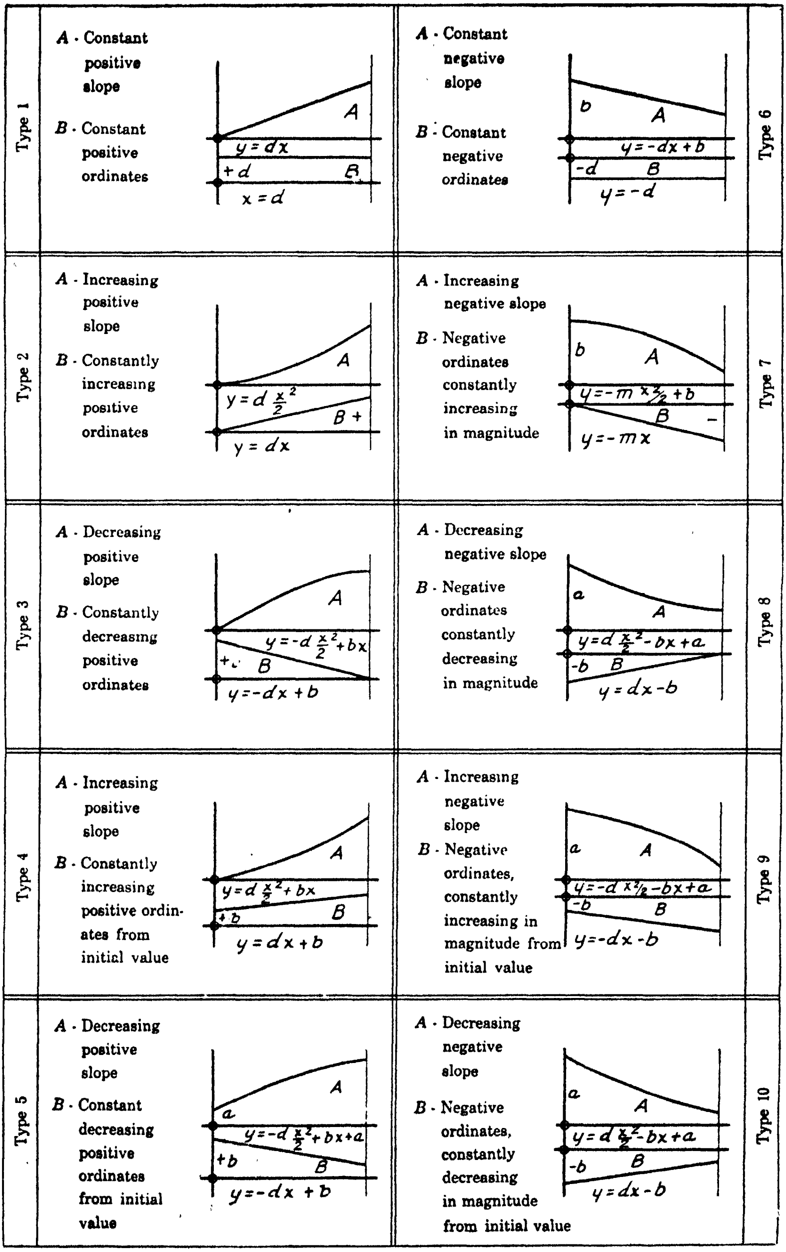


FIG. 20.—Alphabet of derived curves.

and ordinate is true for algebraic signs as well as for magnitudes. Thus, negative slope in the upper curve always requires negative ordinates in the lower curve, and increasing slope calls for increasing ordinates in the lower curve. A table of some of these paired facts follows and can be used as a guide by a beginner.

In Upper Curve		In Lower Curve
a. Positive slope	means	Positive ordinates
b. Negative slope	means	Negative ordinates
c. Constant slope (inclined straight line)	calls for	Constant ordinates (horizontal straight line)
d. Zero slope	means	Zero ordinate (line coincides with x axis)
e. Increasing positive slope (curve rising toward the right and concave upward)	requires	Positive ordinates increasing in magnitude
f. Decreasing positive slope (curve rising toward the right and concave downward)	requires	Positive ordinates decreasing in magnitude
g. Increasing negative slope (curve dropping toward the right and concave downward)	requires	Negative ordinates increasing in magnitude
h. Decreasing negative slope (curve dropping toward the right and concave upward)	requires	Negative ordinates decreasing in magnitude

Each of these pairs of relationships is shown in Fig. 20, page 175, *a* to *h* inclusive. A study of them should enable one to visualize and read a series of simple curves such as those in Prob. 321–353, inclusive, in Chap. 11. After what may be called the "alphabet of curves" (Fig. 20) has been learned, one can proceed to the more difficult problems such as Prob. 353. This type of analysis is especially helpful in the solution of various types of motion problems. These may concern the motions of machine parts in automatic machinery, the action of conveyors, or the construction of time schedules for the operation of transportation systems.

8.18 Notes on Construction of Derived Curves.

When semigraphic derived curves are used to assist in the analysis of any problem, the best plan is to lay out the curve sheet with the base lines and principal ordinates similar to Fig. 4, page 70, or to Form 215 in the Workbook. Do not attempt to draw any curves as yet.

Next, enter all given data on the curve sheet. This includes the values of all x intervals and all known ordinates, whether total or partial values. The partial value is always the change occurring during the interval. Draw any sections of the curve for which the shapes are known, such as straight lines and second-degree sections.

Apply the first law as the first operation, and draw the shapes of the other sections of the set of curves. When there is doubt as to whether or not any value drops to the base line, draw the affected sections lightly, subject to confirmation or change as calculations prove necessary. In general, the shapes of all sections of the set of curves should be completed before *any* numerical values are calculated.

Use the second law as the second operation for computing the values of all missing ordinates. Sometimes the known facts are such that a pair of equations with two unknowns will have to be solved. This is nothing to worry about and is quite a common occurrence. Generally speaking, this happens when the known values are found on the top and bottom curves. As a rule, time will be saved by solving for a value on the center curve rather than for an x interval or a value on the top or bottom curves.

Apply either simple mensuration formulas or analytic geometry to write the equations of the curves. When this has been done, it is apparent that the curves have aided one to go from a few initial or final ordinates and a knowledge of the type of curve involved to the algebraic equation of the lines concerned. Once all the equations are written, substitute the final value for x in the last interval and see if it checks the final ordinate in the highest curve.

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CHAPTER 9

SOME BASIC PRINCIPLES AND NOTES ON THEIR USE

9.1 The Primary Quantities.

The definitions and comments given in this chapter are arranged in the form of concise notes rather than detailed explanations. It is assumed that the necessary preliminary work has been done in the classroom during the discussion and solution of the various study problems. These notes, therefore, are not intended as a textbook for the introduction or development of the fundamentals, but for reference and review when the student is working on problems that involve them. Few symbol formulas have been given, the mathematical relations having been shown in the form of word equations. Words have been used because they reach the mind quicker than mere symbols, as symbols must be translated into words before they can be understood.

Distance, Time, and Force. Many of the review problems in Chap. 11 involve one or more of three fundamental quantities each of which has its own distinctive system of units. These three quantities are frequently referred to as the "primary quantities," as combinations of two or more of them give other quantities such as *work*, *power*, and *velocity*. Another grouping of quantities that is frequently used has *mass* instead of *force* as one of the primary quantities.

a. Distance. Distance is measured in length units such as inches, feet, miles, meters, etc. Areas, volumes, and space are all measured in distance units.

b. Time. Time is measured in seconds, minutes, hours, etc.

c. Force. Force is the action of one body upon another, changing or tending to change their shapes, relative positions, or relative velocities. Force is measured in pounds, tons, grams, etc. Various names are given to forces according to their source or effect.

The most common force is the attraction of the earth for all objects near it. This attraction is known as the *force of gravity.*

Weight is the measure of the force of gravity upon a body. It is a *pulling* force.

A *tension* is a force that pulls, tugs, or drags away from a body.

A *compression* is a force that pushes, thrusts, shoves, or presses against a body.

9.2 Other Definitions.

In the foregoing definitions the word *body* has been used several times, and other words will be used when referring to the properties or actions of bodies. Some of these terms will be defined now.

a. Bodies are definite aggregations of matter, distinct from all others.

b. Inertia is that property of a body by which it resists change of motion or position. To overcome this resistance requires the application of an external force.

c. Motion is the change in the relative position of one body with reference to another. When no change is occurring in the relative positions of two or more objects, they are said to be *at rest* with respect to each other.

d. Deformation is the change in the shape of a body due to the action of a force.

e. A *rate* is a comparison or ratio between two quantities.

f. The *specific gravity* of a liquid or a solid is the ratio of its weight to the weight of an equal volume of water (see Table 16, page 366). Gases are usually compared to an equal volume of hydrogen (or air).

g. The *density* of any substance is its weight per unit of volume. The usual unit is pounds per cubic foot. See Table 16 for the density of various substances.

9.3 Energy.

Energy is the capacity of a body to perform work. It appears in many forms, such as mechanical, electrical, or chemical, and may be changed from one form to another. Mechanical energy may exist in either of two forms:

Potential energy is the energy possessed by the body due to its position or deformation.

Kinetic energy is the energy due to the motion of the body. A body may possess both kinds of energy, and either form can be changed into the other. At all times the energy that a body contains depends upon the work previously done upon it.

9.4 Work.

Work is done on a body whenever a force moves a body against inertia or external resistance or whenever the shape of the body is changed. *Energy* must be expended to perform work. The measure of the work done is the product of the force causing the motion times the distance through which the point of application is moved in the direction of the force. When the proper values and directions are known, the mathematical statement of the relationship is given by the following word equation:

$$\text{Work} = (\text{Force})(\text{Distance}) \tag{9.4a}$$

Thus, work involves two of the primary quantities previously referred to, and its unit of measurement is a compound unit such as foot-pounds and ton-miles.

9.5 Power.

In practice we are concerned not only with the amount of energy expended in performing any task but also with the time in which it is performed, or in other words, the *time rate* at which the work is done. Time is a valuable factor in modern life; so we are accustomed to rate machines by their *power*.

Power is the time rate of doing work, or it may be thought of as the amount of work done in a unit of time. Thus, power is a comparison between the whole amount of work done and the time consumed in doing it. Power involves all three of the primary quantities, distance, force, and time, and is measured in a compound unit such as foot-pounds per minute or ton-miles per year. The mathematical statements showing the relationship of the quantities are:

$$\text{Power} = \frac{(\text{Work done})}{\begin{pmatrix}\text{Time required}\\ \text{to do the work}\end{pmatrix}} \tag{9.5a}$$

$$= \frac{\begin{pmatrix}\text{Distance that load}\\ \text{is moved}\end{pmatrix}}{\begin{pmatrix}\text{Time to move}\\ \text{the load}\end{pmatrix}} \begin{pmatrix}\text{Force required}\\ \text{to move load}\end{pmatrix} \tag{9.5b}$$

$$= (\text{Velocity})(\text{Force}) \tag{9.5c}$$

$$= \frac{\begin{pmatrix}\text{Force to move}\\ \text{the load}\end{pmatrix}}{\begin{pmatrix}\text{Time to move}\\ \text{the load}\end{pmatrix}} \begin{pmatrix}\text{Distance that}\\ \text{load is moved}\end{pmatrix} \tag{9.5d}$$

$$= \begin{pmatrix}\text{Quantity}\\ \text{rate}\end{pmatrix} (\text{Lift}) \tag{9.5e}$$

$$= \left(\frac{\text{Weight}}{\text{Time}}\right) (\text{Distance}) \tag{9.5f}$$

9.6 Law of Work and Energy.

When a force acts so as to change the velocity of a body, work is done by the force. Changing the velocity of a body makes a corresponding change in the amount of kinetic energy that it has. Any force tending to increase the velocity of a body adds to its supply of kinetic energy, and any force tending to reduce its velocity decreases its kinetic energy. The *work and energy law* is a statement of the relation between the work done and the resulting changes in the kinetic energy.

$$\begin{bmatrix}\textbf{The}\\ \textbf{initial}\\ \textbf{kinetic}\\ \textbf{energy}\end{bmatrix} + \begin{bmatrix}\text{Work done}\\ \text{by forces}\\ \textbf{tending to}\\ \textbf{increase}\\ \text{velocity}\end{bmatrix} - \begin{bmatrix}\text{Work done}\\ \text{by forces}\\ \textbf{tending to}\\ \textbf{decrease}\\ \text{velocity}\end{bmatrix} = \begin{bmatrix}\textbf{The}\\ \textbf{final}\\ \textbf{kinetic}\\ \textbf{energy}\end{bmatrix} \tag{9.6a}$$

All forces acting in the direction of the motion tend to increase the velocity. All forces acting in an opposite direction to the motion tend to decrease the velocity. Inertia is not an external force and has no tendency to change the velocity in any manner; hence, inertia does not appear in the work and energy law.

Energy and work are closely related, and one can be converted into the other. They are measured in the same units, such as foot-pounds, inch-pounds, etc.

Kinetic energy is usually expressed in terms of the weight and velocity of the body, but when computed it will be found that the kinetic energy is in work units.

$$\begin{bmatrix}\textbf{Kinetic}\\ \textbf{energy}\end{bmatrix} = \frac{(\text{Weight})(\text{Velocity})^2}{(2)(\text{Acceleration of gravity})} \tag{9.6b}$$

In most engineering computations the acceleration of gravity can be taken as 32.2 fpsps. Note that the velocity and the acceleration of gravity must always be taken in the same kind

of units, preferably feet and seconds. The weight can be taken in any convenient unit. The forces doing the work must be in the same units as the weight.

9.7 Newton's Laws of Motion.

Newton's laws of motion express the relationships between changes in velocity and the forces that produce the changes.

a. The inertia law. A body will remain at rest or in uniform motion in a straight line unless an external force compels it to change its position or velocity.

b. The acceleration law. While a body is having its motion changed by an external force, it receives an acceration that is proportional to that force and in the same direction.

c. The equilibrium law. The action of every force is opposed by an equal and opposite reaction.

It should be kept in mind that the first and second laws refer to external forces only. The third law, however, includes internal forces as well. According to this law, every force must have its equal and opposite reaction; and hence, when the external forces are not in equilibrium and the resultant force is causing a change in the motion of the body, the reactive force to this resultant must be sought within the body. It is recognized as a property of the matter in the body, because force must be applied to overcome the body's resistance to change of motion. This property is known as the *inertia of the body*, and the resistance that it offers is called the *inertia force*. The inertia force is always equal in amount and opposite in direction to the resultant of the external forces. The resultant of the external forces is frequently called the *effective force* because it is effective in overcoming the inertia force of the body. If the inertia force is included in equilibrium sketches with all of the external forces, in accord with d'Alembert's principle, the third law is obviously fulfilled, and the problem is reduced to one in statics.

The second law of motion gives the relation between the acceleration and the force effective in causing the change in motion. Since the forces acting on a body and their resulting accelerations are proportional, it is evident that their ratio is a constant. (In advanced work this constant is known as the *mass* of the object or sometimes as the *mass ratio*.) This relationship can best be shown in equation form, as follows:

$$\frac{\textbf{The effective force}}{\left(\begin{array}{c}\textbf{The acceleration}\\ \text{that it will produce}\end{array}\right)} = \frac{\textbf{Any other force}}{\textbf{Its acceleration}} \tag{9.7a}$$

When the acceleration given a body by any known force has been determined, the value of the ratio can be computed, and then the acceleration produced by any other force can be quickly found. The ratio most readily measured is that between the force of gravity acting on a freely falling body and the resulting acceleration. The force of gravity is the weight of the body as measured on a spring scale. The acceleration of gravity for various localities and altitudes has been determined many times with a high degree of precision. For ease in computation we use the second law in the following form rather than the preceding equation:

$$\frac{\text{The }\textbf{effective force}}{\left(\begin{array}{c}\text{The }\textbf{acceleration}\\ \text{that it will produce}\end{array}\right)} = \frac{\left(\begin{array}{c}\text{The }\textbf{force}\text{ of gravity,}\\ \text{its weight}\end{array}\right)}{\left(\begin{array}{c}\text{The }\textbf{acceleration}\\ \text{of gravity, } g = 32.2\end{array}\right)} \tag{9.7b}$$

Any unit may be used for the forces so long as the same unit is used for both sides of the equation. It is best to keep the accelerations in feet per second per second.

The ratio on the right-hand side of Eq. (9.7*b*) is known as the *mass ratio* or, more simply, as the *mass of the object.* It is represented by the letter m in physics and mechanics textbooks. When the conventional symbols are used, Eq. (9.7*b*) becomes $F = ma$, where F equals the effective force used to produce the change in velocity, m is the mass of the object or $\frac{w}{g}$, and a is the acceleration given to the object by the force F.

In general, use Newton's second law for the first solution of the problem when the acceleration is given or when the time and resulting change in velocity are known. Use the work and energy law for the check method. If the initial and final velocities, also the distance traveled, are given, then use the work and energy law for the first solution and Newton's second law for the check method.

9.8 Rectilinear Motion.

The work and energy law and Newton's laws of motion refer to velocities and accelerations. It is frequently necessary to know

the relation between acceleration, velocity, distance traveled, and the time. The following definitions and equations will give that information for problems involving *uniform* acceleration.

Velocity is the time rate of change of position. The rate may be either a constant or a variable. If constant, the velocity is said to be uniform since equal distances are being traveled in equal time intervals. If the velocity is either increasing or decreasing, the rate is not constant and the velocity is said to be varying. The following equations hold true provided the velocity is changing at a uniform rate:

$$\text{Average velocity} = \frac{\text{Total distance traveled}}{\text{Total time}} \tag{9.8a}$$

$$\begin{bmatrix}\text{Average}\\ \text{velocity}\end{bmatrix} = \frac{(\text{Final velocity}) + (\text{Initial velocity})}{2} \tag{9.8b}$$

Acceleration is the time rate of change of velocity. This rate may be either constant or variable. Acceleration may be either positive or negative, positive acceleration meaning an increase in the velocity and negative acceleration a decrease. Negative acceleration is also called *retardation* or *deceleration*. The following equations are true for uniform acceleration only:

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} \tag{9.8c}$$

$$= \frac{(\text{Final velocity}) - (\text{Initial velocity})}{\text{Time}} \tag{9.8d}$$

Equations (9.8*a*) to (9.8*d*) give some of the relations between distance, velocity, uniform acceleration, and time. Since they are mutually related, we may readily obtain other desired relationships between these quantities such as the ones below.

If Eqs. (9.8*a*, *b*, and *d*) are combined and solved for the distance traveled, we will get

$$\text{Distance} = (\tfrac{1}{2})(\text{Acceleration})(\text{Time})^2 + \begin{bmatrix}\text{Initial}\\ \text{velocity}\end{bmatrix}(\text{Time}) \tag{9.8e}$$

Then if Eqs. (9.8*d*) and (9.8*b*) are combined, we get a useful equation for determining the change in velocity when a certain acceleration has been given to an object for a given distance of travel.

$$\begin{bmatrix}\text{Change in}\\ \text{velocity}\end{bmatrix}^2 = (2)(\text{Acceleration})\begin{bmatrix}\text{Distance}\\ \text{traveled}\end{bmatrix} \tag{9.8f}$$

9.9 Friction.

Whenever an object moves or tends to move across the surface of another, by either sliding or rolling, there is a resistance to motion which is called *friction*. Friction always opposes relative motion between the surfaces. It always tends to decrease the relative velocities of the two surfaces. Thus the friction between a wheel and a brake shoe is used to help stop a train or an auto. Looked at in another way, it can be said that friction tends to *equalize* the velocities of the two surfaces in contact. This effect is used in the case of belt drives and in friction clutches such as are used in automoblies.

Frictional resistances are usually classified under three heads:

a. Static friction is the resistance to motion, other than inertia, that must be overcome in order to *start* one body moving across the surface of another.

b. Sliding friction is the resistance to motion, other than inertia, that must be overcome in order to *maintain relative motion* between the two surfaces in contact.

c. Rolling friction is the resistance to motion, other than inertia, that must be overcome in order to keep one body *rolling* over the surface of another.

In all three cases the main factors governing the amount of the friction force are

a. The nature of the surfaces.

b. The pressure between them.

For static and sliding friction the frictional resisting force may be expressed as a decimal fraction of the normal pressure between the surfaces. This fraction is called the *coefficient of friction.*

$$\begin{bmatrix}\text{Friction}\\ \text{force}\end{bmatrix} = \begin{bmatrix}\text{A}\\ \text{coefficient}\end{bmatrix}\begin{bmatrix}\text{The normal}\\ \text{pressure}\end{bmatrix} \tag{9.9a}$$

The normal pressure is the pressure perpendicular to the surfaces at the point of contact. The friction force is perpendicular to the normal pressure and, hence, is always *tangent* to the surfaces at the point of contact. The coefficient of friction gives the ratio of the friction force to the normal pressure. Its value depends upon the substances in contact, condition of the surfaces, and for high speeds, upon the relative velocities. For moderate pressures and low velocities the following laws are true:

a. Frictional resistance is independent of the areas of the surfaces in contact if the total normal pressure is constant.

b. Friction is directly proportional to the normal pressure.

c. Friction is independent of the velocity for low speeds but decreases as the speed increases.

d. Sliding friction is usually less than static friction.

Rolling friction is computed in various ways; but in the consideration of moving trains, mine cars, automobiles, etc., it is usually given as the number of pounds per ton of weight that must be exerted to keep the object moving and is called *traction resistance.* Equation (9.9*a*) can be written in this form for traction resistance.

$$\begin{bmatrix}\textbf{Traction}\\ \textbf{resistance}\end{bmatrix} = \begin{bmatrix}\textbf{Unit}\\ \textbf{friction}\\ \textbf{force}\end{bmatrix}\begin{bmatrix}\textbf{Total}\\ \textbf{weight}\end{bmatrix} \qquad (9.9b)$$

Values of the friction coefficients are given in Tables 25, 26, and 27, pp. 379–381.

9.10 Efficiency.

Owing to friction in various parts of its mechanism, the final output of work from any machine is always less than that put into it. The *efficiency* of a machine is the percentage of the power input that is obtained as useful power. It is always less than unity and is usually expressed in percentage.

$$\begin{bmatrix}\text{The}\\ \textbf{horsepower}\\ \textbf{output}\end{bmatrix} = \begin{bmatrix}\text{The}\\ \textbf{efficiency}\end{bmatrix}\begin{bmatrix}\text{The}\\ \textbf{horsepower}\\ \textbf{input}\end{bmatrix} \qquad (9.10a)$$

Where the power passes through a series of machines or energy is changed from one form to another, the first output is the input for the second step, and the output of the second is the input of the third, and so on through the series. The net efficiency of the equipment will be the product of the efficiencies for the various steps.

9.11 Resolution of Forces.

In many problems that arise in engineering it becomes necessary to study the effect of a force that acts at an angle to a surface. The computations are usually simplified and the problem made easier to visualize if the original force is replaced by two other

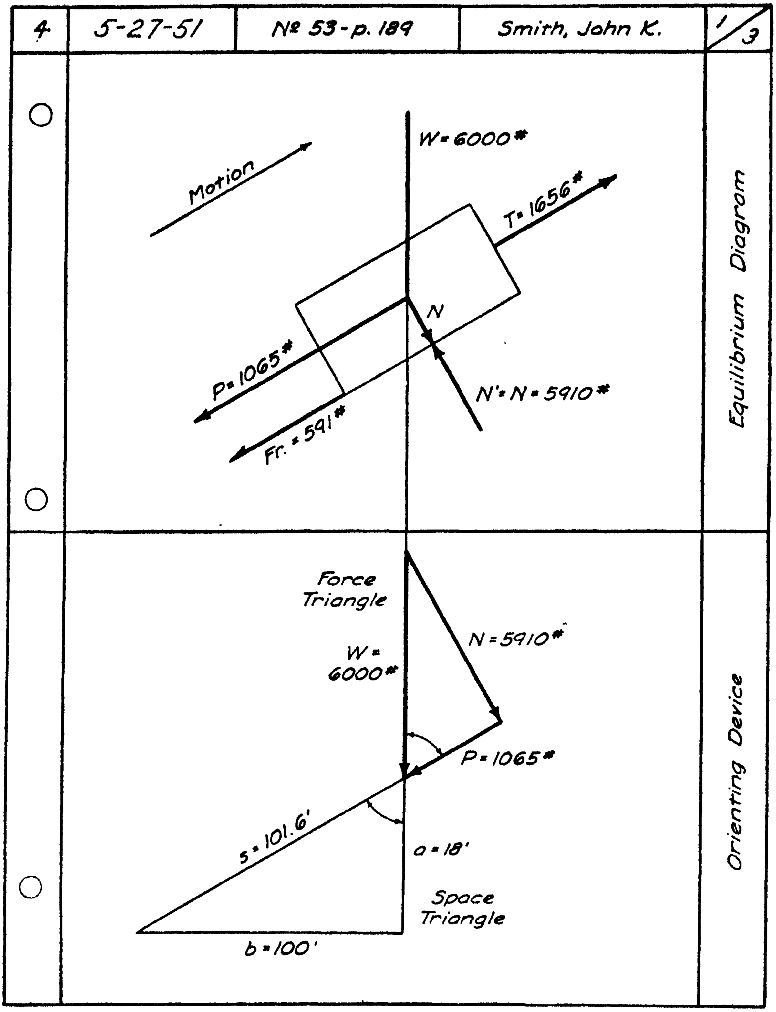

FIG. 21.—Resolution of forces.

forces: one parallel, the other perpendicular to the surface, and the two having the same combined effect as the original force (see Fig. 21, above). The two forces are called the *components* of the original force, and determining the value of the components is known as *resolving the force into its components.*

There are four principal cases:

a. A *vertical* force resolved into components parallel to and perpendicular to an inclined surface.

b. A *horizontal* force to be resolved into components parallel to and perpendicular to an inclined surface.

c. A force *parallel to an inclined plane* to determine its horizontal and vertical components.

d. A force *normal to an inclined plane* to find its horizontal and vertical components.

Inclined forces acting against horizontal or vertical planes are merely special cases of *c* or *d*.

9.12 Computation of Components.

To determine the value of the components of a force in any of these cases, use should be made of what has been called the *orienting device* because it is invaluable in determining the proper directions of the forces. It consists of a *space triangle*, which gives the slope of the plane or inclination of the resultant force, and a *force triangle*, which shows the direction and relative magnitude of the forces involved. The space triangle shows distance only and is usually in the same position for all four of the cases mentioned, but the force triangles (showing forces only), are all different, as the sides of the force triangle *must* be parallel to the corresponding force arrows in the equilibrium sketch. The equilibrium sketch and orienting device are arranged as shown in Fig. 21, page 188.

Directions for drawing the orienting device.

a. Draw the original force and its components on the equilibrium sketch as shown at W, N, and P, in Fig. 21.

b. Draw a light vertical line down the page from the point where the original force is shown as splitting into its components. This locates and forms one side of the space diagram.

c. Draw the space triangle, showing the slope of the incline or of the original force.

d. Draw the force triangle. Each side in it must be parallel to the corresponding force arrow in the equilibrium sketch. Extensions of the sides of the space diagram form the basis of this triangle. Use heavy lines for force arrows.

e. Solve for the value of the unknown components. The space and force triangles are always similar; hence, the unknowns are found by proportion.

9.13 Equilibrium.

A special application of the third law of motion is seen in the case of bodies that are either at rest or moving with uniform velocity. Under either of these conditions the external forces oppose each other in such manner that no change in the motion of the body occurs; if at rest it continues at rest; if in motion it continues moving in the same direction with the same speed. The external forces are therefore said to be *in equilibrium*. So many situations exist in which it is necessary to study the action of such forces that they are commonly grouped together under the name *statics*. In discussing problems in statics it is customary to classify the forces according to the direction and number acting. Some of the terms used and their meanings are as follows:

Concurrent forces are those which intersect at one point.

Nonconcurrent forces do not intersect at a common point.

Coplanar forces are all in one plane.

Noncoplanar forces are in several planes.

Parallel forces have parallel action lines.

Collinear forces have action lines that coincide.

A *couple* consists of two equal and opposite forces with parallel action lines. A couple always tends to cause the body to rotate.

There are several tests that must be applied in order to determine if a body is in equilibrium. The third law of motion says that the action of every force is opposed by an equal and opposite force. In order to apply this law readily its principle is reworded in the two following statements:

a. The algebraic sum of all the forces acting on a body must equal zero.

$$\Sigma F = 0 \tag{9.13a}$$

b. The algebraic sum of the rotating tendencies of all forces must equal zero.

$$\Sigma M = 0 \tag{9.13b}$$

The first condition concerns the magnitude of the forces and their directions. It is usually expedient to resolve the forces into components in two or three directions. When this is done, the summation law becomes

$$\left[\begin{array}{c}\text{The } \textbf{algebraic sum} \text{ of the} \\ \text{components of } \textbf{all} \text{ forces} \\ \text{acting on a body, taken} \\ \text{along } \textbf{any line}\end{array}\right] = 0 \qquad (9.13c)$$
$$\Sigma F = 0$$

When the forces are resolved into components acting parallel to three coordinate planes, it is necessary to apply the test three times to see if the following conditions, expressed in symbol form, are true:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \qquad (9.13d)$$

The second condition stated above, $\Sigma M = 0$, concerns the magnitude of the tendency of each force to cause the body to rotate. The body may not actually rotate, but each force *tends* to cause such motion. For equilibrium, therefore, the following must be true:

$$\left[\begin{array}{c}\text{The } \textbf{algebraic sum} \text{ of the} \\ \textbf{moments} \text{ of } \textbf{all} \text{ the forces} \\ \text{acting on a body, with} \\ \text{respect to } \textbf{any} \text{ axis}\end{array}\right] = 0 \qquad (9.13e)$$
$$\Sigma M = 0$$

The measure of the tendency of a force, or couple, to cause rotation is called its *moment*. It is the product of a distance and the magnitude of the force. The line that indicates the position and direction of a force in space is called the *line of action* of the force. It is of unlimited length. The axis about which the rotating tendencies are to be computed is the *center of moments.* The perpendicular distance from the line of action of the force to the center of moments is called the *lever arm* of the force. The product of the lever arm and the force is the *moment* of the force about the given center of moments. The last two definitions are more readily used if put into equation form as below:

$$\left[\begin{array}{c}\text{The } \textbf{lever} \\ \textbf{arm} \text{ of} \\ \text{a force}\end{array}\right] = \left[\begin{array}{c}\text{The } \textbf{perpendicular distance} \text{ to} \\ \text{the } \textbf{line of action} \text{ of the force, from} \\ \text{the center of rotation}\end{array}\right] \qquad (9.13f)$$

$$\left[\begin{array}{c}\text{The } \textbf{moment} \\ \textbf{of a} \\ \textbf{force}\end{array}\right] = \left[\begin{array}{c}\text{The} \\ \text{force}\end{array}\right]\left[\begin{array}{c}\text{Its} \\ \textbf{lever} \\ \textbf{arm}\end{array}\right] \qquad (9.13g)$$

If the forces are resolved into components in two or three planes, it becomes necessary to apply the moment summation law to each set of components in turn. If the body is in equilibrium, the following is true:

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \qquad (9.13h)$$

If the computer remembers that the three laws of motion apply to every problem involving forces, and that the above laws of equilibrium apply especially to all stationary structures, he will find his approach to the analysis greatly simplified. It is true that texts and handbooks divide the situations into a great many special cases, but they can all be analyzed by applying the simple fundamental laws as given above. Even a problem involving accelerated motion can be reduced to a problem in equilibrium, in accordance with d'Alembert's principle, provided the inertia of the body be shown as an inertia force.

CHAPTER 10

GEOMETRICAL AND GRAPHICAL APPLICATIONS OF CALCULUS

10.1 Calculus as a Way of Reasoning.

The following notes and suggestions are not intended to take the place of a text in calculus but rather to give a few of the more important applications and to restate certain principles from the engineering viewpoint. This branch of mathematics should not be regarded as a mathematical tool for use only in research or, as some conceive it, a vague subject that teaches hard ways of doing easy tasks. On the contrary, the calculus not only enables us to obtain exact answers to problems that otherwise would have to be solved by trial and error or other long methods but also includes a distinctly different type of reasoning, a mental approach to problems not found in more elementary mathematics. The engineer is constantly handling problems that involve the ideas of change, of growth, of variation, of rates. Now calculus is about just such things; but because the computer may not know the algebraic equation involved, he often assumes that calculus cannot be used in analyzing such problems.

The arrangement of the material in the calculus texts naturally leads the student to think of the calculus as being divided into many distinct parts. He spends a certain number of weeks studying the differential calculus; then he spends some more time on integral calculus, finishing with differential equations. As a matter of fact, the two principal operations are complementary, the same as addition and subtraction or multiplication and division. One is finding the rate of change of some variable; the other is determining the total change. Integral calculus is concerned with the finding of a sum, but differential calculus has to do with determining the elemental quantities that yielded the given total. Throughout the entire subject of calculus the student is really working with differential equations; in fact he cannot begin to reason in this branch of mathematics without

thinking in terms of differential quantities, whether they are called that or not.

It is probable that much of the trouble that most computers have with the calculus is due as much to their fear of it and to lack of systematic methods as to their failure to understand the subject. Because they rely more on memory than on reason, they tend to become panic-stricken when they fail to remember formulas. Then they excuse themselves by saying that "the calculus isn't practical, anyway." A very noticeable fault in most of the men who use calculus infrequently is their habit of trying to set up their problem in one complete equation at one operation, no matter how involved it may be. They do not realize that the effective reasoning about the problem comes in the development of the equation, step by step, beginning with the simplest description of the required property of the elemental quantity, and not after the signs of integration or differentiation have been written. Integration, for example, is a routine, more or less mechanical process having little or nothing to do with the reasoning about the fundamental principles of engineering. Problems involving double or triple integration can be simplified by breaking them up into two or three distinct integration statements, each one consisting of a single, simple summation. Since accuracy is far more to be desired than rigid adherence to the textbook technique of the man who is using calculus day after day, the average computer should not hesitate to adopt all the devices and methods that will safeguard him from possible mistakes.

10.2 Classifying the Problem.

If the many problems to which some phase of the calculus way of thinking might be applied are grouped according to the nature of the data, it will be noticed that they fall into one of the following cases:

Case I. The algebraic equation is known. Use the methods of formal calculus. When the equation that describes the relationships between the variables is completely known, the underlying laws are also known, and hence all of the operations of calculus may be applied with exactness.

Case II. The equation is not known, but its form is known. The data consist of a few experimental constants. Use semi-

graphical methods. A large number of problems are of the type in which the equations are unknown but the laws and principles involved are definitely known. If the geometrical relationships of differentiation and integration are used it is often possible to solve the problem completely and to determine the descriptive equations in a simple, direct manner.

Case III. The equations are unknown, but some definite law connects the variables. A series of experimental readings is available from which a graph may be plotted. Use the methods of graphical calculus. The basic laws describing the phenomena may or may not be known or suspected, but in either case the operations of differentiation and integration may be performed by purely graphic methods, such as are suggested in Topics 10.23 and 10.25.

Case IV. No law or equation is known or probable. Data consist of experimental readings of two independent variables. Use graphical methods or approximate analytical methods. Frequently only pure chance relationships exist between the variables, or else the possible equations are so complex that it is not possible to determine them. For such situations the graphic calculus may be used or else approximate methods such as Simpson's rule, as given in Topics 10.30–10.32.

Case V. The equation is fully known, but its accurate analysis is not feasible. Use graphic or other approximate methods. In a few problems the exact mathematical equation may be known, but the accurate analysis may be too complex, or its computation too costly in time or money. In an occasional case, the integrals of the given equation may never have been determined. In such situations the methods suggested in Case III and Case IV may be used to good advantage.

10.3 The Three Principal Types of Equations.

The large number of differential and integral forms that are found in texts and handbooks lead one to imagine that it is not possible to make a simple classification of calculus problems. Such is not truly the case, however, for it has been said that there are only three broad families of equations, or laws of change, occurring in nature which the engineer will commonly meet. He will find that he can usually place the function that he is studying into one of these three classes:

a. **The parabolic law.** This group includes all of the conic sections and may be represented by equations of the form

$$y = bx^m + C \tag{10.3a}$$

b. **The harmonic law or periodic law** includes the trigonometric curves and is represented by functions of the periodic type, such as

$$y = a \sin (bx + c) \tag{10.3b}$$

c. **The law of organic growth.** Also called the compound interest law, logarithmic law, or exponential law. The equations may be in the form

$$y = be^{dx} \qquad \text{or} \qquad \log_e \frac{y}{b} = dx \tag{10.3c}$$

Sometimes combinations of two or three of these may be encountered. The typical equation for a damped vibration is an illustration

$$y = (be^{-ax})[\sin (cx + g)] \tag{10.3d}$$

It is very probable that the average engineer will find that by far the greater number of the functions that he uses will fall into the first group. This is not saying that all of the equations will be easy to solve; some may be simple, some may be very complex and difficult to handle; but the general method that will solve one problem will at least help in attacking another in the same group.

10.4 Setting Up the Problem.

Problems falling under any of the five cases previously mentioned will be approached with considerably more confidence if the computer will adopt a rather definite procedure in setting up his problems and in their consequent solution. He should not be hasty in trying to throw his work into the conventionalized form of the textbooks. He should organize his thoughts and work somewhat as follows:

a. **Endeavor to get a clear idea of what the problem is about.** What is wanted and why?

b. **Draw a diagram when possible.** Many problems concern facts that can be put into a diagram or simple graph in such manner as to be grasped more readily. This diagram should show all of the essential information, such as axes, curves, limits,

variables, elemental quantities, dimensions, and units (see Fig. 7, page 74).

c. **Determine the nature and number of the variables.** Decide which will be dependent and which independent.

d. **Determine which one of the five cases** listed above properly describes the given data.

e. **Use the computation process applicable** to the given problem, and solve the problem.

f. **Put the answer** into the simplest and most practical form.

10.5 The Formal Calculus (Case I).

If the data are such that the algebraic equation describing the relation between the variables is given or can be determined readily, then the formal calculus applies. This will include all such problems as determining rates of change, maxima and minima, centroids, moments of inertia, and other properties of shapes and masses. This group includes all problems concerning the properties of composite shapes such as built-up girders, flywheels, and machine frames. A vast number of problems in mechanics and design call for the application of formal calculus.

The suggested classification of problems as given in Topic 10.3 above will help the computer to plan his solution.

The engineer generally uses differential calculus for determining rates of change or for finding the maximum and minimum values of a variable. To find the rate of change is merely to determine the ratio of the change in the dependent variable to a specified change in the independent variable. When a graph of the given function is drawn, this rate of change is the slope of the graph. If the algebraic equation is given, the calculus enables one to figure the exact value of this ratio, whereas other methods give merely the approximate value. Determining this rate of change is called *taking the first derivative.* Maximum and minimum points are the points where the first derivative is zero and changes sign.

Without doubt, integral calculus causes the most trouble. If integration is thought of as being the summation process or determining the total change in a variable when the rate of change is given, it will be easier to see the objective of the computations. A way of visualizing it is to think of it as the area under the graph of the derivative curve. Integration is truly

a summation process. In fact, the integral sign is just the old-time, long form of the letter s made more or less ornamental by the type founders. It corresponds to the symbol Σ that is also used as a sign of summation. Finding the area under a curve, for example, requires, first, a summing up of elemental areas to get the area of an elemental strip and, second, a summation of the strips to get the total area. This principle can be applied to all integrations. Experience may be the only guide to choosing the proper integral forms, but many mistakes are made long before the computer is ready for the integration. They occur because he attempts to set his equation up in final form ready for integration without carefully developing it from the simple statement of the rate of change of the variables. Because few, if any, of the texts give a suggestion regarding methods of organizing computations, the average computer blindly imitates the symbolized statements of principle that he finds in the book. The truth is, however, that reasoning about the facts, quantities, and principles that are involved must precede any statement of the mathematical process to be performed. The computer who is "rusty" on his calculus will be wise to follow the suggestions in Topic 3.14, page 73.

10.6 Moments.

When students first meet the word *moment* in an engineering class, they are often confused. It plainly has a technical meaning that has no connection with time, as it usually does in everyday conversation. They are also somewhat bewildered when they discover that even in engineering the word may be applied, and correctly, to widely differing things. In general, the name *moment* is given to a particular group of mathematical products that may or may not be visualized, as the action of forces, the movement of bodies, and similar physical events may be. The idea of moments, however, is one of the most valuable concepts used by the engineer. Nearly all problems dealing with forces or the strength of materials involve this idea in some manner.

A *moment* is the product of some quantity and some function of its distance from a point, line, or plane. The distance is called the *moment arm*, but movement or rotation is not necessarily implied, because the product is usually no more than a

mathematical abstraction that cannot be visualized or represented by sketch or diagram. There are many kinds of moments. However, the following outline suggests only some of the most common.

a. Moment of force—no rotation necessarily implied.
b. Torque—rotation is implied.
c. Moment of impulse.
d. Moment of momentum.
e. First moment of:
  1. Lines.
  2. Areas.
  3. Volumes.
  4. Masses.
f. Second moment of:
  1. Lines.
  2. Areas.
  3. Volumes.
  4. Masses.
g. Higher moments.
h. Product moments.

When the first power of the moment arm is used to get the moment, the product is known as a *moment of the first order* or merely *first moment*. An older but not so accurate a name is *static moment*. If the square of the moment arm is used, the product is a *moment of the second order*, or *second moment*. Through an unfortunate analogy, itself based upon a mistaken idea, the name *moment of inertia* was given to the group of second moments early in the development of the science of mechanics. Since inertia does not enter into the computations in any way, especially when areas are involved, the name is meaningless and confusing. The more accurate term *second moment* is much to be preferred but on account of long usage and the reluctance of many men to abandon old customs, the name *moment of inertia* will be with us for a long time to come. Occasionally, the distances from two perpendicular axes are used in the moment, and the name *product moment* is used for the resulting value. Through an extension of the poor naming previously referred to, the name *product of inertia* is often used for this quantity, which is purely a mathematical abstraction.

The users of these moments should avoid trying to visualize most of them. Torque and moment of force may be pictured, but in most cases nothing is gained by such efforts.

10.7 First Moments.

To obtain the first moment of a line, area, volume, or mass with respect to a chosen moment axis, imagine the shape to be divided into elemental parts whose moment arms differ infinitesimally. The product of the element and its moment arm is the first moment of the element. Moment arms to one side of the axis are considered positive, and on the opposite side, negative. The first moment of the entire shape, with respect to the given axis, is the algebraic sum of the moments of all the elements.

Thus

$$\begin{bmatrix}\text{The first}\\ \text{moment of}\\ \text{an elemental}\\ \text{area}\end{bmatrix} = \begin{bmatrix}\text{First power of}\\ \text{the moment arm}\\ y\end{bmatrix}\begin{bmatrix}\text{The}\\ \text{elemental}\\ \text{area } dA\end{bmatrix} \tag{10.7a}$$

and in symbols

$$= ydA \tag{10.7b}$$

The total first moment of the area is the sum of the moments of the elements, or

$$\begin{aligned}\begin{bmatrix}\text{Total}\\ \text{moment}\end{bmatrix} &= \int ydA\\ A\bar{x} &= \int ydA\end{aligned} \tag{10.7c}$$

10.8 Centroids of Plane Areas.

A centroidal axis is a straight line lying in a given plane area in such position that the algebraic sum of the first moments of all the elements in the area, with respect to it, is zero. Although there may be an infinite number of lines fulfilling this requirement, as a rule only two or three centroidal axes, perpendicular to each other, are used in practical work.

All centroidal axes intersect at a common point known as the *centroid* of the figure.

The following remarks on locating the centroidal axes of plane areas will illustrate the principle to be applied in similar manner to the locating of centroids of lines, volumes, and masses.

Because the moment of the total area must equal the sum of the moments of its elemental areas, we have a tool for use in

finding the location of a centroidal axis running in any desired direction. The first step is to choose a reference axis parallel to the desired centroidal axis. The reference axis is usually the coordinate axis, or often one side of the area of which the location of the centroid is desired. Next the total area is computed, and third, the summation of the moments of its parts with respect to the chosen reference axis is obtained. When these values, total moment and total area, are known, it is a simple operation to solve for the location of the desired centroidal axis.

Putting these facts into word-equation form we have

$$\begin{bmatrix}\text{The moment of}\\ \text{the total area}\end{bmatrix} = \begin{bmatrix}\text{The sum of the}\\ \text{moments of its}\\ \text{partial areas}\end{bmatrix} \tag{10.8a}$$

$$\bar{x}A = \Sigma[M_1a_1 + M_2a_2 + M_3a_3 \cdot \cdot \cdot] \tag{10.8b}$$

Or, in algebraic symbols, where $\bar{x}$ is the distance from reference axis to the centroidal axis, the element of area is dA, and $\bar{x}$ is the distance from reference axis to the element, we have

$$\bar{x}\int dA = \int xdA \tag{10.8c}$$

$$\bar{x} = \frac{\int xdA}{\int dA} \tag{10.8d}$$

$$= \frac{\text{Total moment}}{\text{Total area}} \tag{10.8e}$$

To determine the point called the centroid a similar calculation is made to find $\bar{y}$. The values $\bar{x}$ and $\bar{y}$ are the coordinates of the centroid, with respect to the reference axes used.

10.9 Composite Areas.

When the total area is made up of small, simple areas in which the centroidal axes are already known, the work of determining the location of the centroid need not involve any integration whatever.

If the partial areas are numbered A_1, A_2, A_3, etc., and the distances from the reference axes to their respective centroids called x_1, x_2, x_3, etc., Eq. (10.8a) above becomes

$$\bar{x}\begin{bmatrix}\text{Total}\\ \text{area}\end{bmatrix} = \begin{bmatrix}\text{Sum of the}\\ \text{moments of the}\\ \text{partial areas}\end{bmatrix} \tag{10.9a}$$

$$(\bar{x})(A_1 + A_2 + A_3 + \cdot \cdot \cdot A_n) = (x_1A_1 + x_2A_2 + x_3A_3 + \cdot \cdot \cdot + x_nA_n) \tag{10.9b}$$

Locate the centroid of the composite area with respect to

Part		Area in.^{2}	Moments with respect to		Moments with respect to	
No.	Size		Arm, in.	Moment, in.^{3}	Arm, in.	Moment, in.^{3}
1						
2						
3						
4						
5						
6						
Totals						

$= \dfrac{\text{Total moment, 1st axis}}{\text{Total area}} = \underline{\qquad\qquad} =$

$= \dfrac{\text{Total moment, 2nd axis}}{\text{Total area}} = \underline{\qquad\qquad} = \underline{\qquad\qquad}$

Check on total =

FIG. 22.—From computing and checking axis of composite shapes centroidal.

and as before

$$\bar{x} = \frac{\text{Total moment}}{\text{Total area}} \tag{10.9c}$$

Hence

$$\bar{x} = \frac{(x_1A_1 + x_2A_2 + \cdots x_nA_n)}{(A_1 + A_2 + A_3 + \cdots + A_n)} \tag{10.9d}$$

When a centroidal axis for a complicated shape such as a built-up girder cross section is wanted, the work should be handled in distinct steps as follows:

a. Diagram showing size of every part, the reference axes, and the location of the centroid of each partial area with respect to the reference axes.

b. Computation of total area.

c. Computation of the moment of each part and the sum of the partial moments.

d. Solution for $\bar{x}_1$ (or $\bar{y}_1$ as case may require).

The result should be checked by taking moments with respect to the second reference axis parallel to the first one. This adds three more steps:

e. Computation of total moment with respect to the second reference axis.

f. Solution for $\bar{x}_2$.

g. Checking to see if sum of $\bar{x}_1$ and $\bar{x}_2$ equals distance between reference axes.

When the composite area consists of four or more parts the work should be tabulated (see Form 231 in the Workbook, also Fig. 22, page 202). In offices where much work of this kind has to be done, printed or mimeographed forms with the table already ruled will be a great help, both speeding up the work and reducing chances for mistakes.

10.10 Center of Gravity.

The force of gravity acts upon every particle in a mass, and thus there is a system of forces, which converge toward the earth's center, acting upon the body. The resultant of this concurrent-force system is what we commonly call its *weight*, and the line of action of this resultant passes through a point known as the *center of gravity*.

The center of gravity may or may not lie within the mass. No matter what the position of the body may be, the resultant

force of gravity acts through this point. The intersection of two such gravity lines is sufficient to determine completely the center of gravity.

When the mass has a regular shape such that its dimensions are all known, the location of its center of gravity may be determined by applying the idea of moments as used in finding centroids except that now the moments of forces are involved. For moment of area in Topic 10.8 above we now have to substitute moment of force. However, the center of gravity in a homogeneous mass is in exactly the same position as the centroid of a volume having the same shape as the mass. For this reason it is usually simplest to find the centroid of the volume because questions of density, weight, mass, etc., all drop out.

10.11 Second Moment.

To find the second moment (moment of inertia) with respect to a chosen moment axis, the line, area, volume, or mass is divided into elemental parts as was done in finding first moments. Instead of using the moment arm as it is, however, the element is multiplied by the square of the arm. Second moments appear in the analyses of various problems, such as the design of beams and columns, but in many instances they are lifted out of their context and figured as independent problems.

Thus two whole chapters of mechanics texts and some calculus texts are usually given to the calculation of second moments of area and mass, with little or nothing to explain why this quantity is important. The unit for second moment of area is inches to the fourth power, and for volume, inches to the fifth power. Now neither of these quantities can be pictured, even mentally, as we do length, area, or volume, save by inaccurate and misleading analogies. It is better to accept them as the purely mathematical abstractions that they are, remembering that they take on physical meaning only when restored to their places in the analysis that gave rise to them.

Because the second moment of area (moment of inertia) is so important in the analysis of the load-carrying capacity of beams and columns and also in certain problems in hydrostatics, some special comments on this topic follow.

When the area is bounded by coordinate axes and curves

whose equations are known, the calculation of second moment (moment of inertia) is a routine calculus problem. A clear diagram should be drawn, and the element chosen with care. Usually the element should be in the form of the elemental square $dx\,dy$, but frequently it can be an elemental strip, thus avoiding double integration. The strip should run in such direction as to simplify the integrations as much as possible. Follow the suggestions given on pages 73–75 for setting up such problems. Keep the basic operation clearly in mind. It is

$$\begin{bmatrix}\text{Second}\\ \text{moment of}\\ \text{area, } I_x\end{bmatrix} = \int \begin{bmatrix}\text{The square}\\ \text{of the}\\ \text{moment arm}\\ y^2\end{bmatrix}\begin{bmatrix}\text{The elemental}\\ \text{area } dA\end{bmatrix} \qquad (10.11a)$$

and in symbols,

$$I_x = \int y^2 dA \qquad (10.11b)$$

Since the moment arm is squared, the y^2 term is always positive, even though y may be negative.

10.12 The Transfer Formula.

A common problem is one in which the second moment of a quantity (area, mass, etc.) with respect to its own centroidal axis is known or can be figured from some very simple formula, but the desired second moment is with respect to another axis which does not pass through the centroid. This is a routine situation in the design of composite areas such as beams, columns, and special shapes. The *parallel-axis theorem*, better known to engineers as the *transfer formula*, is the invaluable tool for solving such problems. This law says that the desired second moment is equal to the centroidal second moment plus a correction factor which takes care of the distance between the axes. This correction factor, or transfer term, is always positive, because second moment always varies as the square of the moment arm. A clear statement of the transfer formula is given in the form of a word equation.

$$\begin{bmatrix}\text{Second}\\ \text{moment of}\\ \text{area with}\\ \text{respect to}\\ \text{any given}\\ \text{axis}\end{bmatrix} = \begin{bmatrix}\text{Second}\\ \text{moment}\\ \text{with respect}\\ \text{to a centroi-}\\ \text{dal axis}\\ \text{parallel to}\\ \text{the given axis}\end{bmatrix} + \begin{bmatrix}\begin{pmatrix}\text{The}\\ \text{area}\end{pmatrix}\begin{pmatrix}\text{The square}\\ \text{of the dis-}\\ \text{tance be-}\\ \text{tween the}\\ \text{parallel}\\ \text{axes}\end{pmatrix}\end{bmatrix} \qquad (10.12a)$$

$$I_x = I_c + Ad^2 \qquad (10.12b)$$

10.13 Second Moment of Composite Areas.

If the total area is made up from two or more simple areas, such as rectangles, triangles, or parts of circles, the transfer

Compute the moment of inertia with respect to

Part No.	Part Size	d inches	d^2 $(\text{in.})^2$	Area A sq in.	Ad^2 $(\text{in.})^4$	I_c $(\text{in.})^4$	Total $I_x = I_c + Ad^2$ $(\text{in.})^4$
TOTALS							

Fig. 23.

formula must be applied to each of the small, partial areas. The calculations for areas having four or more parts should be tabulated in a form similar to that shown in Figs. 23 and 25, pages 206–208, or Forms 232 and 233 in the Workbook. Speed will be

gained and fewer mistakes will be made if a completely dimensioned diagram is drawn before any calculations are made. Refer to Fig. 24, below, and to Topic 3.11, page 65, of the specifications.

10.14 Radius of Gyration.

Students in mechanics courses are usually confused when they meet the quantity called *radius of gyration*. They do not

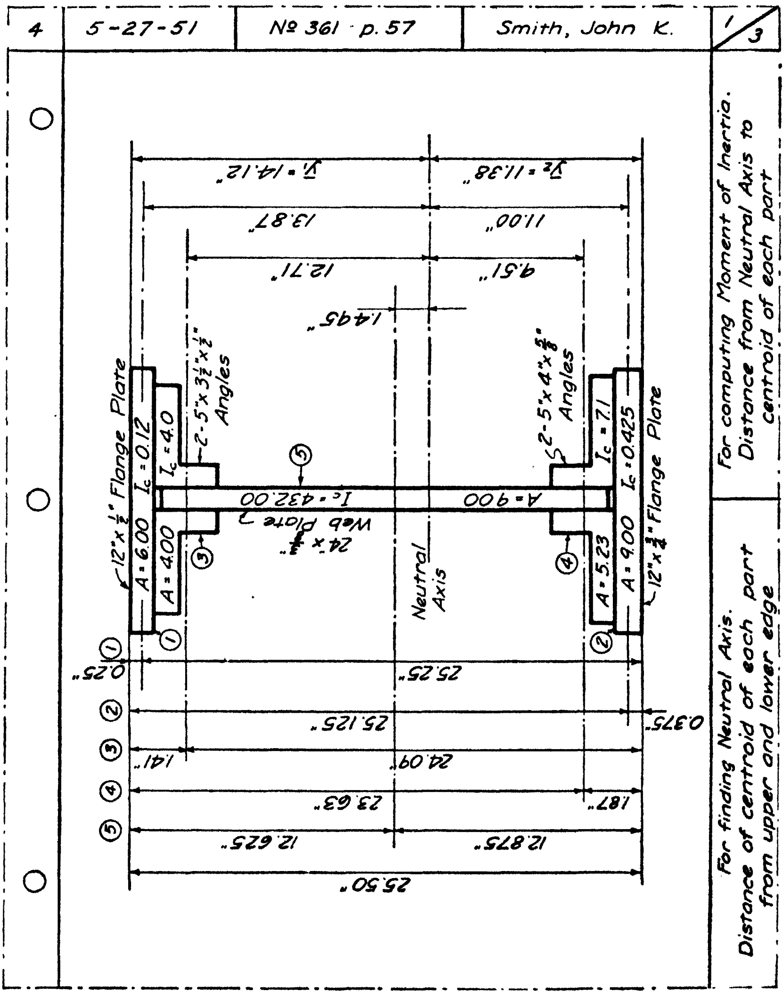

FIG. 24.—Moment of inertia of built-up sections.

Locate the centroidal axis of the composite area with respect to							Compute the moment of inertia with respect to the centroidal axis just computed				
Part		Area "A" sq in.	Moments with respect to		Moments with respect to		d in.	d^2 in.^{2}	Ad^2 in.^{4}	I_c in.^{4}	Total I $=I_c+Ad^2$ in.^{4}
No.	Size		Arm, in.	Moment, in.^{3}	Arm, in.	Moment, in.^{3}					
1											
2											
3											
4											
5											
6											
Totals			$\boxtimes$		$\boxtimes$						

$= \dfrac{\text{Total moment}}{\text{Total area}} = \_\_\_\_\_\_ =$

$= \dfrac{\text{Total moment}}{\text{Total area}} = \_\_\_\_\_\_ =$

Check $=$

Radius of gyration $= k = \sqrt{\dfrac{\text{Moment of inertia}}{\text{Total area}}}$

$\sqrt{\dfrac{\_\_\_\_}{\_\_\_\_}}$

FIG. 25.—Tabular form for complete solution for centroid, moment of inertia, and radius of gyration of composite areas.

realize that the value is another abstract mathematical quantity. Because the word *radius* is used, they vainly attempt to visualize it as they can visualize the radius of a circle. When applied to plane areas the term is especially inappropriate because the value is not a radius and nothing gyrates (rotates). The user of this function of an area will be wise to treat it as the abstraction it is and forego any attempt to picture it. The second moment of area (moment of inertia) was defined above as

$$I = \int y^2 dA \tag{10.14a}$$

We might also say that it is mathematically equal to the area times the square of some distance k, thus

$$I = k^2 A \tag{10.14b}$$

or

$$k = \sqrt{\frac{I}{A}} \tag{10.14c}$$

Through an unfortunate analogy, the value k was called a radius and coupled to another word signifying rotation, although no part of the area was moved or had a tendency to move. The term enters into all column formulas, as a property of the cross-sectional area, and into many other problems of design. It is given in all standard tables of properties of structural steel shapes.

10.15 Geometrical and Graphic Applications of Calculus.

The authors of calculus texts are usually careful to point out that when a diagram of an equation has been drawn, the laws of calculus can be translated into terms of geometrical relationships. In a give function $y = f(x)$, the slope of its graph at any point is given by its first derivative. This new function $y' = f'(x)$ can also be shown, and the length of the ordinate at any point in the new curve is a measure of the slope of the original curve at the corresponding ordinate. The area under any curve represents the integral of that curve, and thus the integral curve can be sketched by breaking the given area in smaller parts, getting the area of the parts, and then plotting those values.

The graphical methods based upon these facts are seldom

taught or given much emphasis in calculus classes. The graphical approach is valuable not only for the original solution of experimental problems but also for clarifying one's thinking when using the formal calculus. This approach is also of assistance in checking the results obtained by mathematical calculation. It is unfortunately true that men tend to go through the algebraic operations with little or no thought about the realities behind the symbols. The graphical approach with its ideas of slope, ordinate, and area helps the mind to focus upon the physical facts described by the algebraic equation.

Engineers by temperament and training are visual minded. They want literally to see what they are doing, and therefore graphic and semigraphic methods are invaluable in engineering offices. This graphic presentation of fact, whether in the form of pictures, diagrams, charts, or graphs, is a far more efficient way of transmitting knowledge than words and symbols. Because he is trained to use and understand such presentations, the engineer will often prefer to think of integration as a means of getting the area under a curve. Unless he makes use of such devices, the average man seldom will use calculus if it can be avoided in any way. He dislikes it, distrusts it, and regards it as a highly distasteful form of mental gymnastics. If circumstances compel him to use calculus, the work is usually done by slavish substitution of values in formulas picked hopefully from some integral table.

This is risky business, and so it is better for the computer to use some of the methods discussed in this chapter if he can thereby get a clearer understanding of what he is doing. He can use these methods either for the original solution or as a check on formal calculus. As pointed out in Topic 10.2 there are situations where the equations cannot be determined or, if known, are too complex or the solution is too time-consuming to justify the cost. In such cases, the graphical methods are the only practicable ones.

10.16 Calculus and Derived Curves (Case II).

A comparison of the first and second laws of derived curves as given in the preceeding chapter, with the comments regarding the graphical relationships of integration and differentiation, shows that they are the same thing. That is, the first law describes a pure graphical way of determining the first derivative of an equa-

tion. The second law simply describes a graphical way of integrating a curve.

This semigraphic method is very helpful in visualizing many problems, even where the equations are known. Because many of the curves needed by engineers are either straight lines or parabolic curves and the areas under them can be computed by simple arithmetical methods, integration is often transformed into the problem of computing the area of a triangle whose base and height are known or perhaps the area of a parabolic spandrel or segment of which the area is figured just as readily. In such cases the laws of derived curves, as they were explained in Chap. 9, are of the utmost value.

There is a third law of derived curves that was not discussed in the previous discussion because it ties in better with the purely graphical viewpoint of calculus. It is essentially double integration expressed in graphical language. This law has been known under various names for about 75 yr. It was first developed in connection with studies in stresses and deflection and is known as the *area-moment method* or *conjugate beam method.* These are merely special-case names for a completely general method. The third law reads,

$$\left[\begin{array}{l}\text{In }\textbf{any}\text{ continuous curve,}\\ \text{the length of the ordinate}\\ \text{from the tangent to the}\\ \text{curve at }\textbf{any}\text{ point (called}\\ \text{the first point) to any}\\ \text{other point on the curve}\\ \text{(called the second point)}\end{array}\right] = \left[\begin{array}{l}\text{The algebraic sum of the}\\ \text{moments of the areas be-}\\ \text{tween the ordinates of the}\\ \text{corresponding points in the}\\ \textbf{second lower}\text{ curve, mo-}\\ \text{ments being taken about}\\ \text{the ordinate through the}\\ \text{second point}\end{array}\right] \qquad (10.16a)$$

This law can also be used as a check on the results obtained by means of the first and second laws.

The whole concept designated in this and other books as *derived curves* has been used for a great many years by the writers of texts on the strength of materials and structural analysis. There are five, sometimes six, curves in the series that they use. Starting with the lowest in the bank, they have the *load curve, shear curve, moment diagram, slope curve,* and lastly, the *deflection curve.* The load curve shows the actual loads on the beam; the deflection curve shows the elastic curve, or bent neutral axis of the beam itself. The other curves indicate what is going on inside the beam. The third law is especially useful in beam

analysis, as it is regularly used to hurdle one or more curves in going from the load to the deflection curves.

10.17 Comparison of the Calculus and the Graphical or Derived-curve Methods.

The complete agreement of the two methods can be seen from a study of the ways of describing a given condition in each of the two approaches. Listed in parallel columns below are certain comparisons of the conventional calculus and the derived-curve presentation of the same facts.

Calculus	Graphical Calculus
The first derivative of the given equation is positive.	The next lower derived curve has positive ordinates. The given curve has increasing ordinates. Tangents to it point up and to the right.
The first derivative of the given equation is negative.	The next lower derived curve has negative ordinates. The given curve has decreasing ordinates. Tangents to it point down and to the right.
When the first derivative of a given equation is zero, it denotes a maximum or minimum in the original equation.	If the next lower curve crosses its base line, then the upper curve has zero slope at that point and it indicates a maximum or minimum in the upper curve.
When the first derivative is zero and is changing from + to − (that is, the second derivative is negative), it denotes a maximum value in the given equation.	If the next lower curve is crossing from + to − side of the base line, it indicates a maximum point in the original curve. The second lower curve has − ordinates.
When the first derivative is zero and is changing from − to + (that is, the second derivative is +), it denotes a minimum value in the given equation.	If the next lower curve is crossing from − to + side of the base line, it indicates a minimum point in the original curve. The second lower curve has + ordinates.
If the first derivative of an equation is zero but it not changing sign, while the second derivative is changing sign from + to − or − to +, it denotes a point of inflection.	When the next lower curve touches but does not cross its base line, the original curve has a point of inflection at that ordinate.
The constant of integration in the equation is the value of y when $x = 0$.	The value of y when the original curve touches or crosses the y axis is the constant in the equation of the curve.
When an equation is integrated, the value of the constant of integration C must be entered as part of the integral equation.	The y intercept of any curve must be included as a constant in the equation of the curve.

10.18 Graphical Calculus (Cases III, IV, and V).

The need and value of graphical methods of solving problems has been felt for many years. The application of graphical methods has been especially valuable in experimental work. Chemical engineering, for example, has been using graphical methods for years. Not many books have been written on the use of graphic methods in calculus, but one of the earliest was "Graphical Calculus" by Barker in 1896. Frequent references to one or another operation are to be found in periodical literature, but few recent texts are available. The two operations described below are those of more general interest, but reference should be made to various library indexes in order to get special applications.

There are cases in which enough data or equations are available to permit conventional algebraic analysis, but the operations are so time-consuming that the graphical solution is much the quicker procedure. At other times the visual presentation is wanted in addition to the other analysis because of its value in showing the whole picture at a glance. Lastly, more often than teachers like to admit, it is true that students forget large sections of what they "learned" in college. Out on the job there is seldom time for a man to go back and dig out a forgotten skill. He is expected to get the job done promptly even if he is "rusty," and hence the graphical approach will often provide a satisfactory solution in much less time than any other. The solution may be approximate, lacking all the niceties of rigorous analysis and the refinement of detail. If, however, it gives a workable solution, precise enough for the job in hand, who cares if it lacks the beauty of a classroom demonstration?

10.19 Equipment Needed in Graphical Calculus.

Very simple tools are required for graphical methods of solving calculus problems. These few should be of good quality, but far more important is the exercise of skill and care on the part of the draftsman who does the work. Surprisingly good precision may be obtained. Integration constructions on 8.5- by 11-in. coordinate sheets can almost approach the slide rule in precision. Because of the difficulty in drawing the original graph, it is harder to get high precision in differentiation. Errors of 1 or 2 per cent may be expected, but the use of the principles of curve fitting will

sometimes reduce this. Since errors of this magnitude are not uncommon in experimental work, however, graphical methods can fill a very useful place in an engineer's list of tools.

The equipment needed is as follows:

Coordinate papers, preferably 10 lines per inch.

A 5H or 6H drawing pencil with long, fine point.

A steel needle point for finest work.

An 8-in. French curve (a spline is very useful if the curve is rather flat).

A pair of 7- to 10-in. transparent triangles.

For graphic differentiation the above tools are needed, and in addition a plane, surface-plated mirror 5 to 10 in. in length.

10.20 Approximate Methods of Getting the Area under a Curve.

As stated above the total area under a curve is the same as the value of the definite integral for that curve and limits. There are various ways of determining this area. They differ somewhat in speed, precision, and convenience. The best known, most commonly used methods are described below.

a. **Counting squares.** The simplest, but also the slowest, method of getting the area under a curve that has been plotted on rectangular coordinate paper is that of counting the squares. It is laborious, trying on the eyes, hard on the patience, and none too precise. It should be abandoned for better methods.

b. **Planimeter.** The method that appeals to many, especially to those who are fascinated by mechanical gadgets, is the use of the planimeter or mechanical integrator. A cheap tool gives results that are little, if any, more precise than those obtainable with a lead pencil and a couple of triangles. The results are always affected by play in the pivots of the instrument, by the adjustments, by the surface of the paper, and most of all by the skill of the user. A really fine planimeter is an expensive instrument. Its purchase is seldom justified in the average engineering office. Even with a good instrument, a keen eye and a steady hand are essential. Regardless of the type and quality of the planimeter, each measurement should be made at least twice, and the average value used. For important work the average of six or more measurements should be used. The planimeter is not, therefore, a cure-all for area determinations.

c. **Average ordinate of total area.** A quick approximation of the total area under a graph may be made by the visual balancing of areas so as to estimate the average ordinate. This method is shown in Fig 26, below . A transparent triangle is placed horizontally on the graph sheet so the area under the triangle between the straight edge and the curve is equal to a similar area outside the triangle and lying between the curve and the straight edge. Make a short mark across the mid-ordinate where the straight edge crosses it. This is the average ordinate. Check

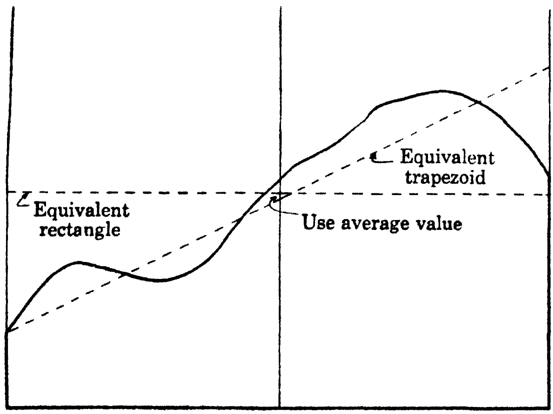

FIG. 26.—Approximate average ordinate.

it by placing the straight edge at either the initial or the final value of the curve; then swing it about this point to form an equivalent trapezoid (see Fig 26). Tick the mid-ordinate again where the straight edge crosses it. The average of these two settings should give a fair approximation of the height of a rectangle of the same width as the graph and having the same total area. This method should yield results having a probable error of less than 5 per cent.

d. **Average height of strip.** The method just suggested can be applied to the individual strips with greatly increased precision. By this method the area is first divided into a series of vertical strips, usually but not necessarily of equal width. Then the average height of each strip is estimated as in *c*. If the strips are not all of equal width, the area of each is figured and the sum computed. If they are of equal width, the sum of the estimated

average ordinates is obtained. Then this sum multiplied by the width of one strip gives the total area. This is a good method, reasonably fast and of higher precision than most users suspect. Errors of less than 1 per cent are to be expected. It seems that the normal eye is quite sensitive in balancing areas, and slight differences in the included and excluded areas are detected with surprising ease.

e. **Weighing.** If chemical or other sensitive balances are at hand, a rather unusual but neat method of estimating the area under a curve is that of weighing. The curve is plotted carefully on paper of uniform thickness, and the curve is drawn as a fine, light line. Cut along the straight sides of the area with a keen knife and a straightedge. Then use fine scissors or a knife, and cut along the graph. Also cut a 2-in. square from the waste area outside the graph. Weigh the desired area, then the 2-in. square. By proportion the area under the curve can be calculated from the weight of the known area of 4 sq in.

f. **The string polygon method.** This method is one of great usefulness not only because it is of relatively high precision but also because it gives a close approximation to the shape of the integral curve. If the work is done carefully, it will give results as good or better than the average planimeter and in not much more time. The constructions used are practically identical with those long familiar to engineers who use graphic statics to determine the resultants of forces and load distributions in trusses. Texts on strength of materials or graphic statics refer to the method either as the *funicular polygon* or *string polygon method,* the first name being Latin for string or cord. Since the application of this method to the problem of area determination is new to many engineers, it will be discussed in detail in the following topics.

10.21 Description of the Diagrams.

The data curve is shown in Fig 27*a*, page 217. It consists of three sections; each one is a horizontal straight line limited by the given ordinates. These ordinates are numbered 0 for the initial value, then 1, 2, 3, etc., for all following values. The intervals between the ordinates are the abscissa lengths x_1, x_2, x_3, etc. The values of the given ordinates are represented by the notations y_0, y_1, y_2, etc. This figure will be referred to as the *data curve* or as

the *lower curve*, this name having the same significance as it did in the study of derived curves in Chap. 9.

The projection diagram, shown in Fig. 27*b*, is, in several respects, the equivalent of the force diagram used in graphic statics. The projection diagram consists of a vertical line called the *projection axis;* a point on the base line of the data curve, called the *pole;* and a series of lines, called *rays,* that connect the

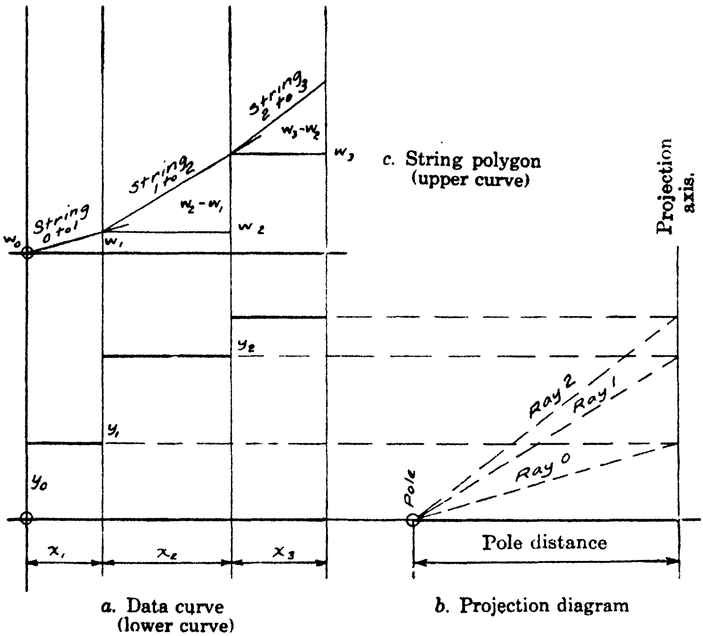

FIG. 27.

pole and the points on the projection axis. The distance from the pole to the projection axis is known as the *pole distance*, *d*.

The string polygon is shown in Fig 27*c*. It is constructed so that each of its several segments is parallel to the corresponding ray in the projection diagram and extends from ordinate to ordinate. As will be seen in the following topic, it is the same thing as "the upper curve" referred to in the topics on derived curves in a previous chapter. It will also be shown that the ordinates in this string polygon give the total amount of the area under the lower curve, up to that particular ordinate. Thus the final ordinate in the string polygon indicates the total area under the

original (or lower) curve. Calculus teaches that the string polygon is, in fact, the integral of the lower curve.

The values w_0, w_1, w_2, etc., are the values of the ordinates in the string polygon at ordinates 0, 1, 2, etc., corresponding to the same x intervals as used in the lower, or original, curve.

10.22 Construction and Mathematical Significance of the Diagrams.

The value y_0 of the initial ordinate is projected, by means of a line parallel to the x axis, to the projection axis, and the projected point is marked 0. Then the pole and the projected point 0 are connected, thus determining the location of ray 0. Next the string 0 to 1 is drawn, as in Fig. 27*c*, so that it is exactly parallel to ray 0 and connects ordinates 0 and 1 in the upper curve. In similar manner, y_1, y_2, etc., are projected, the rays located, and the strings 1 to 2, 2 to 3, etc., are drawn parallel to the corresponding rays, each in turn starting where the preceding string ended.

From the constructions just made it is seen that

$$\text{Slope of ray } 0 = \frac{y_0}{d}$$

$$\text{Slope of string 0 to 1} = \frac{w_1 - w_0}{x_1}$$

or since

$$w_0 = 0, \qquad \text{the slope} = \frac{w_1}{x_1}$$

But by construction these slopes are equal; hence,

$$\frac{y_0}{d} = \frac{w_1}{x_1}$$

and

$$y_0 = d\left(\frac{w_1}{x_1}\right)$$

That is, the ordinate y_0 in the lower curve equals the slope of the string 0 to 1 multiplied by a constant d. Thus y_0 represents, to some scale, the slope of the upper curve. This gives, therefore, an exact graphical counterpart of the first law of derived curves, which states that

$$\left[\begin{array}{c}\text{The slope at any Point in}\\ \text{any continuous Curve}\end{array}\right] = \left[\begin{array}{c}\text{The length of the Ordinate}\\ \text{at the corresponding point}\\ \text{in the next lower curve}\end{array}\right] \qquad (10.22a)$$

In the second interval the slope of ray 1 is $\frac{y_1}{d}$ and the slope of string 1 to 2 is $\frac{w_2 - w_1}{x_2}$. The slopes are equal by construction, and hence,

$$\frac{y_1}{d} = \frac{w_2 - w_1}{x_2}$$

or

$$y_1 = d\left(\frac{w_2 - w_1}{x_2}\right)$$

Hence, as before, y_1 equals the slope of the string multiplied by the constant d, and the first law still applies.

If the proportions shown above are written so as to solve for w values instead of y values, an area relationship is obtained, thus:

First Interval	Second Interval
$\frac{w_1 - w_0}{x_1} = \frac{y_0}{d}$	$\frac{w_2 - w_1}{x_2} = \frac{y_1}{d}$
$\therefore w_1 - w_0 = \frac{1}{d}(x_1 y_0)$	$\therefore w_2 - w_1 = \frac{1}{d}(x_2 y_1)$

Since x_1 is the base of a rectangle in the first section of the given curve and y_0 is its height, the product $(x_1 y_0)$ equals the area under this section of the curve. The difference in ordinates $(w_1 - w_0)$ in the string polygon is therefore equal to the area under the lower curve divided by the pole distance d. Likewise for the second interval, the product $(x_2 y_1)$ is the area under the lower curve, and therefore the difference in ordinates $(w_2 - w_1)$ is equal to this area.

This construction is, therefore, a truly graphical application and verification of the second law discussed in Chap. 9, which reads as follows:

$$\left[\begin{array}{c}\text{The } \textbf{difference} \text{ in the} \\ \textbf{length} \text{ of } \textbf{any} \text{ two ordi-} \\ \text{nates in } \textbf{any} \text{ continuous} \\ \text{curve}\end{array}\right] = \left[\begin{array}{c}\text{The } \textbf{total net area} \text{ between} \\ \text{corresponding ordinates in} \\ \text{the } \textbf{next lower} \text{ curve}\end{array}\right] \qquad (10.22b)$$

These laws can be applied in several ways to obtain a scaled drawing that approximates the shape of the integral curve. The precision will be affected by the width of the strips used and the care taken in the work.

It should be obvious that the actual positions of the various diagrams on the sheet, the lengths of the ordinates, and the pole

distance must be entirely dependent upon the scales used for plotting the curves. When the given lower curve has been plotted, the choice of a pole distance will completely determine the slope of the rays in the projection diagram and, hence, the steepness of the string polygon. By the proper choice of the pole distance, therefore, the string polygon (or integral curve) can be made to agree with any predetermined scale for its y axis. If this is done, the values of its ordinates can be read directly from this calibrated axis without any calculations. The proper scale for the integral curve can be computed by making an estimate of the total area under the lower curve, using method c in Topic 10.20. When the scale for the new curve has been chosen, the pole distance is computed from the following equation:

$$d = \frac{F_w}{F_x F_y} \qquad (10.22c)$$

In this equation F_x is the scale (units per inch) used in plotting the x values in the lower curve, F_y is the scale (also in units per inch) used in plotting the y values in the lower curve, and F_w is the computed scale (in units per inch) from which the values of the ordinates in the integral curve will be read.[1]

10.23 The Graphic Integration Process.

The foregoing discussion and development of relationships was based on a situation in which the given curve was made up of horizontal straight lines. When used in engineering offices, however, this graphical method has its greatest usefulness in cases where the known data, when plotted, yield an irregular curve of no identifiable shape or combination of shapes. The first phase of the construction, therefore, is the use of a method of resolving the complicated curved area into a stepped, straight-line diagram in which both the individual strips and the total area are replaced by a series of equivalent rectangles. When this is done, the process outlined in Topic 10.22 is used. The detailed operations will be described step by step, in the proper sequence for fast, accurate work in getting the area under a curve (integrating)—a curve so irregular that its equation might or might not ultimately be determined.

[1] For proof of the formula for the pole distance see Forms 218 and 219 in the Workbook.

Step 1. Choose desirable x and y scales for the given data. Remember that on 10-line paper scales are limited to 1, 2, or 5 times 10^n. As a rule ignore the fact that other constructions will be made on this same sheet. Choose scales so they fit the paper used and will produce a graph of ample size.

Step 2. Plot the data carefully. Use a hard pencil with a sharp point; or better still, use a needle point. See that all points are tiny, clean, round dots, the smaller, the better. Circle the dots with a clear, sharp circle a sixteenth of an inch in diameter. Do not let the circle touch a dot, and see to it that the graph and any and all construction lines never cut through these circles. The dots constitute original data and must be available for reference and checking at all times.

Step 3. Draw the graph. Use a hard, sharp pencil with a long point, and keep it that way from start to finish. Use a spline or French curve, and draw a smooth fine line for the graph. Important—make all splices between the plotted points, not at the point as so many workers do. Place the curve ruler so that there is always an overlap of at least one point as each new section of the graph is spliced to the ones already drawn. The object of this is to make sure that each piece of the graph is tangent to its neighbors on either side. Failure to do this will result in troubles later on, especially if graphic differentiation is also to be used.

Step 4. Determine the approximate total area by estimating the height of a rectangle equivalent to the total area. Refer to Topic 10.20, method c.

Step 5. Compute a desirable scale for the coming integral curve. This scale is computed thus:

$$\begin{bmatrix}\text{Scale for the} \\ w \text{ axis}\end{bmatrix} = \frac{\begin{bmatrix}\text{Estimated total area} \\ \text{under curve to be integrated}\end{bmatrix}}{\begin{bmatrix}\text{Length of axis available} \\ \text{for showing this total value}\end{bmatrix}} \qquad (10.23a)$$

Revise this scale, if necessary, in order to secure a scale suitable for the paper used. Remember that for 10-line paper this scale must be 1, 2, or 5, multiplied or divided by multiples of 10 as needed to get into the proper range. See Specs. (87), (88), page 58. When the scale is computed, calibrate the w axis. It frequently is advisable to calibrate it along the right-hand side of the sheet.

Step 6. Compute the pole distance. As stated above the value of the pole distance will depend upon the scales used for the x, y, and w axes, thus;

$$d = \frac{F_w}{F_x F_y} \tag{10.23b}$$

All these scales are known by this time; so d is readily computed. It will always be in inches. If it is necessary to make any unit conversions as one goes from curve to curve, it should be done here. Changing area in square feet to acres will, of course, affect the value of F_w and, hence, the value of the pole distance.

Step 7. Place the projection axis. In practice the projection diagram is superimposed upon the data curve. This is for convenience only and to simplify laying off the pole distance. The projection axis should be inside the ruled surface, but do not use any of the heavy coordinate rulings or data ordinates such as the 1- or 1.5-in. lines. The heavy rulings are so wide that precision is lost whenever they are used for the projection axis. To identify this line in order that mistakes will not occur later on, it should be heavily reinforced for about 0.5 in. a little way above the highest level reached by the data curve and in a like manner just below the lowest level of the data curve. Never retrace the printed line.

Step 8. Mark the pole. Lay off the proper pole distance by counting squares and any necessary fractional squares on the actual graph sheet being used. Never bring in an outside scale such as a ruler or another sheet of paper. All papers change in length and width with temperature and humidity changes; hence, the scale of the paper will seldom, if ever, agree exactly with a ruler or other external scales. All scales and measurements must, therefore, be self-contained in the work sheet. For work of the highest precision the graphic constructions should be completed without a break once they have been started. An overnight break with the sheet open to the air for many hours is almost certain to introduce noticable errors.

The pole is marked by drawing a short (0.5-in.) vertical line across the x axis of the data curve at the proper distance from the projection axis. Check this distance carefully, as an error here ruins the scale for the variable w which was computed in Step 5

and substitutes an unknown (and probably very awkward) scale for the one first planned upon.

It is usually a good idea to label the pole and the projection axis and use a dimension line between them to record the pole distance. This not only helps prevent mistakes but is an aid to the checker.

Steps 9 and 10. Projecting the y ordinates and establishing the height of the equivalent rectangular strips. These two operations should always be performed together without lifting the triangle, since each extra movement of the tools will introduce an unnecessary loss in precision.

The long edge of a triangle, which is big enough to cross the sheet, is placed so the edge is parallel to the horizontal coordinate rulings. Slide it up or down until it lies on ordinate 0. Check to see that it is still horizontal; then draw a fine line from the circle marking point 0 to a place approximately two-thirds of the way across the first vertical strip. Without shifting the straight edge, raise the pencil and move over to the projection axis. Draw a short line, less than a 0.10 in. long, across the projection axis. This has projected the length of ordinate 0 onto the projection axis. Number this "tick" with the ordinate number 0. Refer constantly to Fig. 28, page 224, for constructions from this point.

Steps 11 and 12. Determine the ray, and draw the corresponding string. Use one triangle to connect the pole and the projected point 0 corresponding to the initial ordinate. Then use the second triangle to move this line parallel to the ray until the string 0 can be drawn. Draw this string from the value of w_0, the initial ordinate in the integral curve across about two-thirds the width of the strip. Start the integral curve with an initial value of zero unless it is known to be some other positive or negative value. This initial ordinate w_0 usually is the same as the constant of integration encountered in formal integration. Do not draw any of the rays, as they serve no useful purpose once the strings have been located. This phase of the work must be done carefully and accurately, as an apparently minor error in the slope of a single string can result in a startling loss of accuracy in the final result.

Steps 13 and 14. Now draw a light horizontal line through ordinate y_1 (but staying out of the circle). Let this line run to the left as well as to the right so that it crosses about two-thirds of

the way through each strip. Without moving the triangle, project this ordinate as in Step 10.

Steps 15 and 16. Place a triangle vertically in the strip between ordinates y_0 and y_1, and move it slowly right and left until its

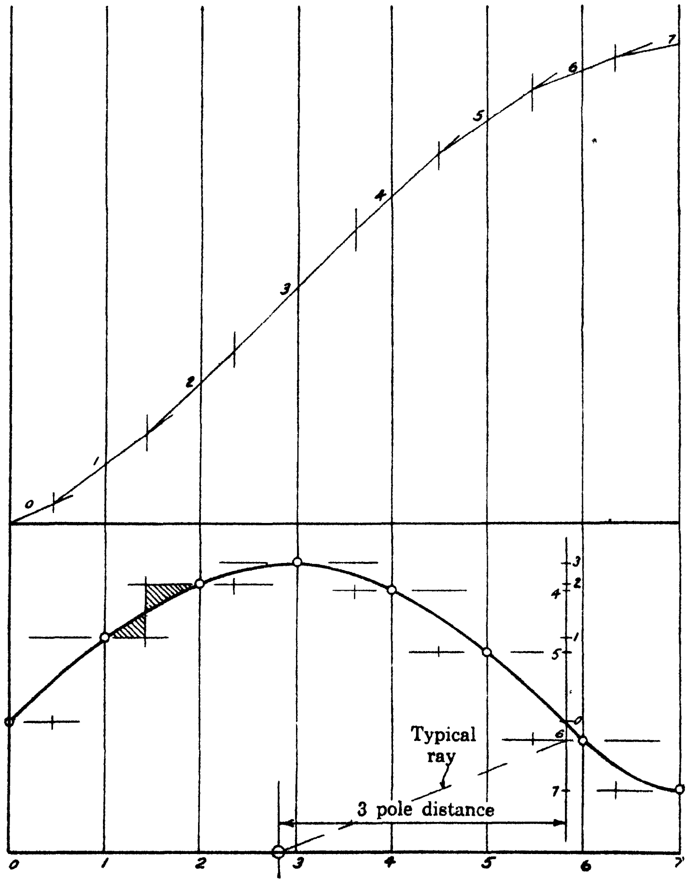

FIG. 28.—Constructions for graphical integration.

edge forms two small equal areas, nearly triangular in shape. One of these areas lies under the straight edge and between the graph and the horizontal line through ordinate y_0. The other little area lies on the opposite side of the curve and with the horizontal line through y_1 as its other side (see Fig. 28, above). When the straight edge is placed so that these areas seem to be equal, draw a very short (0.05- to 0.10-in.) vertical line through

either of the horizontal lines previously drawn through the ordinates. Without shifting the straight edge move along its vertical edge to the place where it crosses the string from w_0 which was drawn in Step 12. Draw another short vertical line through this string. This intersection marks the starting point for string 1 whose slope is fixed by y_1.

Step 17 and following. Proceed as in Steps 13–16, inclusive, picking up each ordinate in turn, and complete the cycle of operations step by step until the string polygon is completed. Each string should be numbered 0, 1, 2, 3, etc., and checked to see that it crosses the ordinate that determined its slope. This will forestall mistakes due to the omission of an ordinate or the use of any ordinate a second time.

Note. The operations performed in Steps 9, 13, 15, and following have converted the original curve-bordered figure into a stepped diagram consisting of a series of rectangular strips of unequal height and width but whose total area is the same as that under the given curve. Since the strips are now of unequal width, the strings will have varied lengths; hence, it is necessary to know the splicing points for the various strings. The vertical line forming the two small equal triangles on either side of the curve thus served two purposes: first, the formation of the equivalent rectangles; second, locating the splicing point of the strings. These splicing points, as Figs. 27*a* and 27*c* show, are vertically above the sides of the vertical, rectangular strips formed by the balancing of the small triangular areas.

Final Step. As a general rule the string polygon, made up of a series of straight-line segments, will be accurate enough to yield all the necessary information, especially when the vertical strips are relatively narrow. With 0.5-in. strips to start with on a sheet of 8.5- by 11-in. coordinate paper it is easily possible to secure results having a probable error of less than half of 1 per cent for the value of the total area under the given curve. Since this is less than the probable error in much experimental work, the method is a satisfactory tool.

When a still closer approximation to the true shape of the integral curve is desired, it may be drawn as follows: Place a French curve so that it lies tangent to two adjacent strings at the points where each string cuts the ordinates directly in line with the original data ordinates y_0, y_1, etc.; then draw in a short

section of the curve. The strings not only have the correct slope of the integral curve where they cross the ordinates that gave rise to them (see first law of derived curves) but also show the correct value of the accumulated area under the lower curve up to the point where the string intersects the ordinate (or ordinate extended). When the smooth curve is completed, the shape of the integral curve is approximated as closely as it could have been plotted were its equation known.

10.24 Graphical Integration on Unruled Paper.

Sometimes it is necessary to integrate curves that have been drawn by automatic recording instruments. At other times it may seem desirable to construct the graph to a large scale on plain sheets of detail paper. Since there is no prepared grid of coordinate rulings ready for use, it becomes necessary to rule in the essential vertical ordinates to form the series of curved end strips that are the starting point for the work. These ordinates should be drawn at all important points in the original graph, such as points of inflection, changes in slope, or places where it is obvious that two different types of line are joined (such as straight-line sections and curved sections). The ordinates should be relatively far apart where the slope of the curve changes slowly but much closer together where the slope is changing rapidly. An ordinate should be drawn through all maximum or minimum points. The strips obviously are of varied widths instead of constant width as when squared paper is used, but the method of constructing the curves will give as good or even better results, because the ordinate spacing takes into account the important changes in the original graph.

When the ordinates have been drawn, the constructions are made exactly as in the preceeding topic.

10.25 Graphical Differentiation.

Greater care and skill must be used in graphical differentiation, because the worker must measure the slope of a changing curve at the exact point where the curve crosses the specified ordinates. This is not easy even when the shape of the curve is most favorable and can be a patience-taxing affair when the given curve is highly irregular.

As previously stated, the derivative curve can be plotted if the

slope can be determined at enough points and these slope values plotted. When the given graph is the record made by a recording instrument, it may be a highly irregular curve consisting of many fine, saw-toothed variations from an unknown average line which probably represents the real trend of the curve. In such cases the worker may decide to use his best judgment and draw in a

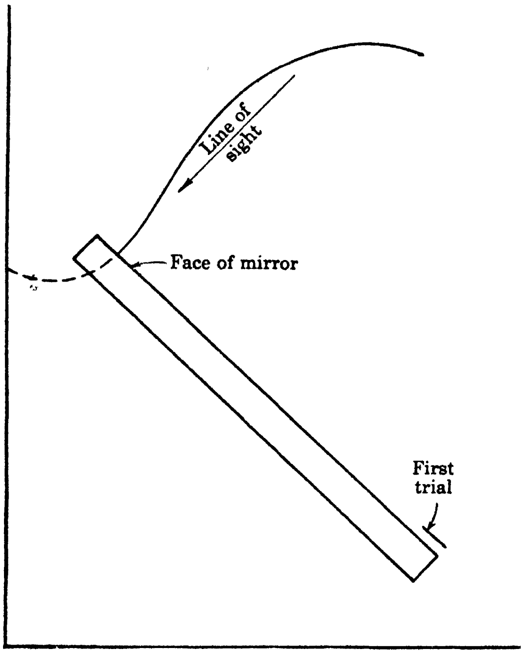

FIG. 29.

smoother curve which he believes will represent the average condition. This new curve can then be worked upon, and usable results obtained.

There are several ways of estimating the slope of a curve, but it must be realized that none will give the high precision of graphic integration. The most commonly used methods are as follows:

a. **Drawing tangents to the curve.** One method is that of guessing at the position of a line tangent to the curve at the specified point. This is the least reliable of all methods and is especially lacking in accuracy when a curve is changing its slope rapidly. It should not be used for anything more than a rough approximation.

b. **Constructing very small-slope triangles.** Some engineers have used high-power magnifying glasses or microscopes to enable them to construct very small triangles along the curve. The triangles are taken so small that the portion of the curve used as the hypotenuse can be considered to be a straight line. When the triangles are formed, the altitude and base of each are measured as accurately as possible; then the slope is figured. This is an extremely slow, patience-trying job. It is hard on the eyes, requires an unusually steady hand, and calls for good equipment.

c. **Use of mirrors and normals.** The best way to determine the approximate slope of a curve at any given point is by the use of a mirror. A mirror is set vertically on the curve at the point where the slope is desired, then rotated slowly about a vertical axis until the image in the mirror and the real curve in front of it make a smooth line (see Fig. 29, page 227). When placed correctly the curve and its image are tangent to each other, and the face of the mirror lies normal to the curve. The true slope of the curve is the negative reciprocal of the slope of this normal.

10.26 Mechanical Differentiators.

This mirror method has been known and used by workers in other fields for many years with excellent success. Engineers should be more familiar with it, because their skill with drafting instruments gives them a decided advantage over untrained workers.

Various types of differentiators have been devised and put on the market, nearly all having a mirror as an essential part of the instrument. Some such devices are supposed to draw the derivative curve as the operator moves the mirror along the curve, always keeping it normal to the line. Others permit the operator to draw a line tangent to the curve. Still others are made so that the slope angle can be read. From the angle and a table of functions the slope can be found. Since the true slope can be determined so easily with nothing more than a simple mirror, there is no need for these complicated gadgets.

10.27 Types of Mirrors.

Three types of mirrors have been used for this work. The simplest is a strip of ordinary mirror made of either common glass or plate glass. The strip should be about 1 in. wide and 6

to 12 in. long. If this type of mirror is used, the constructions must be made along the back of the mirror because the image lies at that surface. A piece of plate glass that is surface-plated can be used, but the mirror surface is easily scratched. All glass mirrors must be handled with care, as the strips of glass are broken so easily.

A more durable mirror can be made from a strip of brass, nickel- or chromium-plated. The material should be from 0.25 to 0.75 in. thick, 0.5 to 1.0 in. finished width, and from 6 to 10 in. long. The mirror surface does not need to run the full length of the material. It is better to limit it to a section about 0.75 in. in length at one or both ends of the material. The brass stock must be planed and ground straight, without bend or wind. If the mirror is made by electroplating, the stock should be cut about 0.5 in. oversize for width and length. This is because in electroplating the material builds up at the edges and corners of the stock. This bead distorts the image and affects the accuracy of the work. Hence the edges and ends of the mirror strip must be sawed off after the plating is finished, and the edges of the material ground so they are perpendicular to the mirror face.

It has been found that one of the best mirrors, both for accuracy and long life, is one made of stainless steel. For office work the bar should be 0.5 in. thick, 1.0 in. wide, and 10 or 12 in. long. The bar must be ground smooth and true, with edges and mirror face perpendicular to each other. It is not necessary to polish more than 1 in. length of the face at each end of the bar.

Some designers have gone to the trouble of mounting a small mirror perpendicular to a straight edge so the tangents may be drawn directly. It is both an expensive and difficult manufacturing operation to machine three planes so that they are mutually perpendicular to each other. This arrangement is an unnecessary complication, because results of equal or higher precision can be obtained by means of the simple mirror and two triangles.

10.28 To Draw the First Derivative of a Graph.

The curve is carefully plotted as in Steps 1, 2, and 3 of Topic 10.23; then the work proceeds as below.

Step 1. Place the mirror on the initial ordinate so that the mirror faces the curve and the image is about half an inch in

from the end. See that the mirror surface lies centered on the dot that shows the value of y_0. Now rotate the mirror slowly with the dot as a pivot point until the graph and its image seem to form a smooth curve. If correctly placed, the face of the mirror is normal to the curve or, in other words, coincides with the radius of curvature at that point. The line of sight should be at about 15° with the horizontal in a plane perpendicular to the mirror face.

Step 2. Hold the mirror firmly in this position; then using a very sharp pencil, draw a line, about 0.1 in. long, at the far end of the straight edge of the mirror. Use the front surface of a metal mirror and the back side of glass mirrors plated on the back. Get as long a normal as possible in order to retain precision. Check this normal carefully, using the average of two or more observations.

Step 3. Now carefully place the longest edge of one triangle so that it passes through the dot showing y_0 and the average of the normals.

Step 4. Then place the second triangle so that its longer leg is perpendicular to this normal. This edge, then, is parallel to the tangent to the curve at point y_0, and the line can be moved parallel to itself to any desired position.

Step 5. Now slide this second triangle along the first triangle so that the tangent line is moved parallel to itself until it passes through a bottom corner of the graph sheet. All tangents having positive slope will pass through the lower left corner, and tangents with negative slope will pass through the lower right corner.

Step 6. Do not draw in the whole length of the tangent, because it would clutter up the sheet. Merely draw a line about half an inch long where the tangent crosses the far edge of the coordinate rulings. This line may cross either a side or the top edge, depending on the slope of the tangent. Label this short line, just inside the rulings, with the ordinate number 0, 1, 2, etc., that gave rise to it.

Step 7. Figure and record the slope thus: Using the same scales with which the original graph was plotted, read the altitude and base of the slope triangles formed in Step 6. Compute the ratio of altitude to base, and record the result as the value of the slope at ordinate y_0.

Step 8. Now move the mirror to ordinate y_1, and place the

mirror on it so that it faces toward the initial ordinate (see Fig. 29, page 227). Locate the normal as in Step 1. Check the observation by facing the mirror away from the initial ordinate, and locate the normal from this viewpoint. Ignore any part of the curve that is behind the mirror. Do not be influenced by the first placing of the normal. Accept the observations only when two normals, not less than 4 in. long, lie within 0.10 in. of each other at the far end. Repeat the observations until a check is obtained.

Step 9. Use the average of two acceptable normals, and construct the tangent as in Steps 5 and 6.

Step 10. Compute and record the slope as in Step 7. Then continue as above until the slopes at all specified points have been measured.

Step 11. Note the over-all range in the slopes obtained and compute a suitable y scale for plotting the slopes. Lay out and plot the values of the slopes as determined above. Do not draw the graph just yet.

Step 12. Before drawing a line through the points just plotted, sight across them to see if any definite trend can be detected. If it seems as though a straight line might best fit the points, then divide the slopes into two nearly equal groups and compute the average x and y values for each group. This gives the coordinates of two points through which the line can be drawn.

Sometimes it will be advisable to proceed with one of the curve-fitting methods outlined in Chap. 9. When this is done, the equation of the slope (or derivative) curve can often be calculated. With known equations the integration by formal calculus methods will yield the whole series of equations of the higher curves.

In some cases it may prove to be necessary to get a smooth derivative curve, then obtain a second derivative by repeating the above process before the equations can be determined.

10.29 Approximate Integration.

When only the total area of an irregular figure is required and the shape of the integral curve is of no importance, this total area may be obtained by several formulas that give more or less close approximations to the correct result. They are of great service in cases where it is not feasible to use either graphic methods or the formal calculus. All these rules for

getting the area are based upon the idea of dividing the area into a series of strips, *all of the same width*, then totaling the strips in various ways. The formulas of most value to the engineer are known as Simpson's rule, Durand's rule, and the trapezoidal rule.[1]

10.30 Simpson's Rule.

Simpson's rule is generally considered to give the best results of the three methods and may be used with confidence in all

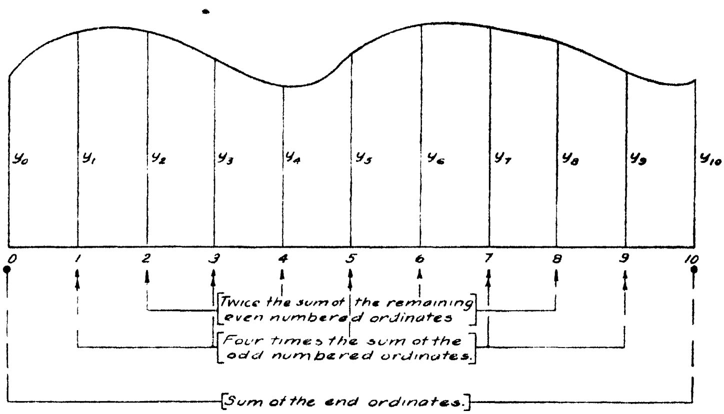

FIG. 30.—Simpson's rule.

situations that the average engineer will encounter. The rule assumes that the curve between any three successive ordinates is a parabola. It is necessary, therefore, to have an even number of strips, since they are really taken in pairs, although this fact is not apparent on casual inspection of the formula.

The ordinates should be numbered in turn, the initial ordinate being numbered 0. If the number for the final ordinate is even, then Simpson's rule may be used; if odd, then either a value must be dropped or another added, or another rule must be used. The extreme ordinates (initial and final) will always be "even numbered" if Simpson's rule can be used.

[1] WHITED, WILLIS, "Methods of Approximate Integration," *Eng. News*, vol. 73, pp. 840–842, Apr. 29, 1915, gives an interesting analysis of the accuracy of various methods.

$$\begin{bmatrix}\text{Approx.}\\\text{total}\\\text{area}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}\text{Width}\\\text{of}\\\text{one}\\\text{strip}\end{bmatrix}\left[\begin{pmatrix}\text{Sum}\\\text{of}\\\text{the}\\\text{end}\\\text{ordi-}\\\text{nates}\end{pmatrix} + (4)\begin{pmatrix}\text{Sum of}\\\text{the odd-}\\\text{num-}\\\text{bered}\\\text{ordi-}\\\text{nates}\end{pmatrix} + (2)\begin{pmatrix}\text{Sum of}\\\text{remain-}\\\text{ing even-}\\\text{num-}\\\text{bered}\\\text{ordi-}\\\text{nates}\end{pmatrix}\right]$$

If the ordinates are tabulated in three columns as called for in the formula, the work goes very quickly (see Fig. 30 for the ordinate groupings).

10.31 Durand's Rule.

Durand's rule does not give quite so accurate results as does Simpson's; but as it has the advantage that it may be used

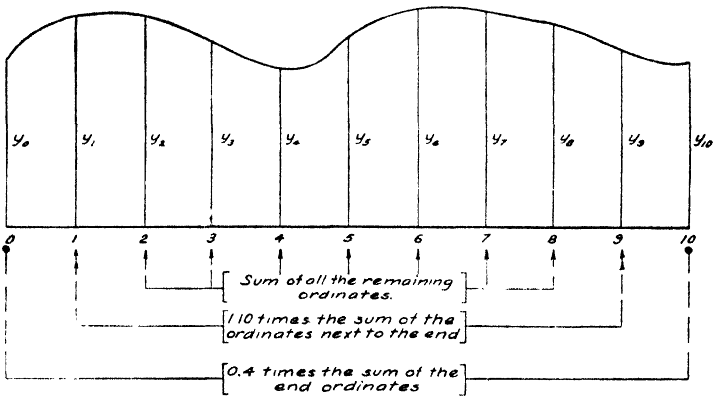

FIG. 31.—Durand's rule.

for any number of strips, it is a good tool to use when Simpson's rule cannot be applied. It is essentially a combination of Simpson's rule and the trapezoidal rule.

$$\begin{bmatrix}\text{Approx.}\\\text{total}\\\text{area}\end{bmatrix} = \begin{bmatrix}\text{Width}\\\text{of}\\\text{one}\\\text{strip}\end{bmatrix}\left[0.4\begin{pmatrix}\text{Sum}\\\text{of}\\\text{the}\\\text{end}\\\text{ordi-}\\\text{nates}\end{pmatrix} + 1.1\begin{pmatrix}\text{Sum of}\\\text{ordinates}\\\text{next to}\\\text{the ends}\end{pmatrix} + \begin{pmatrix}\text{Sum of}\\\text{remain-}\\\text{ing ordi-}\\\text{nates}\end{pmatrix}\right]$$

At the time that the data are copied, tabulate them in three columns ready for summing, and the labor will be reduced (see Fig. 31 for the ordinate groupings).

10.32 Trapezoidal Rule.

This rule is based upon the assumption that the curve is made up of a series of straight lines running from ordinate to ordinate. The error, therefore, may be considerable if the curve has a short radius between any two ordinates. The

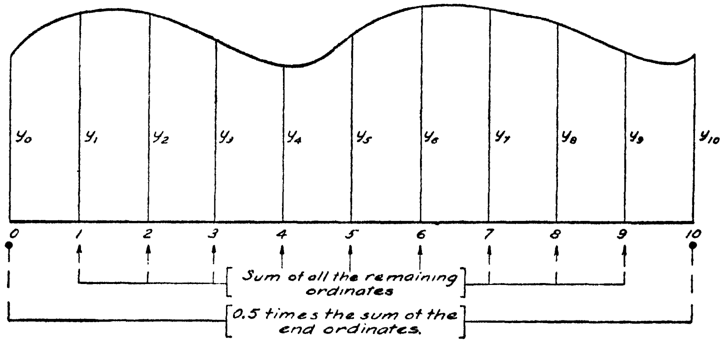

FIG. 32.—Trapezoidal rule.

computed area will be too high when the curve is concave and too low when it is convex. To overcome this fault and give the same relative accuracy as Simpson's rule gives, it is necessary to narrow the strips considerably or, in other words, increase the number of ordinates.

$$\begin{bmatrix}\text{Approx.}\\ \text{total}\\ \text{area}\end{bmatrix} = \begin{bmatrix}\text{Width}\\ \text{of one}\\ \text{strip}\end{bmatrix}\left[0.5\begin{pmatrix}\text{Sum of}\\ \text{the end}\\ \text{ordinates}\end{pmatrix} + \begin{pmatrix}\text{Sum of}\\ \text{remaining}\\ \text{ordinates}\end{pmatrix}\right]$$

The trapezoidal rule may be used for either an odd or even number of strips. It is generally a good plan to use this rule to check the results obtained by either of the other rules. The results will not be identical, but the comparison will serve to detect any bad blunders (see Fig. 32 for the ordinate groupings).

When two of the rules are being used, it is advisable to tabulate the data ready for summing, thus avoiding the labor and danger of mistakes that ensue if the values are recopied in the left-hand column of the work sheet. Do not forget that every ordinate must be used and that *no ordinate is to be used more than once.*

10.33 Conclusion.

The value of graphical and semigraphical methods of computation has long been appreciated by engineers. especially

in the study of forces. They are now making more and more use of similar methods in the study of other problems. Alignment charts, special slide rules, graphs, and mechanical instruments are being used in many branches of engineering. In a large number of investigations the data are obtained as a series of readings, or measurements of related quantities. No mathematical equation may be known, but it is desired to study rates of change of the variables or, perhaps, the total change. Under these circumstances graphic methods are of great value.

The semigraphic methods of derived curves described in Chap. 9 and the extension of this concept to the exact graphical constructions described in this chapter furnish very powerful tools for the "cracking" of many otherwise difficult problems. They will also enable the man who may have forgotten most of his college calculus still to think in terms of the two fundamental operations of the calculus, namely:

a. The determination of the rate of change of two variables with respect to each other.

b. Determination of the total change when the rate of change curve is known.

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CHAPTER 11

MISCELLANEOUS PROBLEMS

MENSURATION

1. A concrete girder with a rectangular cross section by in. is ft long. How much does it weigh?

2. A white-oak beam is 16 ft long and 8 by 10 in. in cross section.

a. What is its weight?

b. At $110 per 1000 fbm what is its value?

3. A rectangular fuselage panel section is 180 in. long and 11.5 in. wide. The panel section is made of aluminum alloy and is 0.020 in. thick.

a. Compute its area.

b. Compute the weight of the panel on the basis of the weight per square foot of sheet aluminum, using footnote below Table 17, page 372.

c. Compute its volume in cubic inches.

d. Check its weight on the basis of the weight per cubic inch or cubic foot, using Table 16, page 366.

4. A rectangular floor panel made of 0.25-in. balsa is 72.4 by 24.6 in. The panel is covered on each side with a layer of aluminum alloy which is 0.012 in. thick.

a. Compute the area of the panel.

b. Compute the volume.

c. Compute the weight. (Balsa weighs 0.0056 lb per cu in.)

5. A triangular piece of steel armor plate is in. long, in. high, and in. thick.

a. Compute its area.

b. Compute its weight, using Table 17, page 372.

c. Compute its volume.

d. Compute its weight, using Table 16, page 366.

6. A triangular piece of micarta 36.5 in. long has a height of 6.0 in. and a thickness of 0.032 in.

a. Compute the area.
b. Compute the volume.
c. Compute the weight by two methods.
(1) Weight per square foot = 0.226 lb.
(2) Weight per cubic inch = 0.048 lb.

7. Two triangular lots lie as shown in Fig. 33. $AB =$ ft; $BC =$ ft; $BD =$ ft. What is the area of each lot?

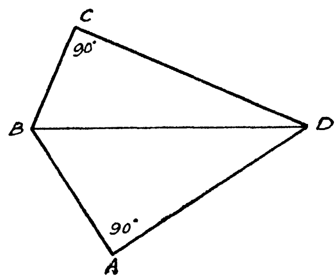


FIG. 33.

8. How many rods of fence will be required to fence each lot in Prob. 7? How many rods of fence will be required to fence both lots in Prob. 7?

9. A trapezoidal windshield section (see No. 5, page 354) is made of plexiglass 0.125 in. thick, weighing 0.043 lb per cu in., or 0.0054 psi.

$AB =$ in.
$DC =$ in.
$h =$ in.

a. Compute the area.
b. Compute the volume.
c. Compute the weight by two methods.

10. In preparing the estimate of the cost of a steel crane it is necessary to compute the weight of a trapezoidal steel plate in. thick (see No. 5, page 354). The parallel edges of the plate are in. apart; the long edge is

in.; and the short edge parallel to it is in. long. How much does it weigh?

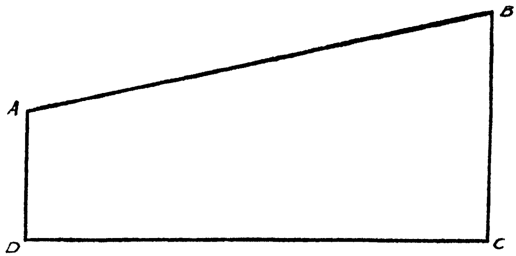


FIG. 34.

11. A tract of land having four sides has two adjacent right angles (see Fig. 34). $CD =$ ft; $AD =$ ft; $BC =$ ft.

a. What is the area of the tract in acres?

b. How many rods of fence would be needed to enclose the tract?

c. How long are the diagonals?

12. A four-sided section $ABCD$ (see No. 6, page 354) is made of plexiglass 0.062 in. thick weighing 0.043 lb per cu in.

$$CF = 6.0 \text{ in.} \qquad AE = 2.0 \text{ in.}$$
$$DF = 3.0 \text{ in.} \qquad AF = 41.0 \text{ in.}$$
$$ED = 36.0 \text{ in.} \qquad BE = 9.0 \text{ in.}$$

a. Compute the area.

b. Compute the volume.

c. Compute the weight.

13. A circular ring 1 in. long is cut from a tube of aluminum alloy. The outside diameter of the tube is 2.000 in. and the inside diameter is 1.965 in. The tube weighs 0.516 lb per lin ft.

a. Compute the cross-sectional area.

b. Compute the volume.

c. Compute the weight.

14. A control system sector (see No. 9, page 355) is an aluminum-alloy forging 0.375 in. in thickness. The length $BC = 7.0$ in. The angle ACB is 122°.

a. Compute the area of the sector.

b. Compute the volume of the sector.

c. Compute the weight.

15. A brass segment (see No. 10, page 355) is 0.375 in. in thickness. The radius AC is in., and angle ACB is deg.

a. Compute the area.

b. Compute the volume.

c. Compute the weight by two methods, using Table 10, page 355, and footnote to Table 17, page 372.

16. A wing fillet (see No. 11, page 355) is an aluminum-alloy forging 0.25 in. in thickness. The radius is 30 in.

a. Compute the area.

b. Compute the volume.

c. Compute the weight by two methods, using Table 10, page 363, and footnote to Table 17, page 372.

17. An elliptical leather hinge (see No. 12, page 355) is 0.0625 in. thick. The long diameter $AB = 26.3$ in., and the short diameter $DE = 7.2$ in.

a. Compute the area.

b. Compute the volume.

c. Compute the weight.

18. A right cylindrical tank is ft diameter inside and holds gal. What is its height?

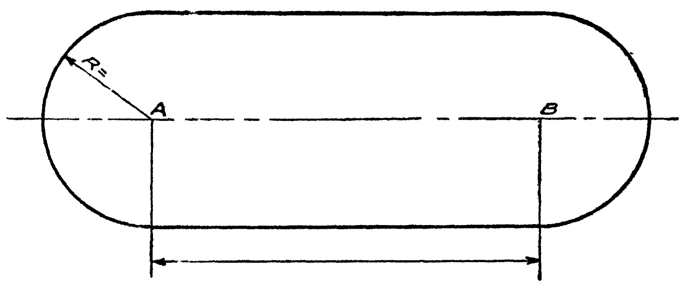


Fig. 35.

19. A stock-watering tank in. deep has a cross section as shown in Fig. 35. How many gallons of water will the tank hold when filled if $AB =$ and radius $R =$?

20. A vertical cylindrical gas tank holds 75 gal. The area of the base is 500 sq in. How high is the tank?

21. A cylindrical tank with open top is ft in diameter by ft high (see No. 18, page 356). It is made of steel plate in. thick, and all seams are butt welded. What is the total weight of the tank and contents in tons when filled with water to a line 1 ft below the upper edge?

22. The supercharged cabin in a high-altitude plane is 20 ft long and 11 ft in diameter. The air in the cabin must be changed every 5 min. Compute the capacity of the ventilating system in cubic feet per minute.

23. A cubical tank contains cu ft. What is the diameter of a cylindrical tank having the same volume and the same height?

24. A tank of rectangular section is ft long and ft high and has a volume of cu ft (see No. 18, page 356). What would be the height of a cylindrical tank having the same volume and the same diameter as the width of the rectangular tank?

25. Compute the piston displacement of a 9-cylinder engine that has a 5-in. bore and a 6-in. stroke.

26. A right conical hard-rubber bumper is 3.7 in. high and has a base diameter of 2.5 in.

a. Compute the surface area.
b. Compute the volume.
c. Compute the weight.

27. A firm making glass novelties is filling an order for 2000 flint-glass paperweights in the form of a right pyramid (see No. 19, page 357). The base is a square in. on a side, and the perpendicular height is in.

a. Compute the weight of one paperweight in ounces.
b. Compute the weight of the full order in pounds.

28. A bridge abutment is made of concrete. Its shape is that of a truncated pyramid ft high (see No. 21, page 357).

	Length	Width
Rectangular top.		
Rectangular base.		

Using the mixture described in Prob. 55, page 245, what quantities of each material will be required. Round off final answers to the next larger unit.

29. A patrol bomber flies in a circle of miles radius. The observer can see 45 miles horizontally in each direction. Compute the area observed.

30. A copper-ball float is 3.50 in. in diameter and is made of material in. in thickness.

a. Compute the surface area.

b. Compute the weight.

31. A rubber bumper is made in the shape of a spherical sector (see No. 24, page 358). The radius is in., and the height h is in.

a. Compute the surface area.

b. Compute the volume.

c. Compute the weight.

32. The nose of a bomber is made of 0.125 in. thick plexiglass and is in the shape of a spherical segment. The diameter of the base of the segment is 50 in., and the height is 30 in.

a. Compute the surface area.

b. Compute the weight.

33. A radio direction-finder loop is made of 1-in. diameter copper and has an outside diameter of 1 ft.

a. Compute the surface area.

b. Compute the volume of copper.

c. Compute the weight.

34. In one of the problems assigned in the forge shop it is necessary for the student to compute the length of round steel rod that will be needed to make a close-fitting collar to be forged onto another round rod. If the collar is to be made from stock in. in diameter and the other rod is in. in diameter, how long should the collar stock be cut?

35. A cylindrical tank 10 ft long and ft in diameter lying in a horizontal position has in. of gasoline in it. Compute the quantity of gasoline in gallons.

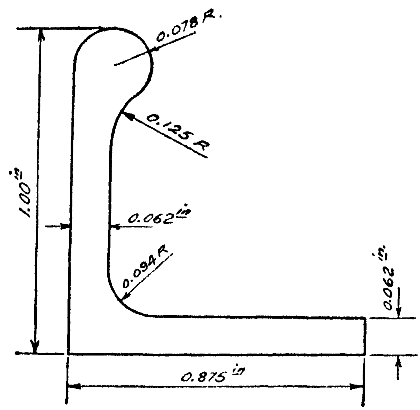


FIG. 36.

36. The extruded bulb angle shown in Fig. 36 is made from aluminum alloy. Compute the weight of lin ft of the bulb angle.

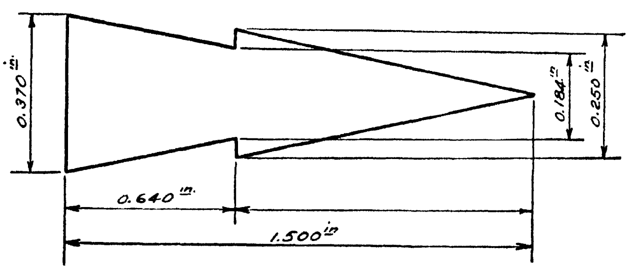


FIG. 37.

37. The extruded trailing-edge wing section shown in Fig. 37 is made from aluminium alloy. Compute the weight of lin ft of the trailing-edge section.

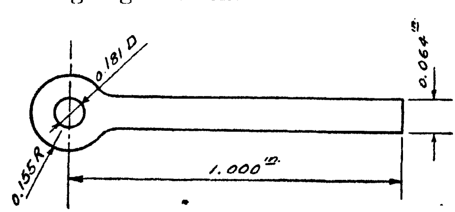


FIG. 38.

38. The extruded elevator-hinge section shown in Fig. 38 is made from aluminum alloy. Compute the weight of lin ft of the hinge section.

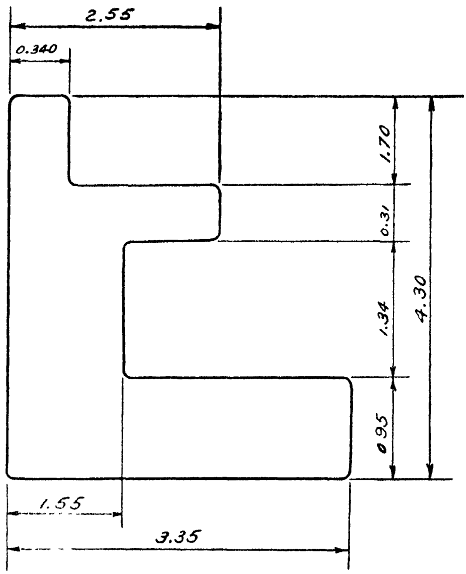


FIG. 39.

39. A cross section of a lower spar cap is shown in Fig. 39. All dimensions are given in inches, and all fillets have a radius of 0.100 in. The spar cap is made from aluminum alloy. Compute the weight of lin ft of the spar cap.

40. It is desired to construct an open steel tank having a capacity of 1200 gal. The depth of liquid is to be 5.00 ft, but the tank is to be 6 in. deeper to allow some reserve space. Material is to be 0.25-in. steel plate for sides and bottom.

a. Determine the weight of steel needed if the tank is to be square.

b. Determine the weight of steel needed if the tank is to be circular.

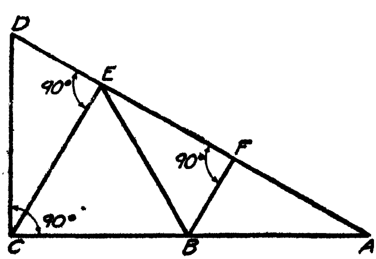


FIG. 40.

41. A roof-truss outline is shown in Fig. 40. Find the length of each piece not given. $CD =$; $CA =$ Point B is the mid-point of CA.

42. A section of a highway is on a 4.5 per cent grade. What is the grade length of the pavement for each 100 ft measured horizontally?

43. A pump is delivering water at the rate of gal per 24-hr day. What is the equivalent amount in cubic feet per second?

44. A pipe in. in diameter is delivering water to a tank. If cu ft enters the tank each second, what is the velocity of the water in the pipe?

45. A reservoir having a capacity of gal is two-thirds full. Water is being pumped in at the rate of gpm, but it is also being drawn off at the rate of cfh. How long will it take to fill the reservoir?

46. A reservoir has two supply pipes. One can fill it in hr, and the other in hr, working separately. How many hours will it take to fill the reservoir if both pipes are filling at the same time?

47. A certain make of compressor has a list price of $. If it is sold at chain discounts of per cent and per cent with an additional 2 per cent off for cash, what is the lowest cost of the equipment?

48. What diameter grinding wheel should be used on a spindle running at rpm so that the rim speed will be fpm?

49. A belt transmits power from one pulley to another. The driven pulley is in. in diameter and runs at rpm, and the driver runs at rpm. What diameter must it be?

50. Two shafts are connected by gears. If one gear has teeth and is turning rpm and the other has teeth, what speed in revolutions per minute will it have?

51. A belt transmits power from a pulley on a line shaft to a pulley on an idler shaft, and from another pulley on the idler

another belt transmits the power to a pulley on a machine. If the machine pulley is in. diameter and is to run at rpm and the line shaft is running at rpm, what diameter pulleys must be used on the idler and on the line shaft to have the speed of the idler a mean between the others?

52. In a certain factory one of the departments was producing pieces in 8 hr. Another machine was added, and the total production was found to be pieces in 5 hr. What was the percentage of increase?

53. Coal is frequently purchased on the basis of the heat units furnished. Two samples were tested. Sample 1 was priced at $6.25 per ton of 2000 lb, and tested 12,660 heat units per pound. Sample 2 was priced at $6.40 per ton and tested 13,810 heat units per pound. Which was the more economical coal to buy, and what was the cost per 1000 heat units of each?

54. A certain electric generator can deliver kw. How many kilowatts are delivered to the consumer if losses are as follows:

Powerhouse, transformers, etc., per cent.
Line losses, per cent.
Losses in local distribution system, per cent.

55. The proportions of cement, aggregate, and water required to secure concrete of maximum strength and density and of proper consistency for forming and finishing will vary considerably, as they will depend upon the materials available and the type of concrete wanted. In a given case experiment indicated that a medium-rich concrete required a 1:2:3 mix by volume. That is, 1 sack of cement (1 cu ft), 2 cu ft of sand, 3 cu ft of coarse crushed rock, and 5.5 gal of water is used for each bag of cement. To make 1 cu yd of this concrete required 7 sacks of cement, 0.52 cu yd of sand, 0.78 cu yd of stone, and 38.5 gal of water. Compute the total units of each material required to make cu yd of concrete.

Note: Bags of cement are not broken. Use next larger whole unit for each material used.

56. As indicated in Prob. 55, the materials used in making concrete should be proportioned carefully. In another locality it

was found that a medium-rich mixture required the volumetric proportions 1:2.5:3.5 and 6.0 gal of water per bag. For 1 cu yd of concrete the following quantities were needed: cement, 5.9 bags; sand, 0.55 cu yd; gravel, 0.76 cu yd; and water, 35.4 gal.

Compute the total units of each material needed to make cu yd of concrete.

NOTE. Do not assume fractional bags of concrete. Go to next larger whole unit for each material used.

57. Refer to Prob. 55, page 245, and Table 16, page 366, for data, and then compute the quantities of cement, sand, and gravel per cubic yard of concrete as percentages of the total weight. Allow 1 cu ft per sack of cement.

58. Refer to Prob. 56 for data, and then compute the quantities of cement, sand, and gravel per cubic yard of concrete as percentages of the total weight. Allow 1 cu ft per sack of cement.

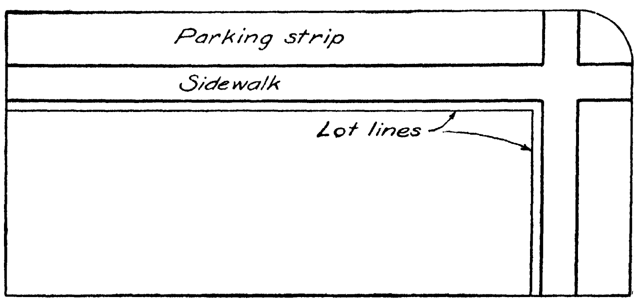


FIG. 41.

59. A concrete sidewalk is to be laid along a side and an end of a rectangular corner city lot. The lot is ft wide by ft deep, the lot lines being 1 ft inside the sidewalk (see Fig. 41). The parking strip between the sidewalk and the curb is ft wide. The sidewalks are to be ft wide and in. thick and are to extend to the curb at the corner.

a. Compute the number of cubic yards of concrete needed.

b. Compute the quantities of cement, sand, and crushed stone needed (see Prob. 55). Round off quantities to the next larger whole unit.

c. Compute the cost of the sidewalks (see Prob. 65).

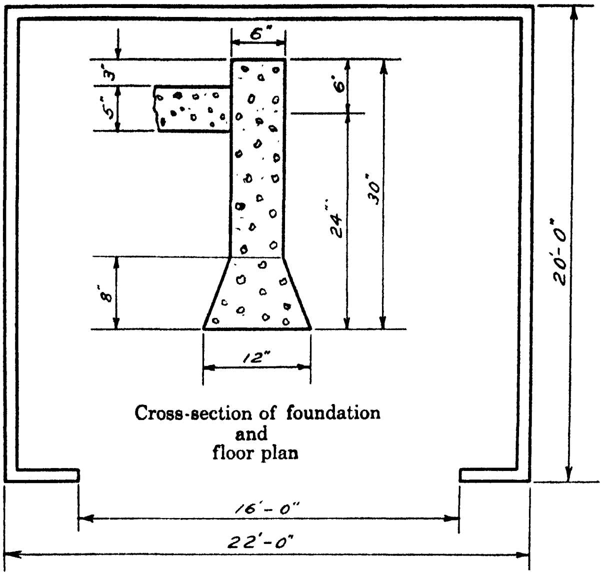


FIG. 42.—Garage floor plan and foundation.

60. Compute the quantities of cement, sand, stone, and water needed to construct the garage foundation and floor as shown in Fig. 42. Consult the figure for all dimensions and Prob. 56, page 245, for the materials required.

61. A man plans to build a cylindrical cistern ft in diameter and ft deep inside. The walls are to be of concrete; side walls and cover in. thick; and bottom in. thick.

a. Compute the area of the forms required for double forming the walls.

b. Find the quantity of concrete required for the cistern. Use the mixture specified in Prob. 56.

c. Find the capacity of the cistern in gallons.

62. The cistern mentioned in Prob. 61 is to be made of concrete proportioned as in Prob. 55.

a. Compute the quantities of material required.

b. Compute the cost of materials according to costs used in Prob. 65*a*.

c. Compute the labor cost (see Prob. 65*b*).

63. The cistern mentioned in Prob. 61 is to have 2 ft of earth over the top, and the excavation is to be such that there is an 18-in. clearance all around the cistern so that workmen can waterproof the outside surface. How many cubic yards of earth are to be removed?

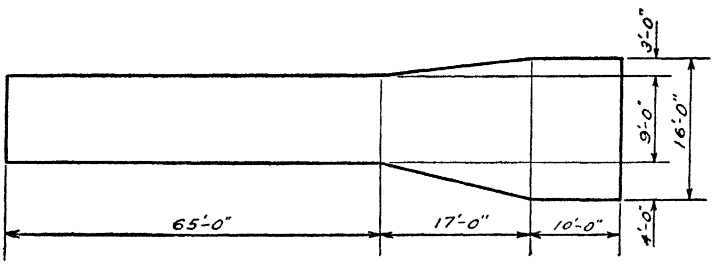


FIG. 43.—Driveway plan.

64. It was decided to put in a concrete driveway, 5 in. thick, leading to the garage mentioned in Prob. 60. The shape and dimensions are shown in Fig. 43.

a. Compute the quantity of materials needed if the mix is the same as for the garage floor (see Prob. 56).

b. Compute the cost of materials (see Prob. 65*a*).

c. Compute the labor cost (see Prob. 65*b*).

d. How much could be saved by using two 24-in. runways for the 65-ft long driveway?

65. Compute the cost of the concrete work in Prob. , if the unit costs are as follows:

a. *Materials*:

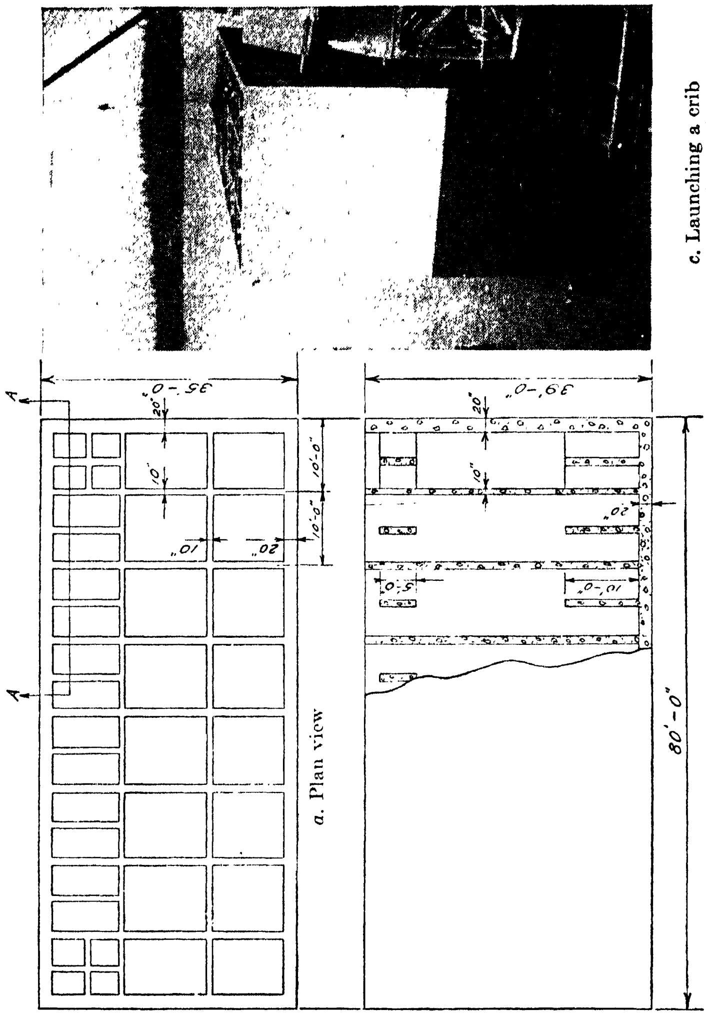
Cement @ cents per sack.
Sand @ $ per cubic yard.
Gravel @ $ per cubic yard.

b. *Labor*:

Foreman at $ per 8-hr day.
2 laborers at $ each per 8-hr day.

This crew can mix and place 1 cu yd of concrete per hour with a small 3.5-cu ft mixer.

66. The "outer wharves" in a certain Canadian port were built in an unusual manner. A series of reinforced concrete caissons or cribs were built several miles from the pier site. They were launched by means of a marine railway, towed to the harbor, then



c. Launching a crib on marine railway

a. Plan view

b. Side elevation and section A-A

FIG. 44.

sunk on a prepared foundation. When all of the cribs were in place, sand and gravel were filled in behind them and in them, making a firm foundation for the dock structures.

The details of the size and construction of a crib are shown in Fig. 44.

Compute the following:

a. Weight of the crib (see Table 16, page 367).

b. Amount of "freeboard," or height of sides above the water surface, when the crib was floating in sea water.

LOGARITHMS

67. Solve the following problems, using five-place logarithms. See and use Form 35 in the Workbook.

$$A = (16.54)(287.5)$$
$$B = \frac{190.3}{0.4162}$$
$$C = (17.23)^{2.4}$$
$$D = \sqrt[3.2]{69.46}$$
$$E = (12.05)(4.167)(6.195)$$
$$F = \frac{(45.75)(0.07142)}{0.006071}$$
$$G = (0.1928)(3.142)$$
$$H = \frac{6.047}{0.07152}$$
$$J = (35.68)^{1.8}$$
$$K = \sqrt[2.3]{94.63}$$
$$L = (12.05)(72.63)(4.275)$$
$$M = \frac{(81.91)(75.64)}{0.06143}$$
$$N = (0.06123)(95.08)$$
$$P = \frac{0.07163}{0.0008243}$$
$$Q = (0.08121)^{-0.2}$$
$$R = \sqrt[-0.4]{0.07123}$$
$$S = (0.04612)(7.174)(69.15)$$
$$T = \frac{(0.04215)(71.83)}{85.97}$$

68. Solve the following problems, using five-place logarithms. See and use Form 35 in the Workbook.

$$A = (21.68)(0.1042)$$
$$B = \frac{34.26}{0.06923}$$
$$C = (716.3)^{0.21}$$
$$D = \sqrt[2.4]{64.52}$$
$$E = (69.41)(0.008154)(91.67)$$
$$F = \frac{(0.2731)(74.32)}{2876.}$$
$$G = (0.04123)(46.81)$$
$$H = \frac{0.07164}{0.8916}$$
$$J = (25.58)^{0.08}$$
$$K = \sqrt[3.2]{100.6}$$
$$L = (61.38)(79.67)(0.04217)$$
$$M = \frac{(89.43)(0.006172)}{0.07149}$$
$$N = (0.08125)(12.23)$$
$$P = \frac{0.08176}{0.9973}$$
$$Q = (0.0006173)^{0.023}$$
$$R = \sqrt[-3.7]{0.006405}$$
$$S = (4.162)(3.162)(0.04185)$$
$$T = \frac{(65.65)(0.04211)}{8.167}$$

69. Solve the following problems, using five-place logarithms. Interpolate for the fifth digit. See Form 36 in the Workbook.

$A = (1708.4)(475.39)$

$B = \dfrac{62.755}{0.021673}$

$C = (24.473)^{0.18}$

$D = \sqrt[0.2]{64.329}$

$E = (57.636)(48.217)(0.71532)$

$F = \dfrac{(0.0021305)(74.316)}{82.454}$

$G = (92.763)(16.735)$

$H = \dfrac{4.8123}{72.797}$

$J = (487.38)^{0.021}$

$K = \sqrt[2.2]{8.1421}$

$L = (82.316)(0.041609)(22.802)$

$M = \dfrac{(816.75)(215.02)}{111.11}$

$N = (0.041299)(0.61907)$

$P = \dfrac{0.061937}{0.0042312}$

$Q = (945.39)^{-0.33}$

$R = \sqrt[-0.41]{0.081673}$

$S = (82.107)(4.1222)(69.198)$

$T = \dfrac{(25.412)(6.1715)}{0.042679}$

70. Complete the value of the following by logarithms:

$$x = \frac{(24.3)(902.5)(0.00274)(\qquad\qquad)}{(104.2)(97.3)(\qquad\qquad)}$$

71. In a certain piece of research work it was necessary to measure the amount of water flowing through a line of tile. A series of orifice plates was used for this purpose. They were made of heavy sheet brass, and each plate had a circular hole through its center. The holes were of different sizes, and the areas of the openings were made convenient decimals of a square foot. In order to bore the holes it was necessary to give the machinist the exact diameter of the opening to half a thousandth of an inch.

Compute the diameters of the following orifices in inches to the required degree of precision:

Area 1 was 0.001 sq ft
Area 2 was 0.025 sq ft
Area 3 was 0.100 sq ft
Area 4 was 0.500 sq ft
Area 5 was 0.750 sq ft

72. Steam turbine power problems involve calculations of the quantity of steam passing through an orifice, pressure and weight of the steam, etc. The following formula is often used:

$$w = 0.01296D^2P^{0.97}$$

where w is the weight of the steam in pounds per second, D the diameter of the circular orifice in inches, and P the steam pressure in pounds per square inch. Find the quantity used in pounds during 2.5 hr when $D =$ in. and $P =$ psi. Use logarithms.

73. A formula that is used in designing the ducts for a warm-air heating system reads as follows:

$$p = (0.075)(0.00624)(d)\left[\frac{V^{1.9}}{2g}\right]\left[\frac{c^{1.18}}{a}\right]$$

In this formula p is the loss in pressure, due to friction, in pounds per square foot; V is the velocity in feet per second; c is the perimeter of the duct in feet; a is the area of the duct in square feet; and d is the length in feet. Find the loss of pressure when V is fps, c is ft, a is sq ft, and d is ft.

74. The following problems illustrate several ways in which decimal points may be located when raising decimal values to some power by the use of logarithms. Refer to page 88; then solve the problems below, handling the decimal point by each of these three methods:

a. Use the absolute value of the logarithm; that is, subtract the positive mantissa from the negative characteristic, thus getting a negative mantissa.

b. Use negative characteristics.

c. Use the "9 − 10" system.

$A = (0.04273)^{0.24}$	$F = (0.0716)^{0.16}$	$L = (0.0825)^{0.22}$
$B = (0.002965)^{-1.2}$	$G = (0.00416)^{-2.4}$	$M = (0.00718)^{-3.8}$
$C = (0.963)^{-0.15}$	$H = (0.925)^{-0.25}$	$N = (0.754)^{-0.71}$
$D = (0.08725)^{2.645}$	$J = (0.0616)^{3.165}$	$P = (0.0412)^{2.718}$
$E = (0.00643)^{-0.03}$	$K = (0.000607)^{-0.08}$	$Q = (0.00213)^{-0.06}$

COMPOUND INTEREST

75. A note having a face value of $, a term of yr, and calling for interest at per cent per annum is offered for sale. How much should be paid for it if the buyer wishes to realize per cent on his investment?

76. If $ are invested in the bonds of the local public utility company at the end of each year for yr, and the income is reinvested each year to earn interest at the same rate so that the interest is compounded annually, what will be the total accumulation at the end of the investment period when the interest rate is per cent per annum?

77. Many men now provide for old age by building up an annuity fund. If a man pays in $ each birthday for 20 yr and interest is earned at the rate of per cent per annum, compounded annually, how large a retirement fund will he accumulate?

78. A contractor wishes to build up a sinking fund so that he will have money on hand for replacing certain equipment by the time it should be discarded.

How much should he pay into this fund at the end of every months in order to accumulate a total of $ in yr? The interest rate is per cent per annum.

79. A sinking fund for new equipment has had $ deposited in it each Dec. 31 for the past yr. If interest is compounded annually at per cent, what is the accumulation of the sinking fund?

80. An annual deposit in a sinking fund for the purchase of new machinery is made each Dec. 31 for yr. If the annual deposit is $ and the interest rate is per cent per annum, how much will be in the fund

a. At the end of yr?

b. At the end of the sinking fund period?

81. A certain contractor wishes to establish a sinking fund to pay for replacing some machinery. How much money must be laid aside for the fund every months in order to accumulate a total of $ in yr? The fund includes interest compounded at every payment period. The annual interest rate is per cent.

82. A man buys an automobile costing $, delivered. He receives an allowance of $ on his old car and also has $ cash to pay

down. He finds that he can save interest by borrowing from a bank; so he signs a note for the unpaid balance and pays the dealer in full. He pays the note with 12 equal monthly payments including interest figured at per cent on the unpaid balance. What is the amount that he should pay each month?

83. A common method of financing the purchase of property is that of making an initial cash payment, followed by a fixed number of equal installments which include the interest on the unpaid balance and a payment on the principal.

Form a table showing the following facts for the problem outlined below: payment period, amount of principal outstanding at the beginning of each period, amount of periodic payments, and the portion of each payment that is applied on the principal at the end of each period.

A contractor makes a purchase of equipment costing $, . The cash payment is 20 per cent of the selling price. The balance is to be paid in six equal payments extending over yr. The interest is per cent per year.

84. A common way of paying a contractor for local improvements in cities is to give him bonds issued by the city, but to be paid by the special assessment taxes. The contractor must sell these bonds, as a rule, in order to get cash to pay for labor and materials. The bonds are supposed to be paid off by equal annual installments plus interest.

A certain contractor has received $75,000 in bonds, earning per cent. They are supposed to be paid off at the rate of $7500 per year.

a. What is the maximum amount that he can expect to get for the bonds?

b. If the buyer wants to make 5 per cent on his investment, what will he offer for the bonds?

85. A certain engineer signs an annuity contract calling for an annual premium of $, and the term is yr. Interest on his deposits is allowed at the rate of per cent, compounded annually. Then he receives an inheritance and decides to pay the premiums in a lump sum, but interest at only per cent is allowed on all premiums paid in advance. The retirement fund is to be paid back to him

in equal annual installments, beginning yr after the last premium would normally be due.

a. What will be the total accumulation by the time the man retires?

b. What single payment will pay all premiums in advance?

c. What will be the annual income when the fund is paid back?

86. A consulting engineer has not been eligible for old-age income under the 1936 National Security Act. If he wishes to have a retirement fund for his declining years, he must set up an annuity fund in a commercial company. An engineer wishes to accumulate $ in yr.

a. If interest is earned at the rate of per cent per annum, compounded annually, what equal annual year-end payment to the annuity fund should he make?

b. If the fund is to be drawn upon at the rate of $ per year after he is 65 yr old, how long will the fund last?

87. The following formula is used in computing the present value of engineering properties:

$$\left[\begin{array}{c}\text{Condition as decimal}\\ \text{part of original value}\end{array}\right] = 1.00 - \left[\frac{(1+r)^n - 1}{(1+r)^p - 1}\right]$$

where p is the probable life in years, n is the present age in years, and r is the rate of interest, expressed as a decimal, that might have been earned had the money been invested at compound interest instead of being spent for the equipment.

a. If you own a costing $ and having a probable life of yr, compute its value at the end of each yr of its life when r equals per cent, and plot a curve showing how its value varies.

b. What is the total amount of the depreciation up to the time that 75 per cent of the probable life has been obtained?

88. The engineer is frequently called upon to determine the value of various properties. One of the important factors to be considered is the "present condition as percentage of original value" or, as usually stated, "the condition per cent" (see Prob. 87).

A certain property had an original value of $,

a probable life of yr, and the interest rate to be used is per cent.

a. Compute its condition for every yr. Tabulate the calculations.

b. Plot a graph showing its age and condition as computed in *a.*

89. The present value of an operating mining property is frequently determined by a method called *Hoskold's formula,* which is as follows:

$$P = R\left[\frac{1}{r + \dfrac{i}{(1+i)^n - 1}}\right]$$

Where P = the present worth.
R = annual operation return.
r = risk rate or desired rate.
i = going rate on safe investments.
n = number of future annual net returns to be received from the mine.

Compute the value of a mine that has a probable life of yr and an annual operation return of $ when the desired rate r = per cent and the safe rate i = per cent.

SLIDE-RULE EXERCISES

90. If $A = 8.73P$, compute values of A for the following values of P: 0.6480, 3.002, 6.425, 7.125, 8.065

91. A given grindstone can be run safely with a rim speed of 1200 fpm. What is the highest speed, in revolutions per minute, that it can be given if its diameter is ft?

92. Compute the values of y corresponding to the following values of x in the equation $x = \dfrac{0.0527y}{0.219}$

$$x = 0.0005280,\ 0.009735,\ 0.3924,\ 4.803,\ 73.85$$

93. Compute the values of x corresponding to the following values of y in the equation $y = (0.416)\left(\dfrac{3.8}{x}\right)$

$$y = 0.007515,\ 0.06525,\ 0.7255,\ 2.850,\ 6.855$$

94. Change the following speeds from feet per second to miles per hour: 17.5, 23.6, 36.2, 67.8, 143.0 fpm.

95. Table 27, page 381, gives a formula for determining the approximate wind resistance of automobiles. If the projected area (maximum cross-sectional area in a front view) of a given machine is   sq ft, find the air resistance for the following speeds: $V = 30, 40, 50, 60, 70, 75, 80, 85, 90$, and 95 mph. Tabulate the calculations.

The table of squares, Table 31, page 384, may be used for getting V^2, and the slide rule in computing the values. Make a single setting for the entire series.

96. The "second moment" of the area of a triangle is given by the formula

$$I_c = \frac{1}{36}bh^3$$

where I_c is the second moment, b is the base of the triangle, and h is its height.

Use the slide rule to compute the value of the second moment for triangles having these dimensions; all values are in inches.

b	h	b	h
4.500	6.800	0.2500	18.25
6.750	10 35	0.5250	20.75
8.500	10.50	0.6750	24.30
24.65	16.50	0.7500	14.75
45.45	22.35	0.8750	16.25

GRAPHS

NOTE: Suitable scales for a 10-line-per-inch paper are as follows: 1, 2, and 5 units per inch or these values multiplied or divided by 10, 100, 1000, etc. Do not use any prime or fractional numbers such as 1.5, 3, or multiples thereof. Adhere to the ASA code at all times [see Specs. *(83)*–*(104)*].

97. Refer to Table 23, page 377, and plot a graph showing how the working strength of
varies with diameter. Refer to Forms 30–32 in the Workbook for a sample solution of this problem for cast-steel rope.

98. Refer to Table 18, page 373, and plot a graph showing how the weight of round steel rods varies with diameter. Plot values for every 0.25 in. to and including 3.5 in. in diameter.

99. Refer to Table 18, page 373, and plot a graph showing how the weight of square steel bars varies with size. Plot values for every 0.25 in. to and including 3.5 in.

100. Plot a graph showing the relation between and diameter for Manila-rope drives (see Table 22, page 376).

101. Table 24, page 378, gives the power transmitted by turned-steel shafting. Draw a graph showing the relation between power and diameter if the speed is rpm.

102. Draw a graph from the data given in Table 24, page 378, showing how power and speed vary for a in. diameter shaft.

103. Plot a graph showing how the speed (in revolutions per minute) of a grindstone varies with diameter. Use a rim speed of 1250 fpm. Use diameters from 1 to 3 ft, taking 0.25-ft intervals. Tabulate the calculations [see Specs. (*133*)–(*151*)].

104. Plot a curve showing the volumes of spheres to and including in. diameter for every inch increase in diameter (see Formula 23, page 357).

105. Plot a graph showing the relation between the numbers 1, 1.5, 2, 2.5, etc., to 10 and the mantissas of their logarithms. Refer to Table 32, page 401, and round off logarithms to 3 decimal places (see Topic 6.11, page 135).

106. Plot a curve showing the value of the natural sine of an angle from 0 to 90°. Refer to Table 33, page 407, and plot points for every 5°.

107. Plot a graph converting speed in miles per hour to speed in feet per second. Let speed in feet per second be the independent variable. Plot values for speeds from 0 to 175 fps in 25-unit intervals. Tabulate the calculations [see Specs. (*133*)–(*151*)].

108. Plot a graph showing the relation between the temperature of water from 0 to 100°C, and its density.

t°C	Density	t°C	Density	t°C	Density
0	0.9999	20	0.9982	60	0.9832
4	1.0000	25	0.9971	70	0.9778
5	0.9999	30	0.9957	80	0.9718
10	0.9997	40	0.9922	90	0.9653
15	0.9991	50	0.9881	100	0.9584

109. The density of the population per square mile in the state of Ohio for 10-yr periods for 110 yr is shown in the table below.

Year	Density	Year	Density
1830	23.3	1890	90.1
1840	37.3	1900	102.1
1850	48.6	1910	117.0
1860	57.4	1920	141.4
1870	65.4	1930	161.8
1880	78.5	1940	161.2

Draw a graph showing the density as it has been and as it probably will be up to 1970.

110. The enrollment of undergraduate students in the Engineering Division at Iowa State College at 5-yr intervals is shown below. Plot a graph for this data.

Period	No. students	Period	No. students
1879–1880	37	1914–1915	605
1884–1885	65	1919–1920	1027
1889–1890	66	1924–1925	1158
1894–1895	156	1929–1930	1621
1899–1900	220	1934–1935	1190
1904–1905	530	1939–1940	2059
1909–1910	586	1944–1945	1391

111. The population record of two cities as shown by the United States census reports since 1860 are shown below.

Year	Worcester, Mass.	Detroit, Mich.
1860	24,960	45,620
1870	41,100	79,575
1880	58,290	116,340
1890	84,655	205,875
1900	118,420	285,700
1910	145,985	465,765
1920	179,755	993,680
1930	196,395	1,573,985
1940	193,695	1,623,450

Plot graphs of each on the same graph sheet, extending the curves to show the estimated populations in 1980.

112. When testing pumps, water wheels, and various other power-plant equipment, it is necessary to measure the quantity of water being used. One means of doing this is by the use of a weir. For rectangular weirs the quantity is $Q = 3.33bH^{1.5}$, where Q is the number of cubic feet per second, b is the width of the weir in feet, and H is the head of water (height of water on weir) in feet. Plot a graph showing how the discharge varies with the head for heads varying from 0 to 5 ft in 0.5-ft intervals. Width of weir = ft. Refer to and use Form 33 in the Workbook.

113. If a testing weir is a 90° V notch, the quantity is

$$Q = 2.53H^{2.5},$$

where Q is the quantity of water in cubic feet per second and H is the head (height of water on weir) in feet. Plot a graph showing how the discharge varies with the head for heads varying from 0 to 2.5 ft in 0.25-ft intervals. Refer to Form 33 in the Workbook for the arrangement of this problem.

114. A formula that is used in determining the head lost in friction in a pipe line is in the form $H = 0.38V^{1.86}d^{-1.25}$, where H is head lost in feet per 1000 ft of pipe, V is the velocity of the water in feet per second, and d is the diameter of the pipe in feet. Plot a graph showing how the head lost in friction varies with the velocity for the following velocities: 0.0, 0.89, 1.42, 2.37, 3.25, 4.80, 5.60, 9.02, 10.75, 12.1, 13.4, 14.0 Pipe diameter = ft. Refer to and use Form 34 in the Workbook.

115. Table 27, page 381, gives the traction resistance of trains at different speeds. Plot a graph showing the relation of traction resistance to speed.

116. Table 27, page 381, has a table giving a formula for the air resistance of automobiles. Plot a graph showing the air resistance against a car having a forward projecting area (maximum cross-sectional area in a front view) of sq ft. Plot values up to 140 mph.

MOTION EQUATIONS

117. An airplane in taking off started from rest with a constant acceleration of fpsps. After it had traveled

ft with this acceleration, its wheels left the runway. What was its velocity at this instant? How many seconds were required to reach this speed?

118. A certain automobile in starting from rest has enough "pickup" to reach a speed of 50 mph in 20 sec. How much is the acceleration if the car increases its speed at a uniform rate? How far does it travel while increasing speed at this rate?

119. A locomotive starting from rest with an acceleration of fpsps reached a speed of mph. If the acceleration was uniform, how far did it travel during the time that it was gaining speed, and what time did it take?

120. A body starts from rest and accelerates at a uniform rate for a time t. If s equals the distance it traveled during this time and v equals the velocity that it attained, develop an equation that will give the velocity for any instant in terms of the acceleration and the distance. Refer to Topic 9.8 for the basic definitions.

121. If a body is accelerating uniformly for a certain period of time, what will be its final velocity in terms of the initial velocity, the acceleration, and the time?

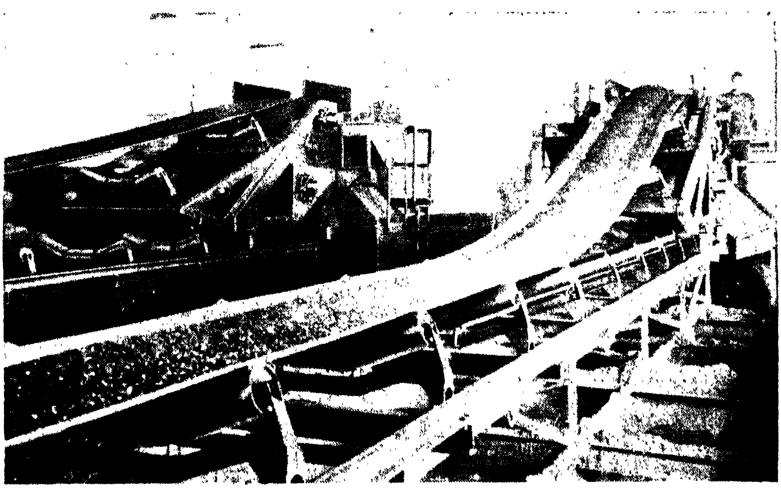


FIG. 45.—Belt conveyor. *(Courtesy of Link-Belt Company, Chicago.)*

122. A coal conveyor (see Fig. 45 above) has a maximum speed of fpm. If a load of coal is dumped

upon the moving conveyor, reducing the speed to fpm in sec, what was the average deceleration?

123. According to a certain chart showing the required efficiency of automobile brakes, it should be possible to stop a car having hydraulic brakes with a uniform deceleration of 24.7 fpsps if the brakes are in excellent condition. How long a time will be required to reduce the speed of a car from 75 to 10 mph? How far will the car travel during this time?

124. Highway engineers are telling automobile drivers that they must allow for their own "reaction time" as well as for the time required by the brakes to stop the car quickly. If it takes sec for a man to start actually applying the brakes after he sees an object dart out in front of his car, and if his automobile can be stopped with a uniform deceleration of fpsps, what is his maximum safe driving speed if his clear view ahead is limited to ft?

125. A chute for delivering shipments of freight is so designed that the boxes, starting from rest, receive an acceleration of fpsps on the first section, which is ft long, and then they are retarded by a fpsps retardation on the next section. If the time consumed in "shooting" both sections is sec, what velocity do the boxes have at the end of each section? How long is the second section?

126. A switch engine was switching some cars on level track. Starting from rest the engine got up to a speed of mph in a distance of ft with uniform acceleration. Four of the cars were then uncoupled and allowed to coast for a distance of ft. If the deceleration was fpsps while they were coasting, what was the final velocity of the cars, and what was the total time consumed?

127. A train runs down a grade for sec and after that on level track for sec. If its velocity was mph at first and mph when it reached the bottom of the grade, what uniform acceleration did it have on the grade, and what distance did it cover on both stretches of track? The speed was uniform on level track.

128. In the design of automatic machinery it is necessary to know exactly the type of motion that the various parts are to have, their velocities with respect to each other, and the distance that any part travels in a given time.

In a given case, one part at a given instant has a velocity of 5 fps and a uniform acceleration of 2.4 fps. Another part has an initial velocity of 8 fps at the time that they begin to move toward each other and a uniform retardation of 3 fpsps.

a. At what time will the two parts have the same velocity?

b. Find the magnitude of that velocity.

c. How far do the two parts travel from the time when they begin to approach each other until they pass if they started ft apart?

129. A train that has been traveling at a speed of 75 mph (110 fps) approaches a "slow signal" ahead of a stretch of track being reconstructed. The speed of the train is reduced at a uniform rate of -1.80 fpsps until the speed is 20 mph. The train travels at this speed until it reaches the "clear signal," which is miles from the first flag. The velocity is now increased at a uniform rate so that it reaches 85 mph in 90 sec.

a. What is the total length of time from the instant that the train first began to slow down until the speed of 85 mph was reached?

b. What distance did the train travel:

(1) In slowing down?

(2) At 20 mph?

(3) In regaining speed?

c. How long a time must the train travel at 85 mph in order to make up for the time lost in not traveling 75 mph continuously?

130. *a.* If a projectile is fired vertically into the air with an initial velocity of fps, how high will it go?

b. If the air resistance causes an average retardation so that the projectile goes to a height of only ft, what uniform deceleration does it have, and what is the average air retardation?

c. If the air causes the same average retardation when the projectile comes down, what will be its velocity as it reaches the ground?

WORK AND POWER

131. A farmer in southeastern Iowa has a series of wells from which he obtains water for irrigation. Each well is equipped with a centrifugal pump, which is 65 per cent efficient and will discharge 500 gpm. The maximum lift is 14.0 ft. He is now using a 6-hp gasoline engine which is moved from pump to pump as each field is irrigated. If he substitutes a portable electric motor, what size should he get?

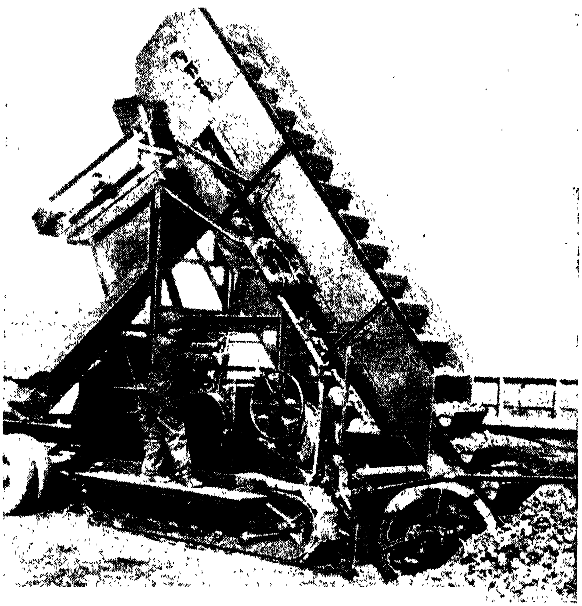


FIG. 46.—Link-Belt loader. (*Courtesy of Link-Belt Company, Chicago.*)

132. One of the manufacturers of conveying equipment makes a portable bucket elevator for loading wagons and trucks (see Fig. 46 above). It will handle 3 cu yd of crushed limestone per minute, raising it to a height of 18 ft. If the hoist is 45 per cent efficient, what horsepower is required to operate it?

133. A pump is handling 500 gal of gasoline per minute, pumping it into a tank 25 ft above the intake of the pump. The specific gravity of the gasoline is 0.66. What horsepower is required to operate the pump if its efficiency is 65 per cent?

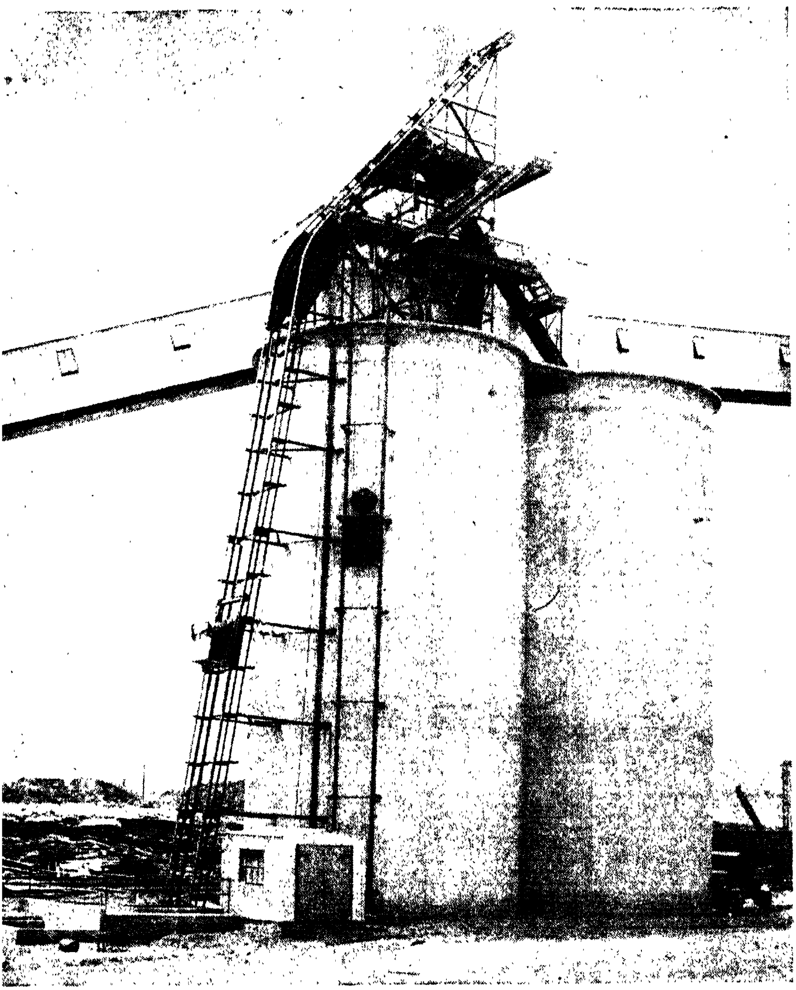


FIG. 47.—Automatic skip hoist. (*Courtesy of Link-Belt Company, Chicago.*)

134. A fully automatic skip hoist for raising crushed limestone to the top of silo-type storage bins is shown in Fig. 47.

Compute the actual horse power required to operate such a hoist under the following conditions:

Capacity of lime rock per hour.
Vertical lift = ft.
Efficiency of equipment = per cent.

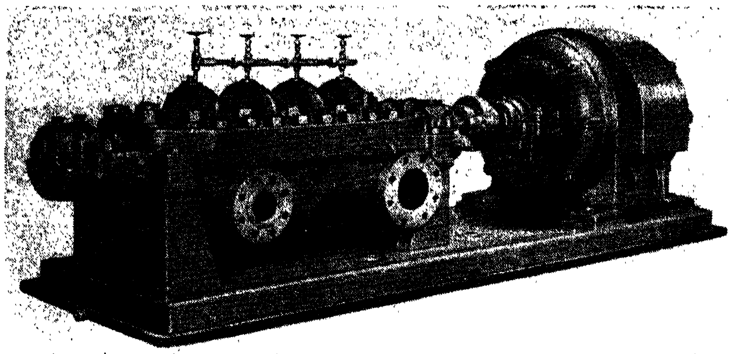


FIG. 48.—Allis-Chalmers Type M multi-stage, ball bearing centrifugal pump. (*Courtesy of Allis-Chalmers Manufacturing Company.*)

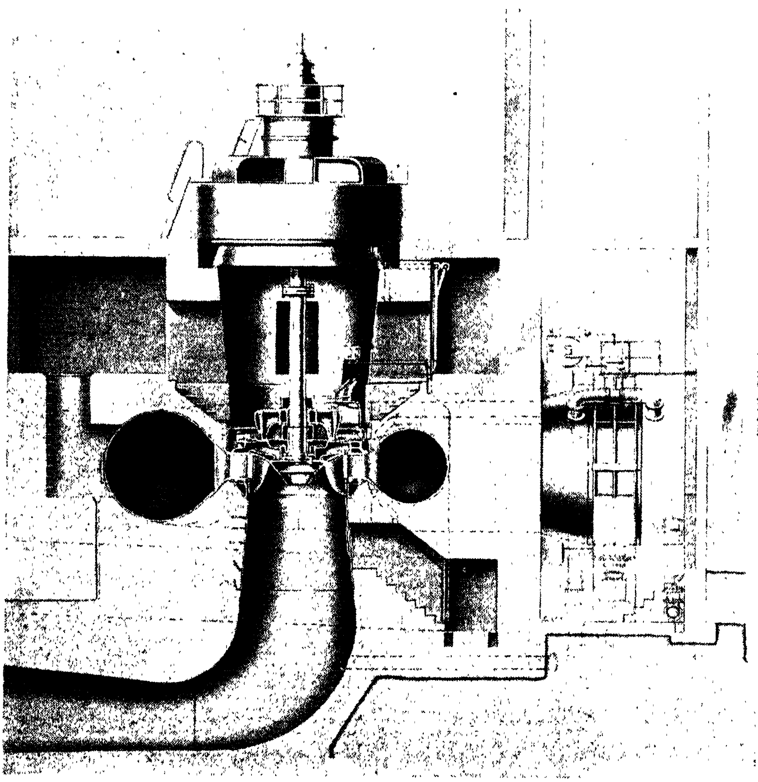


FIG. 49.—Section through one of the main turbines at Grand River Dam, showing 15-ft butterfly valve. (*Courtesy of Allis-Chalmers Manufacturing Company.*)

135. A fire boat used in protecting the water-front property of a coast city is equipped with four centrifugal pumps each having a capacity of 2000 gpm against a head of 250 ft (see Fig. 48, page 266). If the speed of the pumps is increased, they can pump against a head of 350 ft, but the quantity drops to 1200 gpm. Assuming the efficiency of the pumps to be 75 per cent in the first case and 72 per cent in the second, how much horsepower is required in each case?

136. A cross-section view of one of the four main turbines at Grand River Dam is shown in Fig. 49, page 266. The turbines are rated at 20,000 hp, and the operating head is 115 ft. The efficiency is as high as 90.7 per cent. How many cubic feet of water are being used by one of these turbines when carrying full load at above efficiency? If the generators are per cent efficient, how many kilowatts are being delivered to the switchboard?

137. A centrifugal pump is used to circulate calcium chloride brine (specific gravity = 1.2) against a head of 28 ft. The pump is delivering 350 gpm. If the pump is 67 per cent efficient, how large a motor is required?

138. An impulse wheel used on the Santee-Cooper Project is rated at 40,000 hp under 70-ft head. Compute the quantity of water needed to develop this much power if the turbine is operating at an efficiency of per cent.

139. A belt conveyor (Fig. 45, page 261) is built to deliver to the top of a storage bin. The height that the material is to be lifted is ft. The bin is ft long, ft wide, and ft deep and can be filled level full in min. What horsepower motor is required to drive the conveyor with the efficiency of the conveyor at per cent?

140. A municipal pumping plant has a maximum capacity of gpm, pumping water against a head of ft (see Fig. 50, page 268). The pumps are per cent efficient, and the electric motors are per cent efficient. The plant is operating at full load for hr per day and under load for the rest of

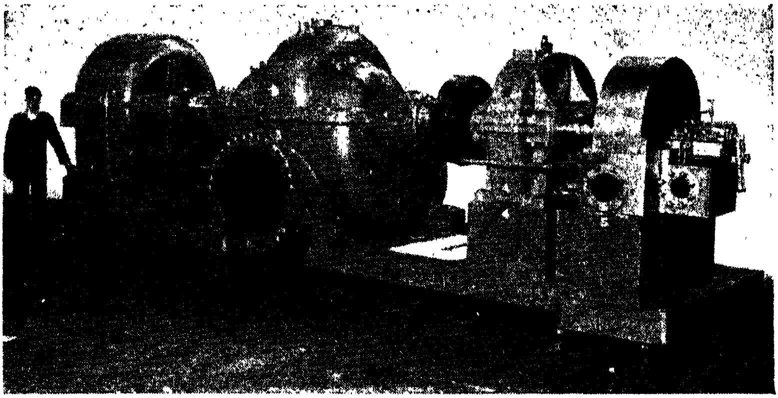


Fig. 50.—Type LS dual-driven circulating pump. (*Courtesy of Allis-Chalmers Manufacturing Company.*)

the day. If the cost of power is cents per kilowatt-hour, compute the monthly power bill.

Fig. 51.—Triplex single-acting pump. (*Courtesy of Goulds Manufacturing Company.*)

141. The water supply for a small town is raised from a well to an elevated tank by means of a triplex single-acting pump (see Fig. 51 above). The surface of the water in the well is

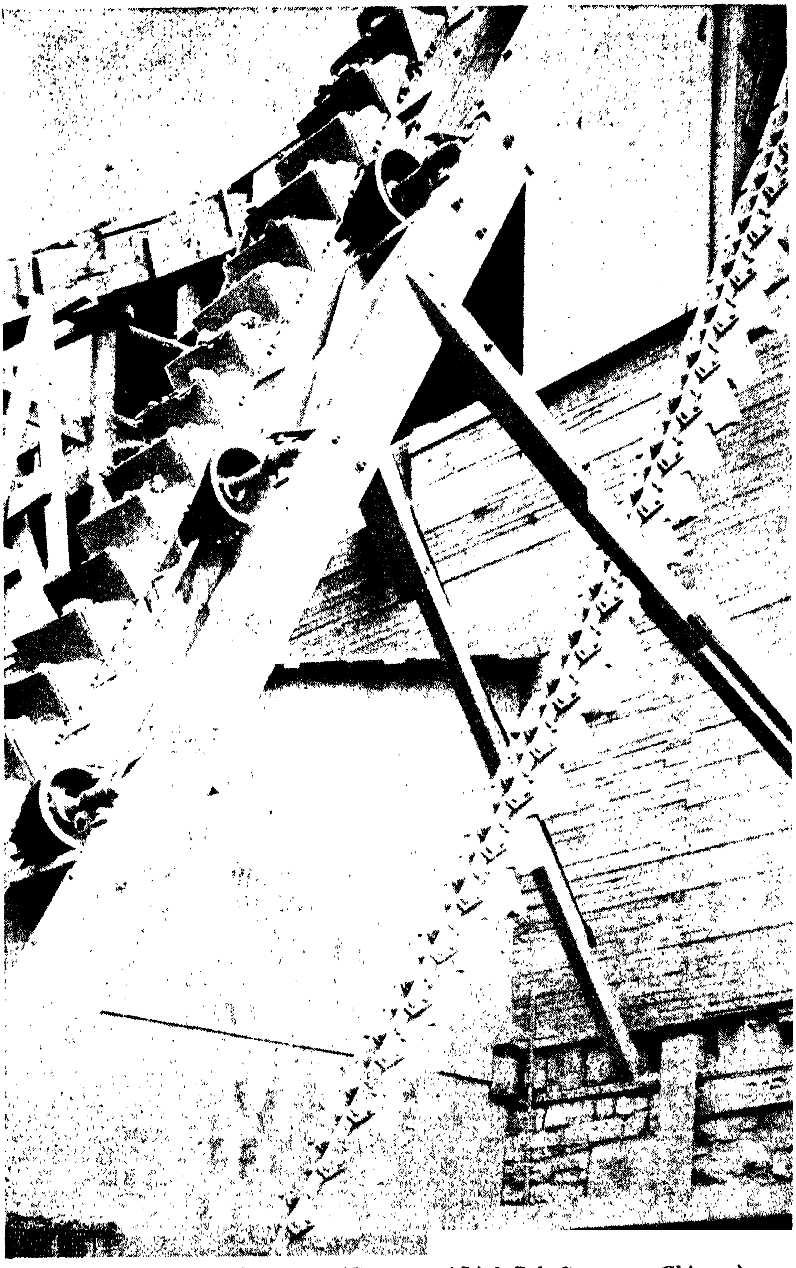


FIG. 52.—Bucket elevator. (*Courtesy of Link-Belt Company, Chicago.*)

ft below the pump, and the base of the tank is ft above it. The tank is cylindrical, ft in diameter, and ft high. Allow ft of head for the friction losses in the pipe line, and compute the following items:

a. How many foot-tons of work are required to fill the tank if the inlet is at the bottom?

b. How many foot-tons of work are required if the inlet is at the top of the tank?

c. What is the capacity of the tank if the bottom is hemispherical instead of flat?

d. If the pump that supplied the water to the tank can deliver gpm, how long will it take to fill the tank when

(1) The base of the tank is flat?

(2) The base of the tank is hemispherical?

142. A small hydroelectric power plant is operating under a head of ft. The turbines are designed for a flow of cfs. If the turbines are per cent efficient and the electric generator is per cent efficient, how many kilowatts can be delivered to the switchboard?

143. A bucket type of elevator (see Fig. 52, page 269) is raising to a height of ft at the rate of . The efficiency of the elevator is per cent. Compute the horsepower needed to operate the conveyor.

144. One of the copper-mining companies in Anaconda, Mont., has an extremely deep shaft to one of its workings. The hoisting engine raises the loaded cage from the mine at a speed of 3000 fpm. If the maximum load on the hoisting cable is tons, what horsepower is being delivered by the hoisting engine? What kind and size of hoisting rope should be used? Efficiency of the hoist is per cent.

145. In a certain type of hoisting equipment, power is transmitted from the driving shaft to the shaft carrying the rope drum by means of friction gearing (Fig. 53, page 271). The "driven" wheel is made of cast iron; the "driver" is made of , is in. in diameter, and has a width of in. The driven wheel is in.

in diameter and is run at a maximum speed of rpm. Assuming that the maximum allowable pressure per linear inch of face is used, what is the maximum horsepower that can be transmitted by this drive (see Table 25, page 379).

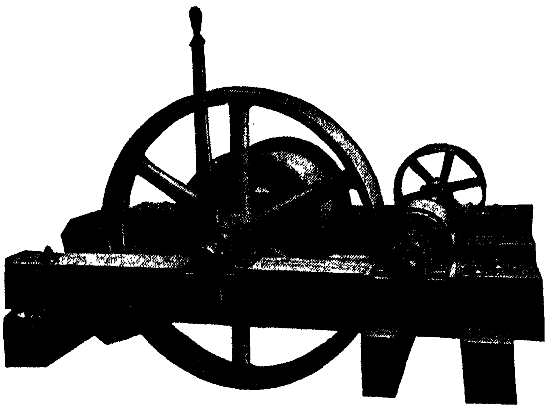


FIG. 53.—Friction hoist, showing drum and friction gearing. (*Courtesy of Allis-Chalmers Manufacturing Company.*)

146. The city of Seattle is developing a large hydroelectric project along the Skagit River in the Cascade Mountains. In order to get materials and equipment to the site of the Diablo Canyon Dam it was necessary to construct the inclined hoist shown in Fig. 54, page 272, and Fig. 55, page 273. The data are as follows:

Slope of the rails	67.5 per cent
Total vertical lift	318 ft
Maximum pay load	79 tons
Hoisting speed	100 fpm along rails
Power required for operation	400 hp

Assume that the counterweight just balances the dead load of the hoist platform. Allow 20 lb per ton for rolling resistance of the hoist.

Determine the efficiency of the hoist when carrying a maximum load.

147. A gasoline-driven hoist is advertised as being capable of raising 1500 lb vertically at a speed of 300 fpm. The gasoline engine delivers 30 hp. What efficiency must the hoist have?

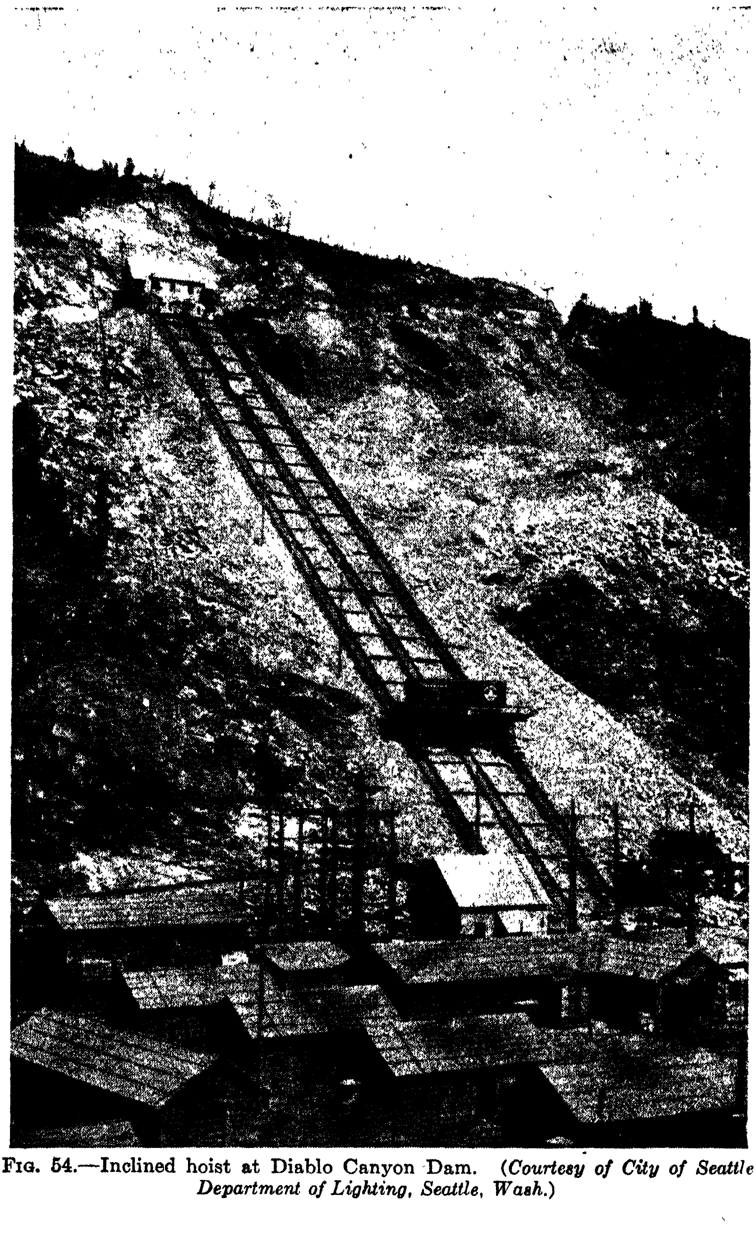


FIG. 54.—Inclined hoist at Diablo Canyon Dam. *(Courtesy of City of Seattle Department of Lighting, Seattle, Wash.)*

FIG. 55.—Inclined hoist at Diablo Canyon Dam. (*Courtesy of City of Seattle Department of Lighting, Seattle, Wash.*)

148. A manufacturer recently began making a new model hoist for contractors' use. It has two speeds and is driven by an electric motor. When operating at a slow speed, it will handle a load of 10,000 lb at 20 fpm. At high speed it will handle a load of 4000 lb at 65 fpm. Efficiency is 63 per cent. What horsepower must the motor be able to deliver in each case?

149. What horsepower must be delivered to the rear wheels of an automobile traveling on a level road in order to maintain a uniform speed of mph? The automobile weighs lb and has a projected area of sq ft (see Table 27, page 381).

150. Manila rope is sometimes used in place of belts for the driving of machinery. What horsepower can be delivered by a rope in. in diameter when its speed is fpm?

151. The head shaft of a bucket elevator is driven by means of a chain drive. The speed of the countershaft is rpm. The speed of the head shaft is rpm, and the chain sprocket on it is in. in diameter. It will require hp to drive the elevator. Compute the working strength of the chain that should be used.

152. It is desired to transmit hp by each of the following means:

a. By a double leather belt at a speed of 2500 fpm.

b. By an 8-in.-diameter spur friction wheel made of straw fiber and having a speed of 600 rpm (see Fig. 53, page 271).

NOTE: The safe load on double leather belting is 1.65 hp per inch of width at 500 fpm. The total horsepower transmitted is proportional to the width and to the speed up to a velocity of about 4500 fpm.

The safe pressure on the friction drive is to be taken as 150 lb per lin in. of face. Find

a. Necessary width of belt.

b. Width of face of the friction wheel.

153. Find the horsepower necessary to keep a ton train moving on level track with a velocity of mph. Refer to Table 27, page 381.

154. A locomotive is hauling a ton train at a velocity of mph. How much kinetic energy would be required to increase the speed to mph in sec?

155. An elevator with its load weighs tons. If it starts at the bottom of the building and reaches a velocity of fps in sec, what pull will be exerted on the hoisting rope? What size plow-steel rope will be required?

156. A skip hoist like that shown in Fig. 47, page 265, starts up from the bottom with a load of lb, including its own weight, and has a constant acceleration of fpsps for a distance of ft. By the law of work and energy, what is the maximum stress in the cable pulling the hoist? Assume 8 lb of friction per ton weight. What size standard cast-steel hoisting rope is required? What is its final velocity?

157. An automobile weighing lb is running at a velocity of mph, traveling on a dirt road. If the automobile takes a spurt and accelerates at fpsps for sec, what is the maximum horsepower required? Projected area for wind resistance is sq ft (see Table 27, page 381).

158. The belt conveyor shown in Fig. 45, page 261, has a load of tons of coal on it. If the belt is capable of sustaining a maximum safe pull of lb and friction amounts to per cent of the load, what maximum acceleration can be developed in starting the loaded conveyor?

159. A ton passenger train was running on level track at mph when an emergency stop was made. The engineer stopped the train in a distance of ft. What resistance was offered by the rails? What time did it take to stop?

160. A conveyor with its load weighs lb. It is moving at fpm. Friction amounts to per cent of the weight. If the order were given to increase the speed to fpm, what extra power would be required if this speed could be attained in ft? How long a time would it take?

161. A lb pile-driver hammer drops a distance of ft due to the action of gravity alone [see the definition of g, page 184, Eq. (9.7b)]. How much kinetic energy will it develop? What will be its final velocity?

162. A motor truck and its load weigh tons. If it is traveling on level ground at a velocity of mph, what braking force must be exerted on the wheels to reduce the velocity to mph in a distance of ft? What will be the force if it is going up a per cent grade? What if on a per cent down grade? Compute friction at lb per ton weight.

163. At a strip mine there is an inclined track for hauling coal to the bunkers. The track has a slope of 30 per cent. If the loaded cars weigh lb, with friction at lb per ton of weight, what tension must the rope pulling the cars sustain in order to start them from rest and get them up to a speed of fps in ft? Draw the equilibrium sketch, and find the work done by means of the work and energy law.

RESOLUTION OF FORCES

164. A coal car and its load weigh tons. The car is being drawn up an inclined mine shaft at a uniform speed of mph. The shaft has a slope of ft vertical to ft horizontal. Frictional resistances amount to lb per ton weight. If the hoisting machinery is per cent efficient, what horsepower must be delivered by the hoisting engine?

165. A heavy safe on skids is being hauled to the second story of a building. The safe weighs 6 tons and is being moved up an oak incline that rises 3.6 ft for every 10 ft horizontal. It is found necessary to slacken the rope used to pull the safe; so a stick of timber is braced between the safe and a convenient wall at the foot of the incline. If the timber is in a horizontal position, what force must it withstand?

166. An automobile is being towed up a per cent grade by another automobile. The first automobile weighs lb, and the frictional resistances amount to

lb per 1000-lb weight. What horsepower is required to maintain a uniform speed of mph up the grade?

167. A crated machine, on steel skids fastened to the crate, is resting on an oak gangplank that stands at a per cent grade. The total weight of machine, crate, and skids is tons.

a. Will it slide down if it is not held?

b. If it will slide, what force parallel with the incline will be required just to prevent sliding?

c. What horsepower will be required to run a hoist used to haul the crated machine up the gangplank at fpm if the hoist is per cent efficient?

d. What horizontal force must be used just to prevent sliding?

168. A chute for sliding wooden boxes from the second floor of a warehouse to the main floor is on a slope of 4 ft vertical to 7 ft horizontal. If the average box weighs 66 lb, what is the force tending to make the box slide downward? If the slide is steel, what is the resultant force down the incline?

169. A heavy stick of timber is being dragged along the level by a steel cable with a constant slope of 25 ft horizontal to 11 ft vertical. If the tension in the cable is 0.75 ton and the stick is being moved at a speed of 150 fpm along the level, what horsepower is being transmitted by the cable?

170. A gable roof has a rise of ft to a total horizontal span of ft. The wind is blowing against the roof to such an extent that each rafter must withstand a total horizontal pressure of lb. What is the normal pressure against the roof?

171. The roof on one of the engineering buildings at Iowa State College has a slope such that snow will just slide off when the coefficient of friction between the snow and the roof is 0.36. What is the slope?

172. A post 18 ft long stands at a slant so that its horizontal projection is 6 ft. If the post carries a load of 4500 lb along its axis, what are the horizontal and vertical reactions at the foot of the post?

173. A stone sled weighing 4 tons, including its load, is being hauled up a 25 per cent grade. If the cable hauling the sled is pulling parallel with the grade with a force of 4000 lb, what is the coefficient of friction between the sled and the grade?

EQUILIBRIUM

174. A Douglas fir beam in. wide by in. deep and ft long between centers of end supports is carrying two concentrated loads. The first is lb applied ft from the left end. The second is lb applied ft from the right end. Compute the reactions.

175. A beam ft long has one support at the left end, but the second is placed at a point ft back from the right end. A load of lb is concentrated at a point ft from the left support. A second load of lb is placed ft to the right of the first load. The third load is at the extreme right-hand end of the overhanging portion of the beam. Compute the reactions at each support.

176. An 8- by 12-in. Douglas fir beam ft long has a uniform load of lb per lin ft on the left ft of its span and a concentrated load of lb applied ft to the left of the right-hand support. Compute the reactions.

177. A white oak beam in. wide by in. deep is bridging a span of ft. A concentrated load of lb is applied ft from the left end. A concentrated load of lb is applied at the center point. A uniform load of lb per lin ft begins at a point ft from the left end and extends to the right-hand support. Compute the reactions.

178. The derrick shown in Fig. 107, page 307, and with dimensions as listed in Prob. 251, page 307, has a load of steel plate suspended from point D. The load consists of 20 plates of flat steel. Each plate is 36 in. wide, 10 ft long, and 0.5 in. thick. Compute the reactions at A and B.

179. Compute the reactions at A and B for the derrick shown in Fig. 105, page 306. A load of 8 lengths of pipe is carried from the point D. Each length of pipe is 4 ft long, 20 in. inside diameter, and 0.5 in. thick. The pipe is butt welded for the longitudinal seam. The derrick dimensions are listed in the statement of Prob. 249, page 307.

180. A crane of the type shown in Fig. 80, page 293, is used in a foundry for handling heavy castings. The maximum load to be lifted is tons hung from point C. The height of the post is ft, and the horizontal projection is ft. Compute the reactions at the supports A and B. All of the vertical load is to be carried at point A.

181. The jib crane shown in Fig. 106, page 307, is anchored to the side of a factory building in such manner that all the vertical reaction has to be taken care of at point A. The dimensions are listed in Prob. 250, page 307. The boom BC is stationary. Compute the reactions at A and B when a load of tons is hung at point C.

182. A heavy crane of the type shown in Fig. 108, page 308, is used for handling crushed rock. The dimensions are listed in Prob. 252, page 308. The load suspended from point F consists of a steel bucket weighing 3600 lb and loaded with 2.5 cu yd of crushed limestone. Compute the reactions at points A and B.

TRIGONOMETRY

183. A tree ft tall stands at the end of one of the runways of an airfield. A plane with a gliding ratio (cotangent of the angle of glide) of 12 to 1, glides in for a landing just clearing the tree. How far from the tree does the plane touch ground?

184. The engine in a plane stalls when the plane is at an altitude of 20,000 ft. How far will the plane glide if the gliding ratio (cotangent of the angle of glide) is 11 to 1.

185. The lower end of a hoist chain 10 ft long is ft to the right of the center line of the hoist. Compute the angle that the chain makes with the horizontal and the distance from the end of the chain to the center line of the hoist.

186. A plane leaves the ground 900 ft from a 50-ft high obstacle. What must be the angle of climb if the plane is just to clear the obstacle?

187. A force AB of 110.5 lb has a line of action that makes an angle of 36° 48′ with the horizontal. Find the horizontal component of the force, AC, and the vertical component of the force, BC.

188. A plane is flying due south with an air speed of mph. A cross wind is due east and has a velocity of 25 mph. Compute the angle of drift.

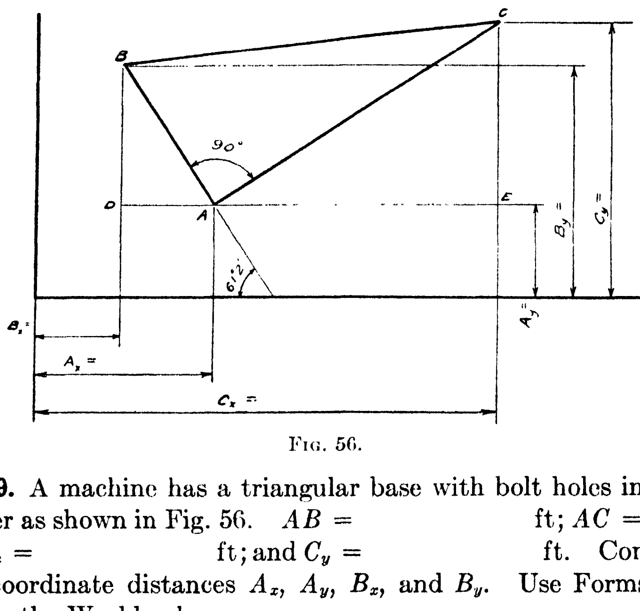


FIG. 56.

189. A machine has a triangular base with bolt holes in each corner as shown in Fig. 56. $AB =$ ft; $AC =$ ft; $C_x =$ ft; and $C_y =$ ft. Compute the coordinate distances A_x, A_y, B_x, and B_y. Use Forms 103, 104 in the Workbook.

190. A cylindrical steel smokestack is anchored by guy wires fastened to the stack three-fourths of the distance to the top (see Fig. 57, page 281).

a. One of these wires is anchored to the earth at the same level as the bottom of the stack and ft out from it. If the wire makes an angle of with the horizontal, how long is the wire and how high is the stack?

b. One of the wires must be anchored to the earth on an

elevation ft lower than the level of the base of the stack, but it makes the same angle with the stack as the short

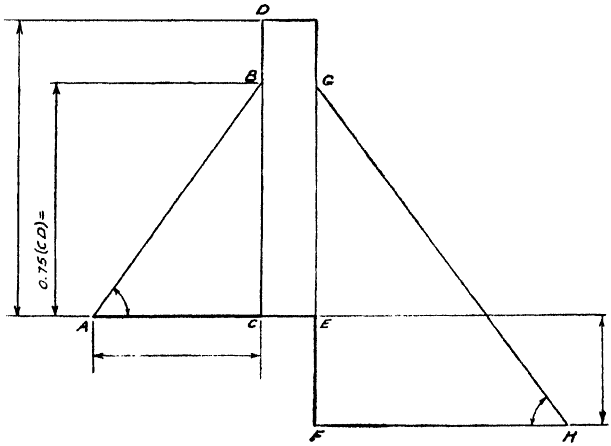


FIG. 57.

wire. How long will this wire be? Refer to and use Form 105 in the Workbook.

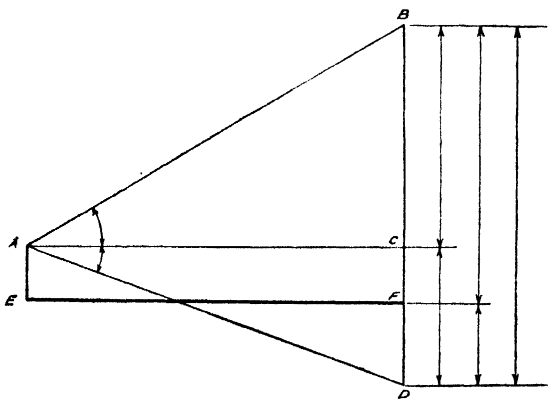


FIG. 58.

191. In a certain preliminary survey it is desired to determine the height of a cliff (see Fig. 58 above). A transit is set up over point E, the height of the instrument A being ft above E. The point E is ft, measured horizontally, from F on the face of the cliff. A sight is taken

on point *B* which is directly above point *C*. The angle *BAC* is measured and found to be . A similar sight is taken to point *D* which is vertically below point *C* and in a ravine in front of and below *E*. The angle *DAC* is measured and found to be . Compute the height of the cliff *BD* and the elevations of *B* and *D* with respect to the monument at *E*. Refer to and use Form 106 in the Workbook.

192. Power station *A* is situated 42.780 miles due west of another station *B*. It is proposed to connect two intermediate stations *C* and *D*. Station *C* is 12.150 miles east and 9.167 miles north of *A*, and station *D* is 11.260 miles west and 9.728 miles south of *B*. Find the length of wire to connect the stations in the following order: *A*, *C*, *D*, *B*. Do not square any sides in finding the desired quantities.

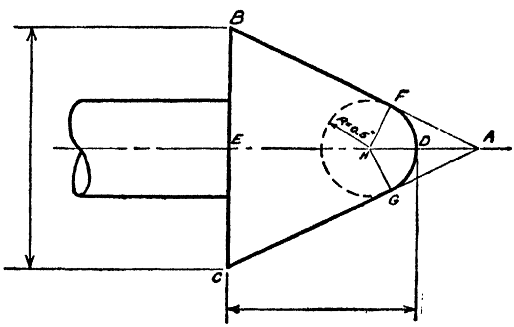


FIG. 59.

193. A ball-point center for a lathe is shown in Fig. 59. If *BC* = in. and *DE* = in., what angle *BAC* must be used in turning the center? Refer to and use Form 107 in the Workbook.

194. Figure 60 represents a machine dovetail for guiding sliding parts. The depth *AM* is to be ; the angle *ADC* is to be ; the angle *BCD* is to be ; and the bottom width *CD* is to be . The angles *ADC* and *BCD* are checked by placing gauge cylinders having a radius of in., as shown and then measuring the distance *FG* between them. Refer to and use Form 108 in the Workbook.

What will be the distance *FG* when both *DC* and the angles are correct?

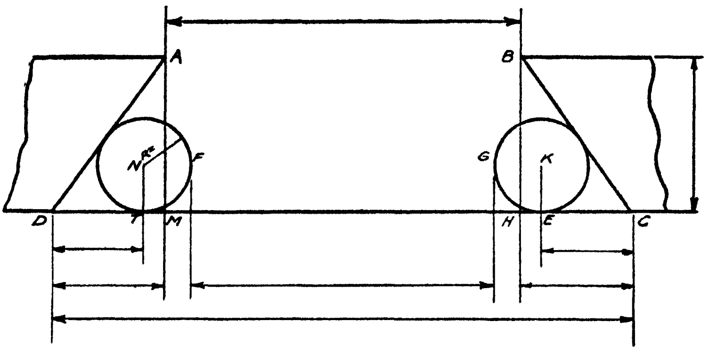


FIG. 60.

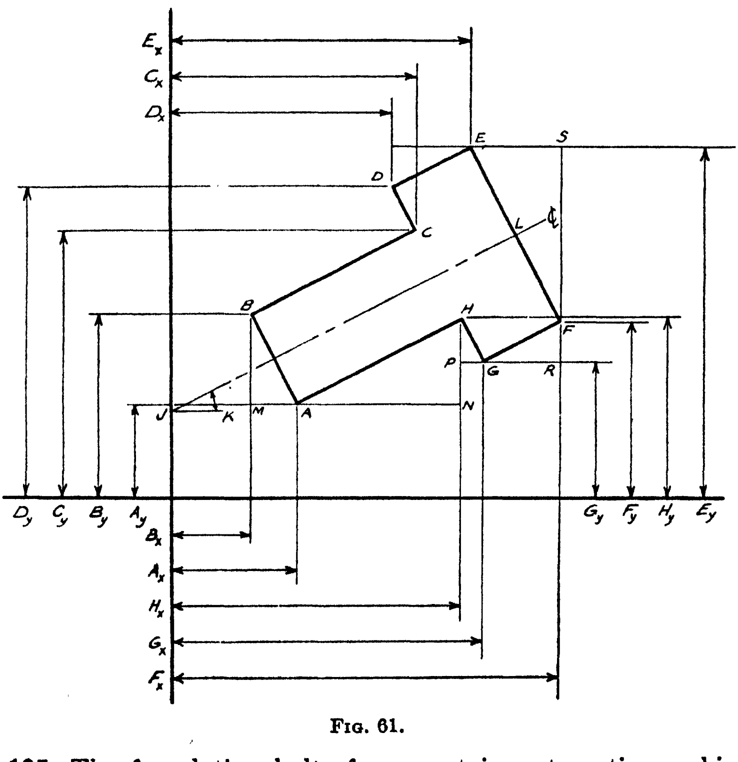


FIG. 61.

195. The foundation bolts for a certain automatic machine are spaced as shown in Fig. 61. Compute the coordinates of

the bolt holes with respect to the two walls as axes, the data being as follows:

$AB =$	Angle $LJK =$
$BC =$	$B_x =$
$CD =$	$B_y =$
$DE =$	

196. A profiling machine has an equilateral triangular base with foundation bolt holes at each corner as shown in Fig. 62.

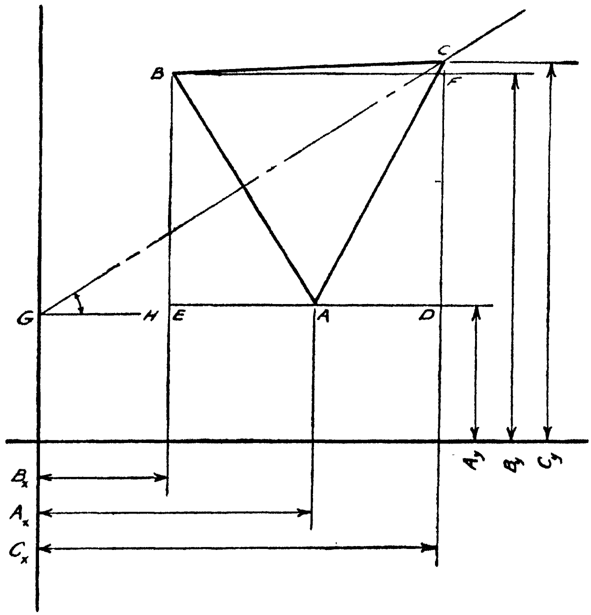


Fig. 62.

The holes are ft in. apart, center to center. Compute the coordinate distances B_x, B_y, C_x, and C_y of the other holes with reference to the walls. Angle CGH equals . The coordinates of hole A are $A_x =$ and $A_y =$.

197. A screw-cutting machine has a rectangular base with holes for foundation bolts in each corner. The holes are ft apart along the front and back and ft apart from front to back, distances measured from center to center. The machine is to stand so that its long side makes

an angle of with an adjacent wall. If the center of the nearest bolt hole is ft from the wall, locate the other holes by coordinates from the wall and this hole.

198. A regular hexagon head used on a turret lathe is in. between the flat sides. What is the length of one of the

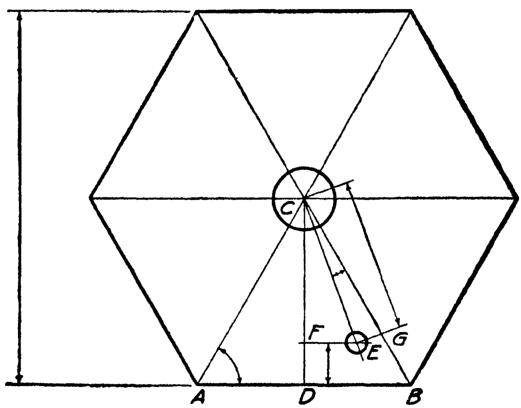


FIG. 63.

sides? If the center of a hole in the bottom of the hexagon turret is located in. from the center of the hexagon and in. from one side, what angle does a line passing through the centers of the hexagon and of the hole make with the nearest diagonal of the hexagon (see Fig. 63)?

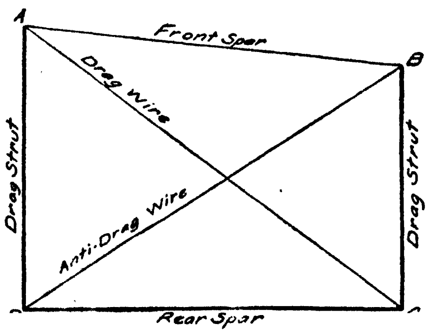


FIG. 64.

199. A portion of the wing bracing in a small plane is shown in Fig. 64. $AD =$ ft; $DC =$ ft;

$BC =$ ft. Angles ADC and DCB are 90°. Compute the length of the drag wire AC and the antidrag wire DB.

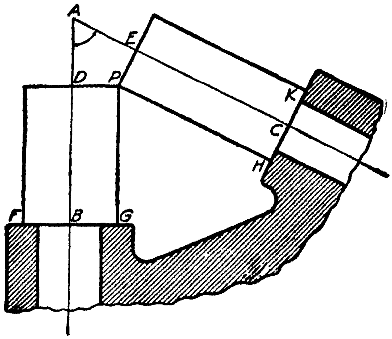


FIG. 65.

200. Figure 65 shows a cross section of a casting holding two shafts at the angle BAC with each other. These shafts are connected by a pair of bevel gears whose hubs bear on the faces of the casting FG and HK. If the gears are to run smoothly, these faces must be machined so that the distances AB and AC are exact. To test these distances plugs of specified sizes are inserted in the holes; and when they just touch at point P, the faces are correctly machined.

If angle $BAC =$, $AB =$ in., $AC =$ in., plug diameter $FG =$ in., and $HK =$ in., what must be the lengths PG and PH of the plugs?

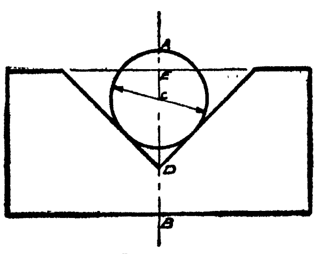


FIG. 66.

201. A 90° notch is to be machined in a block of tool steel shown in Fig. 66. To test the depth of the cut AD a plug gauge, in. in diameter, is laid in the notch, and the total height AB is measured. What should be the correct

reading of AB when the bottom of the notch is to be in. below the top surface?

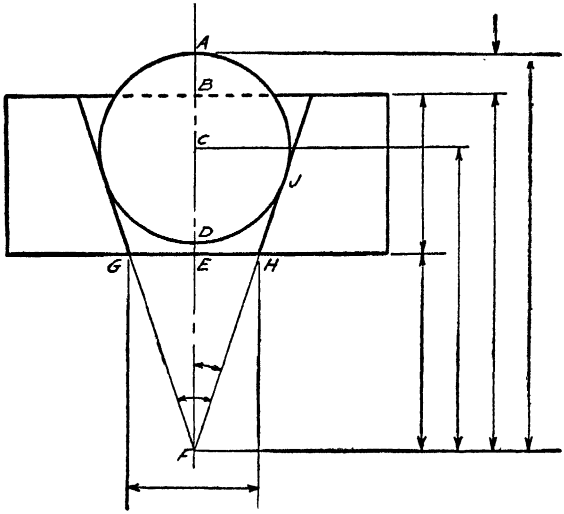


FIG. 67.

202. A tapering hole, as shown in Fig. 67, is to be drilled into a piece of steel BE in. thick. The small diameter of the hole must be GH, and the sides form an angle GFH. To test the size of the hole a ball is frequently used and the distance AB measured. If $GH =$, $BE =$, $AD =$ diameter, and angle $GFH =$, what will be the value of AB?

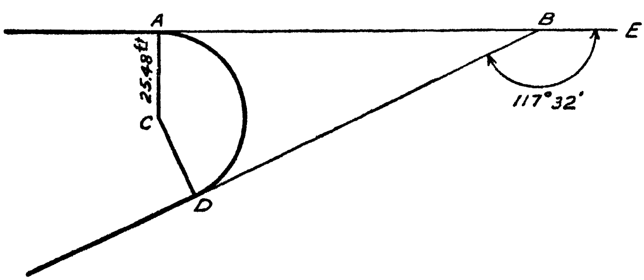


FIG. 68.

203. The curb lines of two streets that cross would make an exterior angle of 117° 32′ with each other if they were extended

to a point of intersection (see Fig. 68). A rounded curb line with a radius of 25.48 ft is built in at the corner. How far from the point of intersection will the curve start?

204. Two forces AB and AC are pulling up and to the right from point A, both forces being in the same vertical plane. Force AB = 150.5 lb, and its line of action makes an angle of 45° 14′ with the horizontal. Force AC = 75.50 lb, and its line of action makes an angle of 15° 10′ with the horizontal. Find the resultant force AR and the angle that it makes with the horizontal.

205. Two forces AB and AC are pulling up and toward the right from the point A, both forces being in the same vertical plane. Force AB = lb and acts at an angle of from the horizontal. Force AC = lb and acts at an angle of from the horizontal. Find the value of the resultant AR and its inclination.

206. Three forces, AB, AC, and AD, are pulling from an anchor. AB is acting upward and to the right; AD down and to the left; while AC is acting downward and to the right, all in the same vertical plane. The forces and their inclinations are as follows:

AB = lb; its angle with the horizontal = .
AC = lb; its angle with the horizontal = .
AD = lb; its angle with the horizontal = .

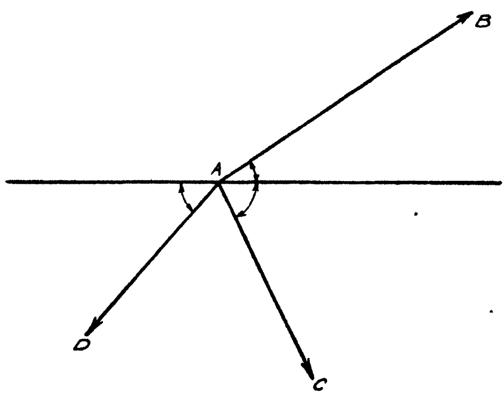


Fig. 69.

Find the resultant force and its inclination by getting the horizontal and vertical components of each force and the algebraic sum of these components (see Fig. 69).

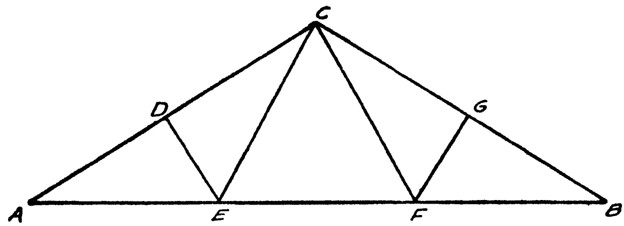


FIG. 70.

207. A given roof truss has the shape shown in Fig. 70. The lower chord is divided into three equal panels, and diagonals are run from the points E and F to C. Members DE and FG are perpendicular to AC and CB. Compute the length of the diagonal CE when $AB =$ and the height is ft. What is the length of the member DE?

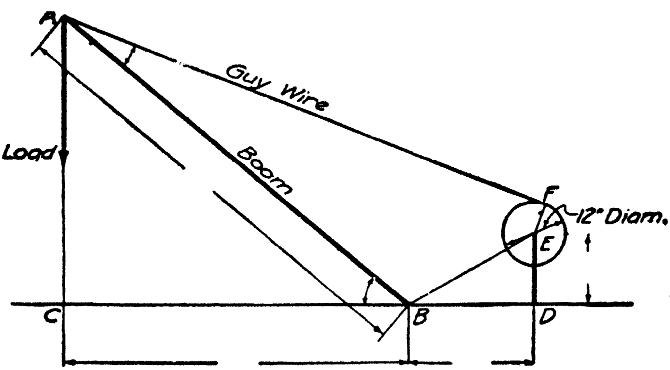


FIG. 71.

208. A Brownhoist locomotive crane has a boom and guy wire placed as shown in Fig. 71. If the minimum angle between the boom and guy wire is limited for safety to 13° 30′, how much overhang has the load, and what angle does the boom make with the horizontal?

209. A bucket made of steel plate is shown in Fig. 72.

a. Find the size required for the side plate if $ED =$, $AB =$, $BC =$, and $FD =$.

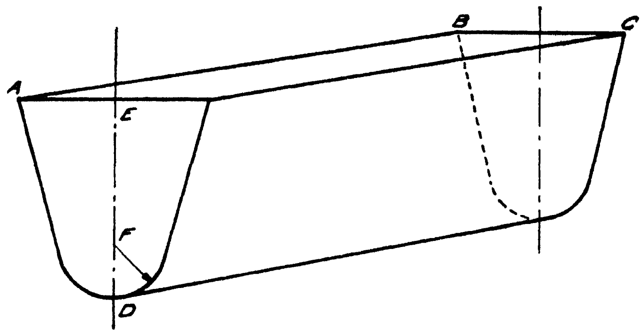


FIG. 72.

b. Find the size required if the end is shaped as shown in Fig. 73 and angle *HAB* is .

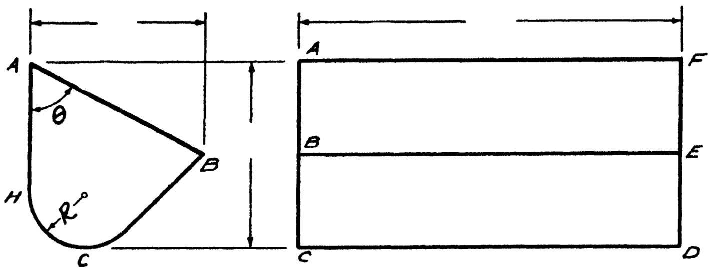


FIG. 73.

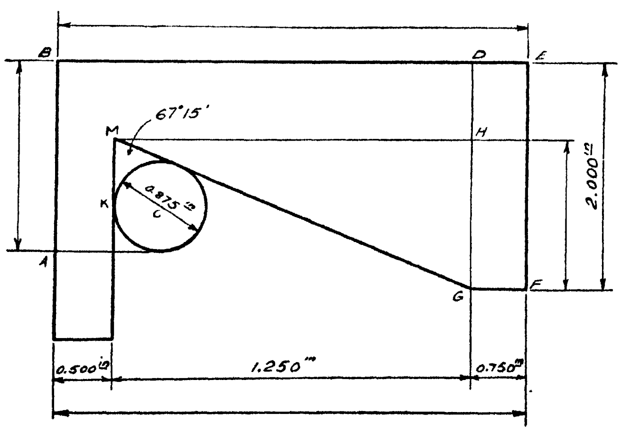


FIG. 74.

210. A special gauge is to be made according to the dimensions shown in Fig. 74 above. Determine the distances *HG* and *AB*.

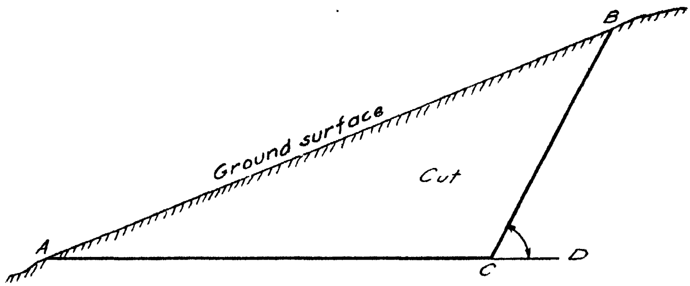


FIG. 75.

211. Triangle ABC in Fig. 75 represents the cross section of a highway cut. $AB =$ ft; $BC =$ ft; and the angle $BCD =$. Compute the length of the slope AC and the cross-sectional area of the excavation.

212. When making a survey the instrumentman discovers that the line that he is running will cross a pond. He can see across the pond; so he has the chainman set stakes at points A and B on the near and far sides. He then picks up his instrument and moves to a point C where he can sight on A and B to measure angle ACB and the chainman can measure the distances AC and CB. Compute AB when $AC =$, $CB =$, and angle $ACB =$.

213. A tunnel is to be bored straight through a hill from A to B, but neither end is visible from the top of the hill. A point C

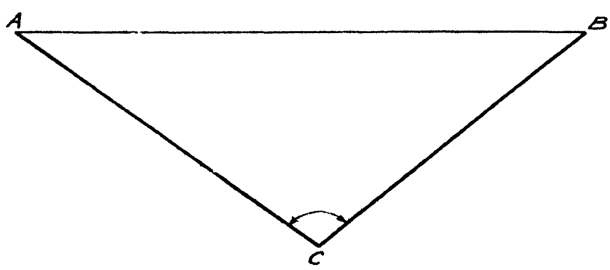


FIG. 76.

is selected over at one side so that both A and B are visible. If the length AC is found to be ft, BC is ft, and angle ACB is , what is the length of the tunnel AB (see Fig. 76)?

214. A certain distance AB must be obtained; but on account of an obstruction in the way, it cannot be measured directly. Instead, a line AC to one side of the obstruction is measured and found to be ft. The line between B and C is also measured and equals ft, and the angle ACB is . Find the length AB.

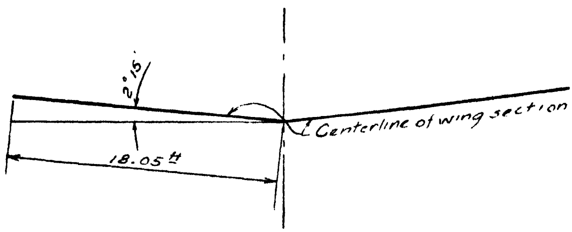


FIG. 77.

215. The wing section of the plane shown in Fig. 77 is 18.05 ft long. The dihedral angle (the angle that the center line of the wing makes with the horizontal) is 2° 15′. Compute the wing span of the plane.

216. An airplane is flying at 275 mph on a north to south course according to the compass. A cross wind of 30 mph is blowing south 47° west and carries the plane west of its course. What is the actual velocity and course of the plane?

217. A certain lot $ABCD$ is being surveyed (see Fig. 78). AB has been measured as ft; BC as

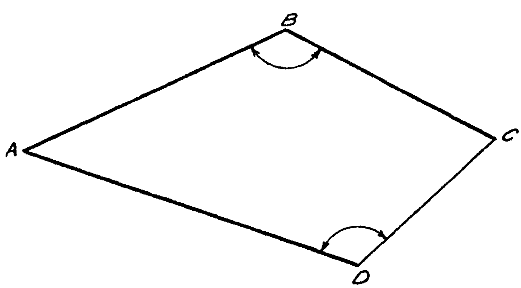


FIG. 78.

ft; AD as ft; angle ABC as ; and angle ADC as ; but it is not practicable to measure CD, owing to obstructions. Find CD and also the area of the lot.

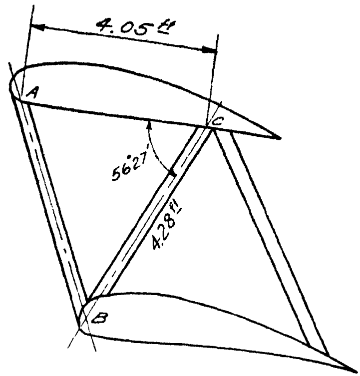


FIG. 79.

218. The interplane bracing of a biplane is shown in Fig. 79. All lines shown in the figure are center lines of the bracing. If $AC = 4.05$ ft, $BC = 4.28$ ft, and angle $ACB = 56°\ 27'$, compute the length of the leading edge strut AB.

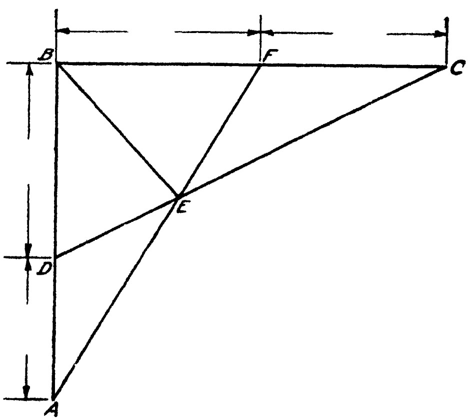


FIG. 80.

219. Figure 80 shows a bracket frame. Find all angles between parts and the length of each part that is not given.

220. The following problem was given in a certain examination for surveyors.

In a highway relocation survey it was found that the route crossed a pond; so the line could not be chained. The chief of the party had a line CD laid off as shown in Fig. 81, and angles

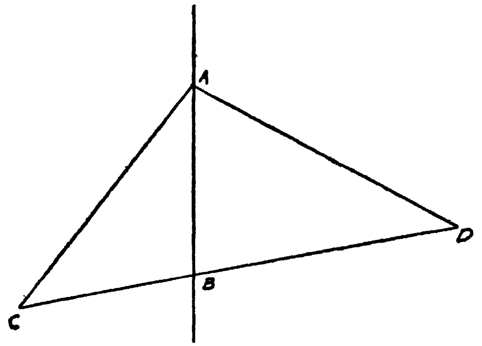


FIG. 81.

ACB and ADB and lengths CB and DB were measured. From these data compute the distance AB.

$CB =$ Angle $ACD =$
$BD =$ Angle $ADC =$

221. A water pipe line cuts through a quadrilateral field $ABCD$ on the diagonal BD. If $AB = 821.5$ ft, $BC = 1234.0$ ft, $CD = 930.0$ ft, $DA = 1578.0$ ft, angle $BAD = 77°\ 30'$, and angle $BCD = 95°\ 24'$, find the length of the pipe line.

a. Solve by triangle ABD.
b. Check by triangle BCD.

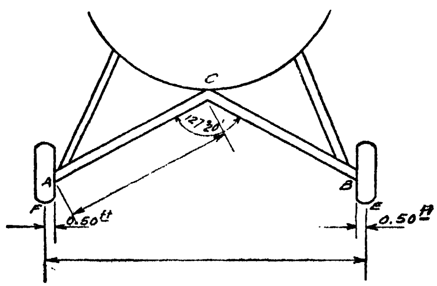


FIG. 82.

222. The landing gear shown in Fig. 82 has the following dimensions:

$AC = BC =$ Angle $ACB = 127°\ 20'$
$FA = BE = 0.50$ ft

Compute the length of the landing-gear tread FE.

223. Three monuments, A, B, and C, are important points in a survey. If $AB = 965.3$ ft, $BC = 557.8$ ft, and angle $BAC = 29° 22'$, find angle ABC, angle ACB, and the distance AC. Use Forms 111 and 112 in the Workbook.

224. A triangular-shaped area ABC on the campus is to be enclosed by sidewalks. $BC =$ ft; angle $ACB =$; and angle $ABC =$ Find lengths AC and AB and the angle CAB.

225. A survey party in making a preliminary survey for the relocation of a highway found that the line passed through a building on the right of way. The following method was used to extend the line through and beyond the building even though the line of sight was broken (see Fig. 83):

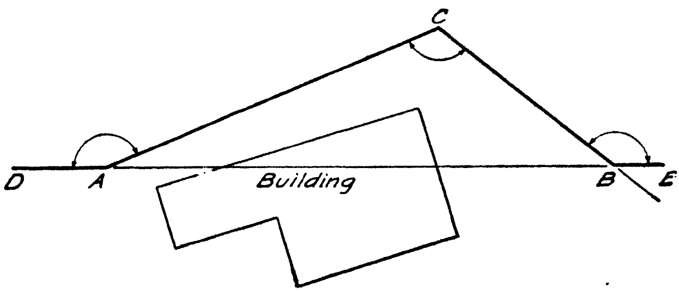


FIG. 83.

A stake was set on the line at A and another at point C. Angle DAC was carefully measured, also distance AC. Then the transit was set at C, and the angle ACB measured. It was then necessary to compute the distance CB that should be laid off in order to have point B on the desired line. Angle CBE was also computed so that the line DE could be extended ahead. The transit was then set at point B and sighted on C; angle CBE turned; and the line continued. Compute the distance CB and AB and angle CBE when AC is ft, angle DAC is , and angle ACB is

226. The wing span of the plane shown in Fig. 84 is 37.25 ft. The dihedral angle (the angle that the center line of the wing makes with the horizontal) is 7° 5′. The center line of the wing strut makes an angle of 24° 45′ with the center line of the wing.

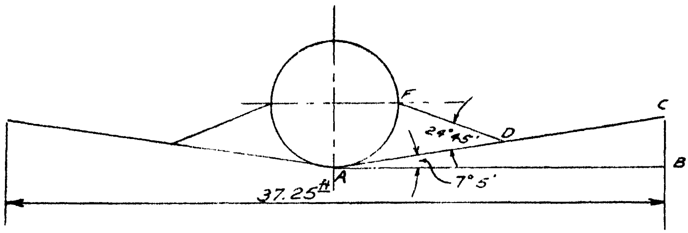


FIG. 84.

The strut is attached at the mid-point of the wing. How long is the strut?

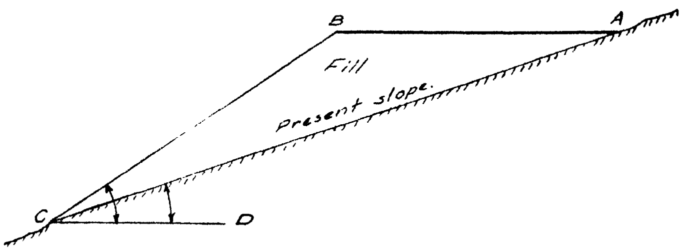


FIG. 85.

227. A level roadway is to be made on top of a fill on a hillside (see Fig. 85). The road is ft wide (AB). The present slope of the hill is angle $ACD =$, and the angle of repose (angle BCD) of the fill material is . How far from A, measured along the line CA, will be the toe of the fill, point C? What is the cross-sectional area of the fill?

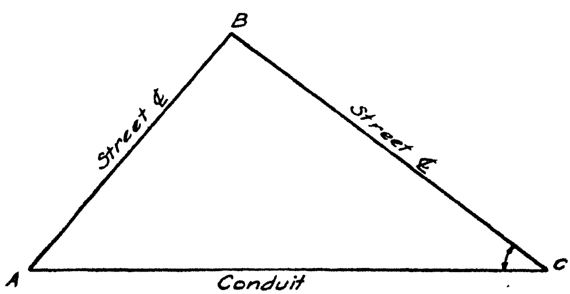


FIG. 86.

228. A covered conduit for a stream crosses the center lines of two streets that intersect at B, making the oblique triangle

ABC (see Fig. 86). The distance AB is ft, and BC is ft. If the conduit line makes an angle of with the line BC, how long is the conduit, AC.

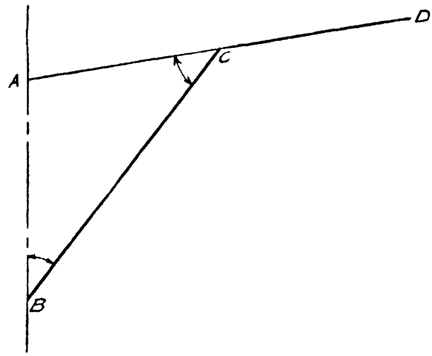


FIG. 87.

229. See Fig. 87, and compute the wing span of the plane. AB is the center line of the plane; AD is the center line of the wing; and BC is the center line of a strut. Angle $ACB =$; $BC =$; angle $ABC =$ Point C is the mid-point of the wing.

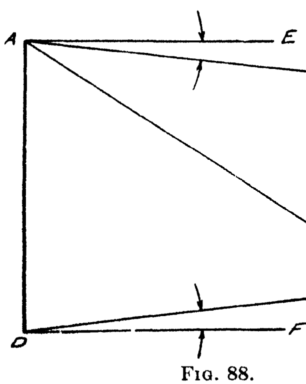


FIG. 88.

230. The fuselage bracing in a small plane is shown in Fig. 88. $AD =$ ft; angle $EAB =$; angle $CDF =$ and cross tube AC bisects angle BAD. AD is perpendicular to AE and DF. Compute the lengths of the tubes AC, BC, and CD.

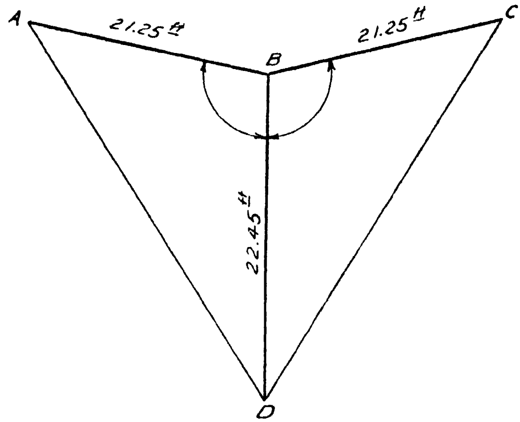


FIG 89.

231. Two radio antennas are attached to the wing and tail of a plane as shown in Fig. 89. Compute the total length of the antennas *AD* and *CD*.

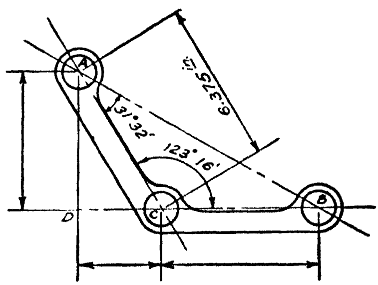


FIG. 90.

232. The bell crank shown in Fig. 90 has the following dimensions: Angle ACB = 123° 16′; angle CAB = 31° 32′; AC = 6.375 in. Compute the lengths *AD*, *DC*, *CB*.

233. A plane is flying on a course N 62° E, angle *FAD*. The pilot gets a radio bearing of 5° right on Wright Field Station, angle *DAB*. The pilot also gets a radio bearing of 38° right on Cincinnati Station, angle *DAC*. Wright Field is located N 20° E, angle *GCB*, and 49.6 miles from Cincinnati (see Fig. 91).

a. How far is the plane from Wright Field?

b. How far is the plane from Cincinnati?

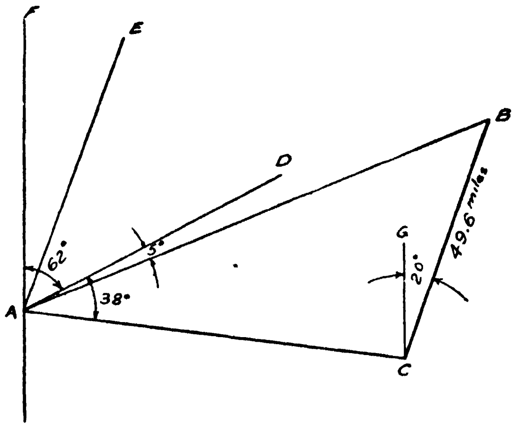


Fig. 91.

234. The bracing of a semicantilever monoplane wing is shown in Fig. 92. Angle $CAD = 2° \ 45'$; $AB =$; and $BC =$. AD is horizontal. Compute the distance AC along the wing spar to the strut joint.

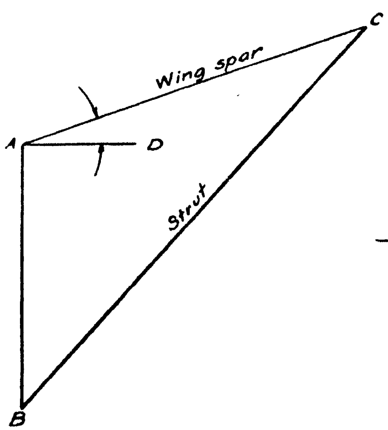


Fig. 92.

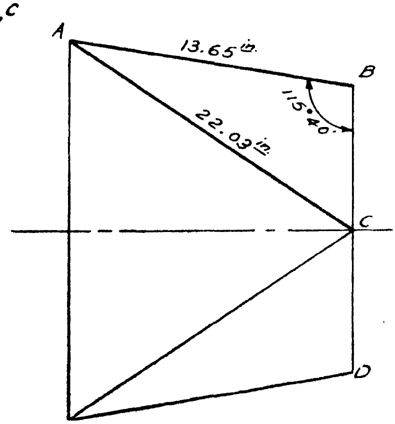


Fig. 93.

235. The engine mount shown in Fig. 93 has the following dimensions: $AB = 13.65$ in.; $AC = 22.03$ in.; angle $ABC = 115° \ 40'$. Compute the diameter of the engine mount BD.

236. The following data are for Fig. 94. Crank arm $AB = 6.25$ in.; connecting rod $AC =$ in.; angle $ABC =$

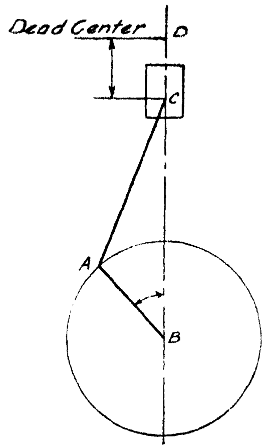


FIG. 94.

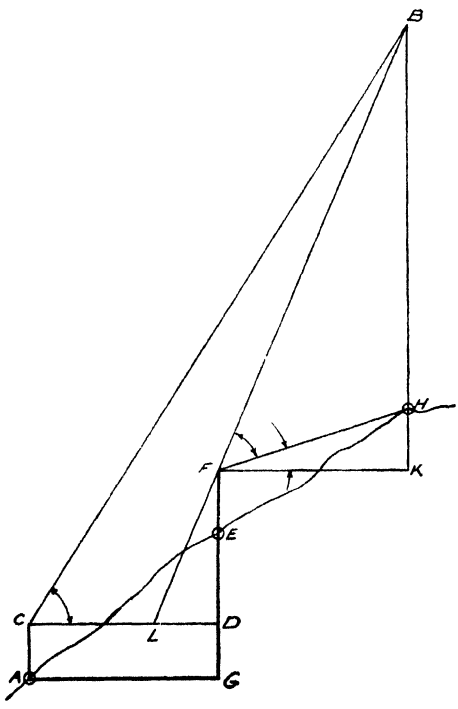


FIG. 95.

When the crankshaft has traveled through 36° 17′, how far has the piston traveled from top dead center?

237. Engineers are sometimes asked to settle questions about the height of an inaccessible structure such as a radio tower or a high smokestack. In a given case it was desired to determine the height of a famous radio tower. The tower stood on a knoll and was fenced in; so no measurements could be made at or close to the base.

A surveyor obtained the following information (see Fig. 95):

A stake was driven at point A, and a transit was set up over it, the instrument height being CA. A sight was taken on the tip of the tower, point B, and the horizontal angle BCL was read. Another stake in line with A and B was set at point E. The transit was then set up over E, the instrument height being FE. The horizontal distance AG between A and E was carefully measured; then the difference in elevation EG between A and E was measured. Sighting on B, the top of the tower, with the transit at E the vertical angle BFH was read; also angle HFK was read.

What was the height of the tower BH when the measurements were as follows:

CA =
Angle BCL =
FE =
AG =
EG =
Angle BFH =
Angle HFK =

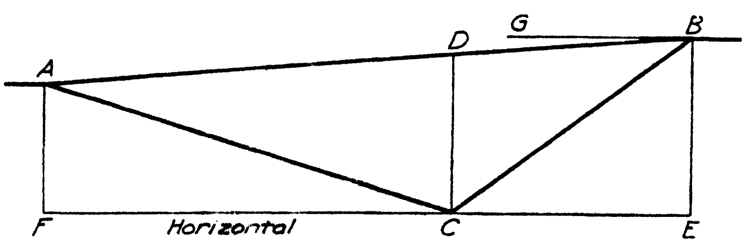


Fig. 96.

238. A depression ACB (Fig. 96) in a road is to be filled to the line AB. If AC has a per cent grade, AB

a per cent grade, and CB a per cent grade, and the total height BE is ft, find

a. The angle of the grades with the horizontal.
b. The angle of the grade AB with AC.
c. The area to be filled.
d. The maximum depth of the fill.

239. In Fig. 96, if AC has a per cent grade, BC a per cent grade, AB a per cent grade, and BC is ft long, find the grade angles, the angles in the triangle ABC, and the lengths AB, BE, and AC.

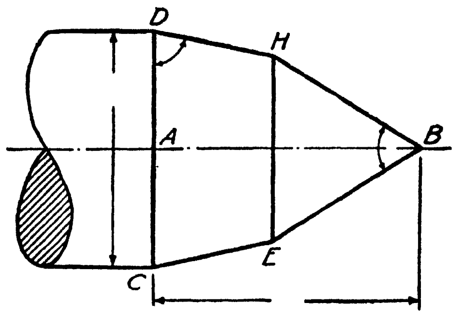


FIG. 97.

240. A double-pointed cone center is shown in Fig. 97. Total length $AB =$ in.; diameter $CD =$ in.; angle $HBE =$; angle $ADH =$ Find the diameter HE.

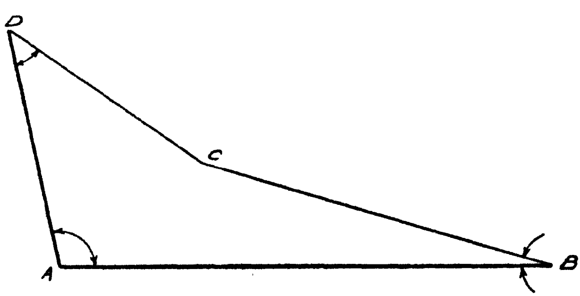


FIG. 98.

241. A machine-testing gauge, made of a flat piece of tool steel, is shaped like Fig. 98. If $AB =$ $AD =$, angle $ADC =$

angle $ABC =$, and angle $BAD =$
find the lengths DC and CB.

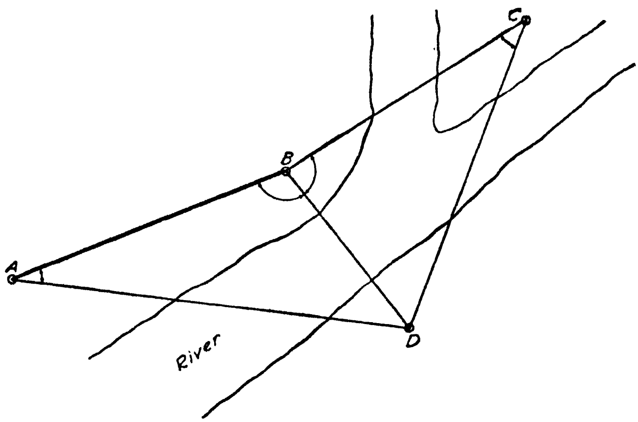


FIG. 99.

242. In laying out the triangulation for a certain region, illustrated in Fig. 99, it was necessary to establish the bench marks A, B, C, and D. The survey showed the following: $AB =$ ft; angle $BAD =$; angle $ABD =$; angle $CBD =$; and angle $BCD =$. Find the distances AD, BD, BC, and CD.

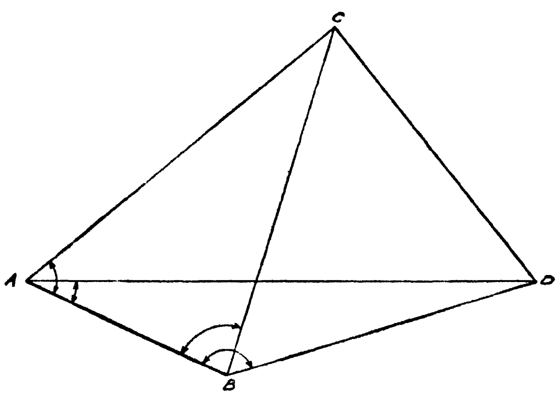


FIG. 100.

243. In Fig. 100, points B, D, and C represent important centers in an electric power-station development, and the

distances between them must be found. Since they cannot be measured directly, triangulation is necessary.

A base line AB, ft long, is measured, and the angles with D and C are measured as follows:

Angle ABC =
Angle BAD =
Angle ABD =
Angle BAC =

Find the lengths BC, BD, and CD.

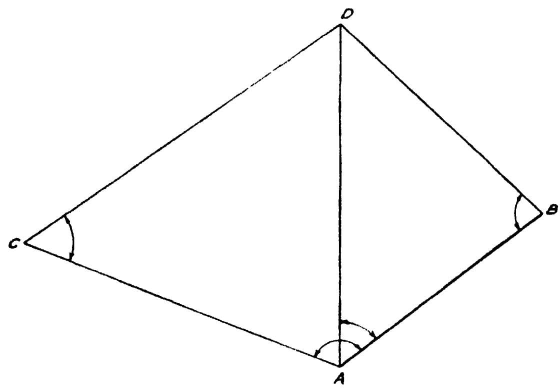


FIG. 101.

244. Referring to Fig. 101, points A, B, C, and D are important location monuments in a power-transmission development, and the distances between them, except BC, must be found. Length AB is measured as ft, and the angles with C and D measured as follows:

Angle BAC =
Angle ABD =
Angle BAD =
Angle ACD =

Find the lengths AC, AD, BD, and CD.

245. Referring to Fig. 102, points B, C, and D are important power-line transmission centers, and the distances between them must be found. Since they cannot be measured directly, a base line AB, ft long, is laid off, and

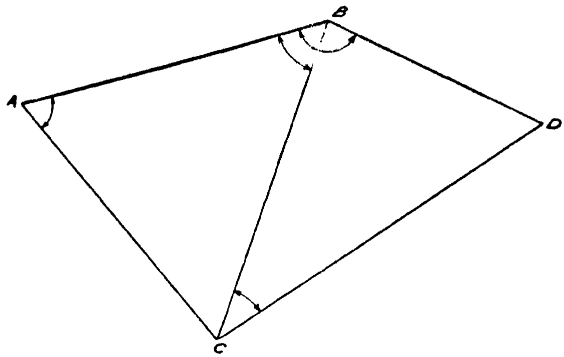


FIG. 102.

the following angles measured:

Angle ABC =
Angle BAC =
Angle ABD =
Angle BCD =

Find the lengths BC, BD, AC, and CD.

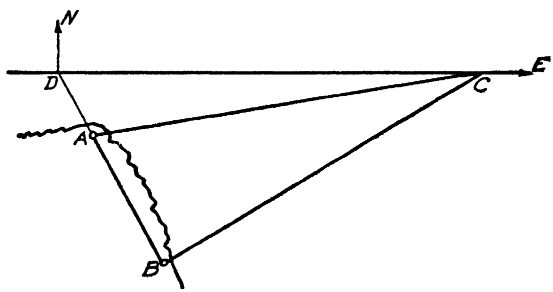


FIG. 103.

246. A and B in Fig. 103 are lighthouses miles apart. B bears S E from A. A ship is sailing due east on the line DE. One hour after crossing point D an observer on the ship at C sights both lighthouses and finds that A bears S W and B bears S W.

Find the speed of the ship, the distances AC and BC, the distances AD and BD, and the nearest distances to A and B as the ship passed north of the lighthouses.

247. Two intersecting streets AB and AC are crossed by a third street BC, forming the triangular enclosure ABC. It is proposed to put a circular park into this enclosure and to pave the remaining parts of the triangle. The curb lines $AB =$ ft; $AC =$ ft; $BC =$ ft. How large a circle can be enclosed; what is its area; and what is the area remaining?

248. The field shown in Fig. 104 has been chained, and the following distances obtained:

$AB =$	ft	$BC =$	ft
$AC =$	ft	$CD =$	ft
$AD =$	ft		

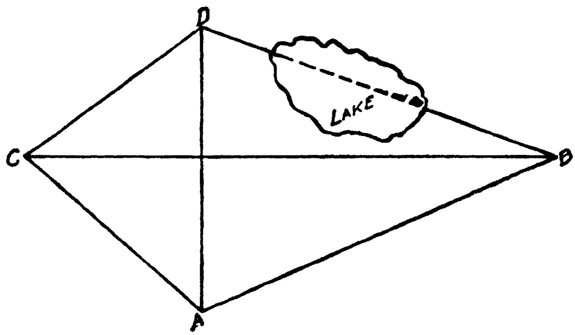


FIG. 104.

Determine all angles, the area, and the length of all sides. Get each of the angles by the specified formula, and check their sum for each triangle.

a. Use the half-angle formula (see Form 118 in Workbook).

b. Use the whole-angle formula (see Form 116 in Workbook).

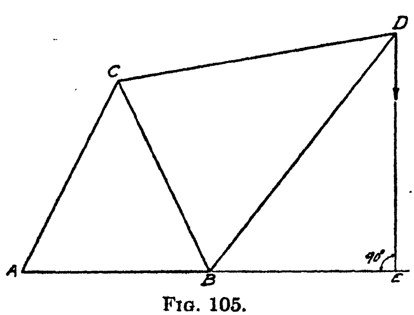


FIG. 105.

249. In the derrick shown in Fig. 105, $AB =$ $AC =$, $BC =$, $CD =$, and $BD =$. Find the overhand BE, the height DE, and also the angle DBE. Get all three angles by specified formula, and check their sums in triangles ACB and BCD.

a. Use the half-angle formula (see Form 118 in Workbook).

b. Use the whole-angle formula (see Form 116 in Workbook).

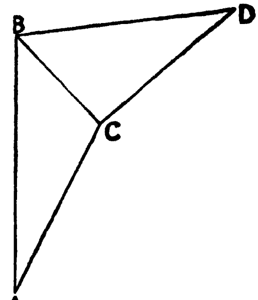


Fig. 106.

250. A certain type of foundry jib crane is shown in Fig. 106, the crane swinging around the line AB. $AB =$ ft; $AC =$ ft.; $BC =$ ft; $BD =$ ft; $CD =$ ft.

Find all three angles in each of the triangles shown, as these angles must be used in laying out the steel framework. Check the sum of the angles.

a. Use the half-angle formula (see Form 118 in Workbook).

b. Use the whole-angle formula (see Form 116 in Workbook).

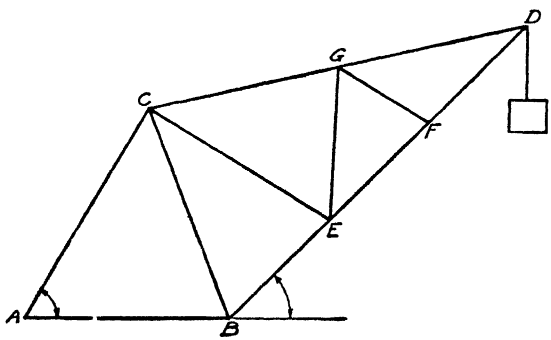


Fig. 107.

251. Figure 107 represents a side view of a crane structure, the lines representing center lines of the parts, all being in the same plane. $AB =$ ft; $AC =$ ft; $BD =$ ft; angle $CAB =$; and BD makes an angle of with AB extended.

G bisects the line CD, and E and F divide the line BD into three equal parts.

Find all angles and the lengths of all lines not given.

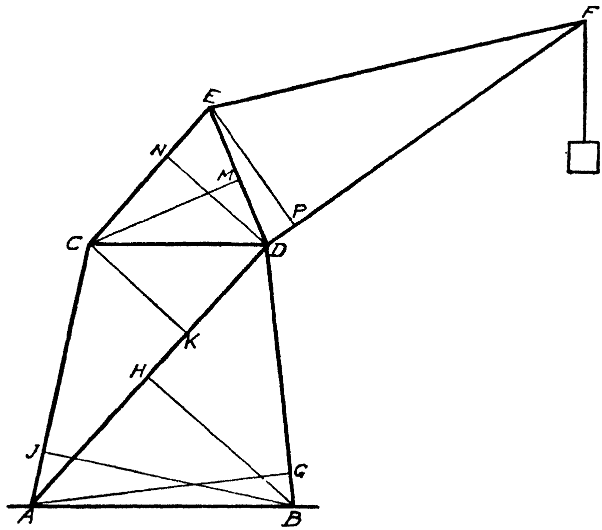


FIG. 108.

252. In the crane shown in Fig. 108,

$AB =$	$CE =$
$AC =$	$DE =$
$AD =$	$DF =$
$BD =$	$EF =$
$CD =$	

Find the perpendicular distances AG, BH, BJ, CK, CM, DN, and EP.

253. Three holes whose centers are at A, B, and C are to be bored into a piece of steel plate. If the distances between them are as given below, find the angles made by the center lines.

$AB =$; $AC =$; $BC =$.

a. Use the half-angle formula to get all three angles (see Form 118 in the Workbook). Check the sum of the angles.

b. Use the whole-angle formula to get all three angles (see Form 116 in the Workbook). Check the sum of the angles.

254. A piece of flat bar steel, shown in Fig. 109, is to be drilled at B and C. The distances shown below are known; but in order to drill these two holes, the coordinate distances AE, CE, AF, and BF must be found.

If $AB =$, $AC =$,
$BC =$, $CD =$;
$AD =$, find the above distances.

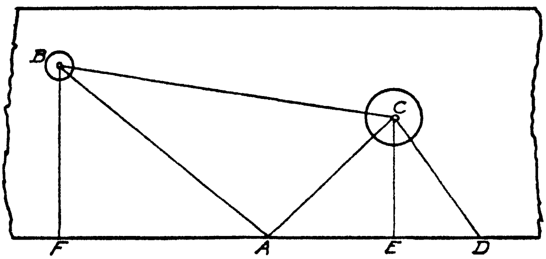


FIG. 109.

a. Use the half-angle formula, solving for all angles in triangles ABC and ACD (see Form 118 in the Workbook). Check the sum of the angles in each triangle.

b. Use whole-angle formula, solving for all angles in triangles ABC and ACD (see Form 116 in the Workbook). Check the sum of the angles in each triangle.

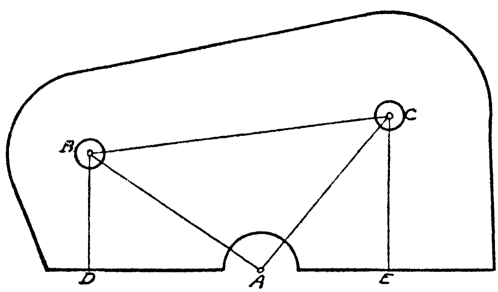


FIG. 110.

255. Figure 110 represents one view of a part of a gear speed-change box, used for changing speeds in automobiles and other machines. The holes A, B, and C are gear centers. The distances between the centers are found when the gears are designed, but to drill the holes the box is placed on the drill table so that it can move only parallel to the line DE or perpendic-

ular to it. It is necessary to compute distances AE, CE, AD, and BD.

If $AB =$, $BC =$, $AC =$, and angle $CAE =$, find AE, CE, AD, and BD.

a. Use the half-angle formula, solving for all angles in triangle ABC (see Form 118 in the Workbook). Check the sum of these angles.

b. Use the whole-angle formula, solving for all angles in triangle ABC (see Form 116 in the Workbook). Check the sum of these angles.

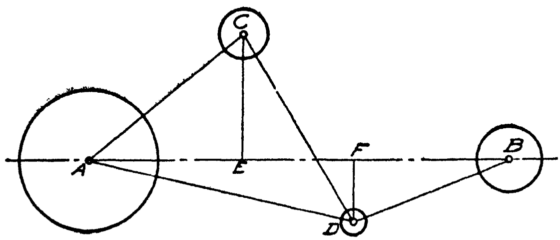


FIG. 111.

256. Figure 111 represents holes that are to be drilled into a casting for a machine.

$AB =$ $AC =$

$AD =$ $BD =$

In order to locate these holes on a boring machine, however, the two holes A and B are placed on the table in line with the motion of the table, and the holes C and D must be located by parallel or perpendicular distances. Find the distances AE, CE, AF, and DF.

a. Use the half-angle formula (see Form 118 in the Workbook). Get all three angles in each triangle, and check the sum.

b. Use the whole-angle formula (see Form 116 in the Workbook). Get all three angles in each triangle, and check the sum.

257. A plot of land is triangular in shape. The three sides have been measured and are as follows: AB is ft; BC is ft; and CA is ft. Find the area of the lot and the corner angles.

a. Use the half-angle formula, solving for all three angles. Check the sum of the angles (see Form 118 in the Workbook).

b. Use the whole-angle formula, solving for all three angles. Check the sum of the angles (see Form 116 in the Workbook).

c. Use the segment solution (see Forms 120 and 121 in the Workbook).

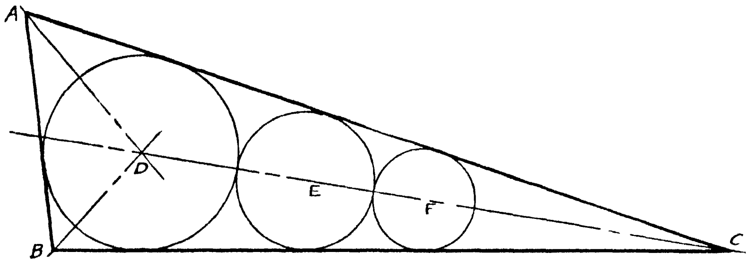


FIG. 112.

258. An ornamental iron bracket in front of a public building is in the form of an oblique triangle ABC (see Fig. 112). Side AB is ft; BC is ft long; and AC is ft long. Three circles, whose centers are at D, E, and F, form part of the ornamental ironwork in the bracket. Circle D is the inscribed circle tangent to all three sides of the triangle. Circle E is tangent to circle D and to sides BC and AC. Circle F is tangent to circle E and the same sides. Compute the radius of each of the circles, neglecting the thickness of the metal from which they are formed.

259. In making a survey in the lake region in northwestern Iowa, it was necessary to determine the distance between two points A and B, both on the farther side of one of the lakes and inaccessible. As a means of finding this distance, two points, C and D, were selected so that A and B could be seen from each point, C being nearer to A. The distance CD was carefully measured as a base line and found to be ft, and the following angles were read: $ACD =$; $BCD =$; $ADC =$; $BDC =$. Find the distance AB.

260. It is desired to connect two points C and D with a transmission line; but on account of the difficulty of measuring the distance directly, an indirect method was employed. A base line AB was measured as ft long, and the angles observed between the points A and B as follows:

Angle $BAD =$; angle $BAC =$;
angle $ABC =$; angle $ABD =$.
Find the distance CD.

261. A railroad line is being surveyed and must bridge across a river and over some tracks that follow alongside the river. Since the length of this bridge cannot be measured directly, a convenient base line CD, ft long, is measured at the side of one of the tracks along the river. Find AB, the length of the bridge required, if CD makes angles as follows:

$ACD =$ $ADC =$
$BCD =$ $BDC =$

262. A locomotive is traveling at the rate of 85 mph. If its drive wheels are 6 ft 8 in. in diameter, what is their angular velocity in radians per second?

263. A steam boiler ft in diameter is to be hoisted by means of rope slings hung from a crane hook. If the angle between the ropes from the hook is limited to , how much rope is needed for each sling, allowing 2 ft for connections?

264. Two pulleys, ft apart from center to center, are and in. in diameter, respectively. What length of belt will be required to drive one from the other if they revolve in the same direction? What length if they revolve in opposite directions?

265. A gasoline storage tank 5 ft in diameter by 20 ft in length rests horizontally on its side. How many gallons will it hold for each foot of depth? What will be the depth when one-third full?

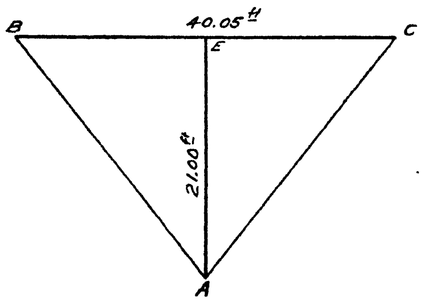


FIG. 113.

266. Two radio antennas have their ends attached to the wing of a plane 40.05 ft apart. Their other ends are attached to the tail of the plane. The antennas are attached 5.00 ft higher on the tail than on the wing. Compute the total length of the antennas AB and AC (see Fig. 113).

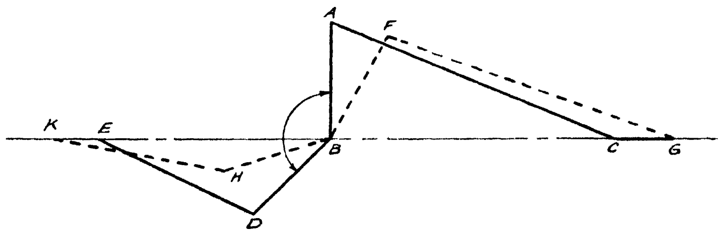


FIG. 114.

267. A portion of the control system in a plane is shown in Fig. 114. $AB = 3.50$ in.; $AG = 39.00$ in.; $DB = 2.75$ in.; $ED = 27.00$ in. If the right end of push-pull tube AG is moved 2.00 in. to the right to the new position FG, what will be the displacement EK of the left end of the pull-push tube ED?

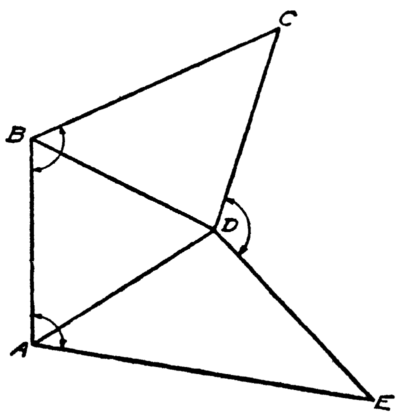


FIG. 115.

268. Three adjoining pieces of land on the campus are sketched in Fig. 115. The following measurements have been made:

$AB =$	Angle $ABC =$
$BC =$	Angle $BAE =$
$CD =$	Angle $CDE =$
$BD =$	

Plot a map of these lots, showing all angles, the length of each side, and the area of each lot.

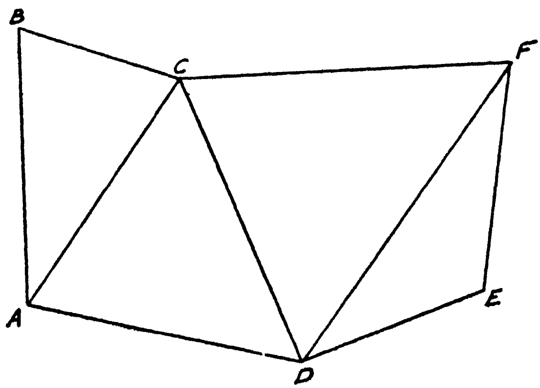


FIG. 116

269. In running a triangulation of a certain area (see Fig. 116) certain lengths and angles were measured. It is desired to compute the missing lengths and angles and also the area shown. The values are as follows:

$AC =$	Angle $ACB =$
$AB =$	Angle $ACD =$
$CD =$	Angle $CFE =$
$CF =$	Angle $FED =$
$FD =$	

CALCULUS

270. Determine the most economical proportions for a cylindrical water tank for the following conditions: Cost of sides is two-thirds of the cost of the bottom, and the cost of the cover is three-quarters the cost of the sides per square foot.

271. The space-time equation of a moving object is given by the equation $s = -0.9t^2 + 0.24t^3 - 0.04t^4$. Determine the maximum and minimum velocities occurring between $t = 0$ and $t = 7$. Also find the times when the object has maximum and minimum velocities.

272. Determine the maximum and minimum accelerations given the object in Prob. 271.

273. Determine the area under the curve, $y = 3.6 - 1.6x + 0.8x^2$, lying between the ordinates $x_1 = 0.35$ and $x_2 = 7$. All values are in inches.

274. Solve for the area lying between the two curves and limits shown below, all values measured in inches.

$$y_1 = 3.6 + 2.4x \qquad x_1 = -2$$
$$y_2 = 1.8 + 0.36x^2 \qquad x_2 = 5$$

275. Draw a diagram of the two curves $x_1 = y_1^2 - 12y_1 + 38$ and $x_2 = 0.2y_2^2 - 2.06y_2 + 5$. Compute the larger area lying between these curves and the parallel lines $x_3 = 3.0$ and $x_4 = 7.0$ All values are in inches.

276. Get the area lying below the curve $y = 3.6 - 1.5x + 0.4x^2$ and between the ordinates $x_1 =$ and $x_2 =$ All values are in inches.

277. What is the area between the curves $y_1 = 6.00 + 0.24x + 1.8x^2$ and $y_2 = 5 - 0.8x$? The area is bounded at the sides by the lines $x_1 = 1.6$ and $x_2 = 7$. All values are in inches.

278. Compute the area between the curves and limits shown below, all values being measured in inches.

$$y_1 = 10 + 2x - 1.6x^2 + 0.4x^3$$
$$y_2 = 2.8 - 2.1x + 0.8x^2$$
$$x_1 = 1.5 \qquad x_2 = 4.5$$

279. Get the area lying between the curve $x = 1.2 \log_e y$, the y axis, and the lines $y_1 = 1.5$ and $y_2 = 30$. Use natural logarithms in solving this problem (refer to Table 34, page 412). All values are in inches.

280. Compute the area between the curves $y = (1.8 \log_e x) + 1$ and $y = e^{0.8x} + 4$. The other limits are $x_1 = 2.5$ and $x_2 = 7.0$ Use natural logarithms (see Table 34, page 412). All values are in inches.

281. Determine the area under the curve $y = 2.4 \log_e x + 1.6$ and between the ordinates $x_1 = 2.5$ and $x_2 = 7.0$ All values are in inches.

282. Compute the positive and negative areas, respectively, between the x axis and the curve $y = (1.4 \sin x) + 0.6$ for one complete cycle. All units in inches.

283. Solve for the positive and negative areas, respectively, between the x axis and the curve $y = 1.1 + (3.5 \sin x)$ for one complete cycle.

284. Locate the centroid of the area lying between the two curves $y_1 = 0.8x + 4.5$ and $y_2 = 0.2x^2 - 1.6$. Use the ordinates $x = 3$ and $x = 10$ for the limits of the figure.

285. Compute the position of the centroid of a circular spandrel in terms of the radius of the circle.

286. Compute the centroid of the quadrant of a circle in terms of the radius of the circle.

287. It frequently becomes necessary to know the location of the centroid of a parabolic spandrel or a parabolic segment.

The equation of a given curve is $y = 0.5kx^2$. Find the location of the centroid of the spandrel between the limits $x = b$ and $x = 0$ in terms of b and h, h being the value of y when x equals b.

288. Compute the moment of inertia of the area bounded by the following lines: $y_1 = 2x^2$; $y_2 = 10$; $x_1 = 4$; $x_2 = 10$.

289. Compute the general formula for the second moment (moment of inertia) for any oblique triangle of height h and base b with respect to a centroidal axis parallel to the base. Any side may be used as the base.

290. The gear-shift lever used in a heavy machine has a cross section in the form of an ellipse. The major axis is in., and the minor axis is in. long. Compute the moment of inertia of the section with respect to both axes.

CENTER OF PRESSURE

291. A tank ft deep is filled to a depth of ft with a liquid having a specific gravity of A rectangular gate in. high and in. wide is in the side of the tank, and the lower edge of the gate is on the bottom of the tank. Compute the total pressure on the gate.

292. If the gate mentioned in Prob. 291 is a circle in. in diameter, what is the pressure on it?

293. A common shape of manhole used in steel standpipes and pressure tanks is that of an ellipse. A certain manhole of this shape has the major axis in. long and horizontal and the minor axis in. long. What is the total hydraulic pressure on the cover when the water is 30 ft deep over the center of the manhole?

294. A dam is closed by a trapezoidal gate whose parallel sides are horizontal and ft apart. The long side is at the bottom and is ft long, and the short side is ft long. The head on the lower side of the gate is ft. What is the depth of the center of pressure?

295. A rectangular gate ft by ft, with the long side horizontal, is used to close the sluice gate in the dam at the outlet of a lake. If the normal water depth to the bottom edge of the gate is ft, what is the depth of the center of pressure?

296. A sluiceway in a dam is closed by means of a circular gate ft in diameter. If the center of the gate is ft below the surface of the water, what is the depth of the center of pressure on the gate?

297. A large steel standpipe has an elliptical manhole in its side. The major axis of the ellipse is horizontal and is in. long. The minor axis is in. long. The center of the manhole cover is ft below the surface of the water. Locate the center of pressure on the manhole cover.

GRAPHIC CALCULUS AND NUMERICAL INTEGRATION

298. The following problem gives a comparison of the relative precision of the approximate integration rules (page 231). Carry data and all calculations to four decimal places.

a. Determine the area lying between the curve

$$y = 1.8000 + x^{-1},$$

the x axis, and the limits $x_1 = 24.0000$ and $x_2 = 3.0000$, by

each of the three methods of approximate integration, Simpson's, Durand's, and the trapezoidal rules. Use $x = 3, 4, 5, 6$, etc., to $x_1 = 24.0000$

b. Compute the area by integration, using natural logarithms (see Table 34, page 412).

c. Using the results from *b* as correct, compute the percentage error in the results by the three methods of approximate integration.

299. The naval architect uses the term *half ordinate* to describe the horizontal distance from the center line of a boat to the shell. Both the cross section of the boat and its deck plans are made up of complex curves. The half ordinates are used in laying out the cross sections and in figuring various structural properties.

Use Simpson's and Durand's rules to compute the cross-sectional area of the mid-ship section of a vessel whose half ordinates, taken 2 ft apart, are as follows, beginning with the keel as ordinate No. 0 and the deck as ordinate No. 18:

Ord. No.	Half ordinate	Ord. No.	Half ordinate	Ord. No.	Half ordinate
0	5.00	6	26.70	12	28.50
1	15.80	7	27.45	13	28.40
2	19.75	8	28.00	14	28.10
3	22.20	9	28.28	15	27.75
4	24.16	10	28.50	17	26.75
5	25.65	11	28.55	18	26.15

300. The coordinates of the corners of an irregular tract of land are given below:

Point	x, ft	y, ft	Point	x, ft	y, ft
A	0.0	0.0	*E*	1400.0	0.0
B	400.0	560.0	*F*	1000.0	−240.0
C	800.0	460.0	*G*	600.0	−100.0
D	1200.0	280.0	*H*	200.0	−180.0

Determine the x location of a north-south line that cuts off acres from the west side of the tract. Use graphic integration.

301. The coordinates of the corners of an irregular piece of land are given below:

Point	x, ft	y, ft	Point	x, ft	y, ft
A	0.0	0.0	F	1400.0	200.0
B	200.0	323.4	G	1000.0	−368.2
C	600.0	548.0	H	700.0	−378.4
D	800.0	347.4	J	400.0	−162.6
E	1200.0	412.6			

Determine the x location of a north-south line that will cut off acres from the west side of the tract. Use graphic integration.

302. In the course of making the survey of a tract of land it is necessary to compute the area between the boundaries of the land and a stream that winds through it. An east-west line is laid off, and the perpendicular distance from this line to the edge of the stream is measured every ft. The first reading is taken along the west edge of the tract. The measurements are as follows:

Line	Length, ft	Line	Length, ft	Line	Length, ft
0	205.5	5	240.5	10	154.0
1	227.0	6	229.5	11	164.5
2	272.0	7	201.5	12	187.0
3	289.5	8	163.0	13	207.5
4	278.0	9	145.0	14	216.0

a. Compute the scale factor for the integral curve, and determine the total area of the tract by graphic integration.

b. Use graphic methods only to determine the location of the north-south property lines needed to divide the tract into three lots all equal in area. Scale the lot widths, and record them on the plot.

c. Check the above results by Simpson's rule and the trapezoidal rule. Tabulate the summing of ordinates.

303. A series of velocity-time readings of a moving part in a machine was determined as follows:

Time, sec	Velocity, ips	Time, sec	Velocity, ips	Time, sec	Velocity, ips
0		5		10	
1		6		11	
2		7		12	
3		8		13	
4		9		14	

a. Plot a graph showing this velocity second by second.

b. Use graphic differentiation to determine the shape of the acceleration-time diagram. Use a mirror, and follow Forms 221–223 in the Workbook. Refer to Topic 10.28, page 229.

c. If possible, determine the equations describing the motion. Use successive differentiation if necessary.

d. Use graphic integration to draw the distance-time curve (see Topic 10.23, page 220).

e. At what time has the object completed 75 per cent of its total movement? Determine by graphic construction.

f. What are its velocity and acceleration at that time?

304. The area under a velocity-time curve between any two ordinates is equal to the total distance traveled during that time interval. Use Simpson's rule and the trapezoidal rule to determine the total distance traveled by an object that has the following velocities, the readings being taken   sec apart:

Reading No.	Velocity, fps	Reading No.	Velocity, fps	Reading No.	Velocity, fps
0		10		20	
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19			

305. An indicator card taken on the crank-end, low-pressure side of a certain Corliss engine is just 3.5 in. long. The net

height in inches of the diagram is measured at 0.25-in. intervals, beginning at the left end of the card. The measured values are as follows: 0.00, 0.48, 0.58, 0.65, 0.71, 0.79, 0.89, 1.01, 1.18, 1.37, 1.38, 1.54, 1.55, 1.67, 1.68

a. Plot a graph using the above values. Use scale factors as follows: F_x = 0.5 in. per inch; F_y = 0.5 in. per inch.

b. Use graphic integration to determine the total area under the curve. What is the average ordinate when figured from this graphic result?

c. Compute the total area of the indicator card by each of the three methods given in Topics 10.30–10.32, and also compute the average ordinate from each of the values of the area thus obtained. Which one seems to give results closest to the graphic method?

306. The area under a force-distance curve represents the work done by the force in moving an object. A variable force moves an object in such a manner as to give the force-distance readings below.

Distance from start, ft	Force, lb	Distance from start, ft	Force, lb
0	662	80	214
10	654	90	200
20	624	100	188
30	566	110	176
40	458	120	168
50	316	130	165
60	265	140	162
70	236		

a. Plot the force-distance curve.

b. By graphic integration determine the total work done.

c. Determine graphically the distances at which the following percentages of the work have been done: 25, 50, 60, 75, and 90 per cent.

d. Use Simpson's, Durand's, and the trapezoidal rules as check methods to obtain the total work done.

307. The ordinates tabulated below represent measurements made at 1.605-ft intervals on a wing panel of an English plane.

Ord. No.	Length, ft	Ord. No.	Length, ft
0	7.865	7	6.640
1	7.865	8	6.035
2	7.815	9	5.245
3	7.760	10	4.275
4	7.600	11	2.940
5	7.405	12	0.268
6	7.085		

Determine the projected wing area by

a. Graphic integration.
b. Simpson's rule.
c. Durand's rule.
d. The trapezoidal rule.

308. The ordinates tabulated below represent an airfoil cross section. The ordinates are measured at 0.3000-ft intervals.

Ord. No.	Length, ft	Ord. No.	Length, ft	Ord. No.	Length, ft
0	0.0000	7	1.0870	14	0.6555
1	0.6400	8	1.0480	15	0.5680
2	0.8645	9	1.0080	16	0.4800
3	0.9680	10	0.9445	17	0.3760
4	1.0400	11	0.8800	18	0.2640
5	1.0880	12	0.8160	19	0.1440
6	1.1030	13	0.7360	20	0.0480

Determine the cross-sectional area of the airfoil by

a. Graphic integration.
b. Simpson's rule.
c. Durand's rule.
d. The trapezoidal rule.

309. The ordinates tabulated below represent an airfoil cross section. The ordinates are measured at 0.3000-ft intervals.

Ord. No.	Length, ft	Ord. No.	Length, ft	Ord. No.	Length, ft
0	0.0000	7	0.7200	14	0.4320
1	0.4555	8	0.7040	15	0.3840
2	0.5755	9	0.6755	16	0.3200
3	0.6640	10	0.6400	17	0.2560
4	0.7115	11	0.5840	18	0.1840
5	0.7280	12	0.5440	19	0.0960
6	0.7280	13	0.4880	20	0.0320

Determine the cross-sectional area of the airfoil by

a. Graphic integration.

b. Simpson's rule.

c. Durand's rule.

d. The trapezoidal rule.

310. The following readings represent a series of sounding at ft intervals across a river. Its cross-sectional area is desired.

a. Plot the sounding downward from the x axis, and draw the cross section. Determine the area by graphic integration, putting the projection axis to the left of the pole. Put the pole at the water surface.

b. Compute the area by each of the following methods: Simpson's rule, Durand's rule, and the trapezoidal rule.

Station No.	Depth, ft	Station No.	Depth, ft	Station No.	Depth, ft
0		5		10	
1		6		11	
2		7		12	
3		8		13	
4		9		14	

311. The following velocity-time readings represent the motion of a moving part in an assembly-line machine:

Time, sec	Velocity, fps	Time, sec	Velocity, fps	Time, sec	Velocity, fps
0	1.500	5	8.000	10	9.507
1	3.195	6	8.696	11	9.201
2	4.705	7	9.202	12	8.697
3	6.005	8	9.500	13	8.000
4	7.098	9	9.598	14	7.095

a. Plot a graph showing the relation of velocity to time.

b. Use graphic differentiation to determine the acceleration-time curve.

c. Draw the "K" curve (rate of change of acceleration).

d. Use graphic integration, and draw the distance-time curve.

312. The following velocity-time readings represent the motion of a moving body.

Time, sec	Velocity, fps	Time, sec	Velocity, fps	Time, sec	Velocity, fps
0	5.000	5	3.244	10	5.991
1	4.290	6	3.431	11	7.080
2	3.759	7	3.800	12	8.350
3	3.409	8	4.350	13	9.801
4	3.239	9	5.079	14	11.431

a. Plot a graph showing the relation of velocity to time.

b. Use graphic differentiation to determine the acceleration-time curve. Refer to Forms 221–223 in the Workbook. Tabulate the work on Form 227 in the Workbook or similar columnar paper.

c. Draw the "*R*" curve (rate of change of acceleration).

d. Use graphic integration and draw the distance-time curve.

313. The following velocity-time readings represent the motion of a moving body.

Time, sec	Velocity, fps	Time, sec	Velocity, fps	Time, sec	Velocity, fps
0	4.60	5	4.20	10	7.80
1	4.20	6	4.60	11	9.00
2	3.96	7	5.16	12	10.36
3	3.88	8	5.88	13	11.88
4	3.96	9	6.76	14	13.56

a. Plot a graph showing the relation of velocity to time.

b. Use graphic differentiation to determine the acceleration-time curve. Refer to Forms 221–223 in the Workbook. Tabulate the work on Form 227 in the Workbook or similar columnar paper.

c. Draw the "*R*" curve (rate of change of acceleration).

d. Use graphic integration, and draw the distance-time curve.

314. The following distance-time readings represent the motion of a moving body.

Time, sec	Distance, ft	Time, sec	Distance, ft	Time, sec	Distance, ft
0	0.0	5	108.6	10	145.1
1	34.1	6	111.5	11	164.2
2	61.9	7	113.6	12	187.3
3	83.7	8	120.0	13	214.5
4	99.3	9	130.6	14	246.0

a. Plot a graph showing the relation of distance to time.

b. Use graphic differentiation to determine the velocity-time curve. Refer to Forms 224–226 in the Workbook. Tabulate the work on Form 227 or similar columnar paper.

c. Draw the acceleration-time curve.

315. The following velocity-time readings represent the rim speed of a rotating part in a machine during the time when it is being brought to full velocity.

Time, sec	Velocity, fps	Time, sec	Velocity, fps	Time, sec	Velocity, fps
0	0.0	5	36.5	10	119.8
1	1.5	6	52.6	11	130.0
2	5.8	7	71.6	12	137.2
3	13.1	8	90.6	13	141.5
4	23.4	9	106.7	14	142.9

a. Plot a graph showing the relation of velocity to time.

b. Use graphic differentiation to determine the acceleration-time curve. Refer to Forms 221–223 in the Workbook. Tabulate the work on Form 227 in Workbook or similar columnar paper.

c. Draw the "*R*" curve (rate of change of acceleration).

d. Use graphic integration, and draw the distance-time curve.

316. The following distance-time readings represent the motion of a moving body.

Time, sec	Distance, ft	Time, sec	Distance, ft	Time, sec	Distance, ft
0	0.00	5	7.75	10	25.00
1	0.79	6	10.43	11	29 60
2	1.96	7	13.50	12	34.55
3	3.51	8	16.95	13	39 90
4	5.44	9	20.80	14	45.65

a. Plot a graph showing the relation of distance to time.

b. Use graphic differentiation to determine the velocity-time curve. Refer to Forms 224–226 in the Workbook. Tabulate the work on Form 227 or similar columnar paper.

c. Draw the acceleration-time curve.

317. The following distance-time readings represent the velocity of a moving part in a machine during the starting period.

Time, sec	Velocity, fps	Time, sec	Velocity, fps	Time, sec	Velocity, fps
0	0.00	5	15.80	10	25.56
1	3.64	6	18.24	11	26.68
2	7.04	7	20.44	12	27.48
3	10.20	8	22.40	13	27.96
4	13.10	9	24.12	14	28.12

a. Plot a graph showing the relation of distance to time.

b. Use graphic differentiation to determine the velocity-time curve. Refer to Forms 224–226 in the Workbook. Tabulate the work on Form 227 or similar columnar paper.

c. Draw the acceleration-time curve.

318. The following velocity-time readings represent the starting velocity of a large conveyor.

Time, sec	Velocity, fpm	Time, sec	Velocity, fpm	Time, sec	Velocity, fpm
0	0.00	5	17.86	10	58.68
1	0.76	6	25.00	11	67.10
2	3.02	7	32.90	12	75.00
3	6.70	8	41.32	13	82.14
4	11.70	9	50.00	14	88.30

a. Plot a graph showing the relation of velocity to time.

b. Use graphic differentiation to determine the acceleration-time curve. Refer to Forms 221–223 in the Workbook. Tabulate the work on Form 227 in the Workbook or similar columnar paper.

c. Draw the "R" curve (rate of change of acceleration).

d. Use graphic integration, and draw the distance-time curve.

319. The moment of inertia of an irregular area can be determined approximately by constructing a new area whose ordinates are equal to the cube of the corresponding ordinate in the figure whose second moment is desired. The second moment of the given area is equal to K times the area of the constructed figure. What is the proper value for K?

320. Use the method described in Prob. 319, page 326, to determine the second moment (moment of inertia) of the shape with ordinates as follows:

Ord. No.	Length, in.	Ord. No.	Length, in.	Ord. No.	Length, in.
0		10		20	
1		11		21	
2		12		22	
3		13		23	
4		14		24	
5		15		25	
6		16		26	
7		17		27	
8		18		28	
9		19			

a. Determine the needed area by graphic methods (see Topic 10.23).

b. Determine the needed area by Simpson's and the trapezoidal rules.

DERIVED CURVES

Study Chap. 8, Topics 8.13–8.18 inclusive, before starting to solve any of the following problems on derived curves. Arrange the diagrams and the calculations in accordance with the specifications in Chap. 3, Topic 3.12. Also refer to Fig. 4, page 70, and to Forms 213–216 in the Workbook.

321. An automobile driver approaching a stop signal at an intersection coasts his car at a velocity of   fps for   sec, then applies the brakes so that the car is brought to a stop with constant negative acceleration. The car travels   ft after he applied the brakes. How long did it take him to stop? See Form 213 and 214 in the Workbook for a solution of this problem.

322. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body. Write equations for each curve.

First time interval:
Time = 20 sec
Initial velocity = 0
Final velocity = 40 fps
Velocity increases uniformly throughout the time interval
Second time interval:
Distance traveled = 1000 ft
Velocity is constant throughout the time interval

323. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body. Write equations for each curve.

First time interval:
Distance traveled = 64 ft
Velocity is 8 fps throughout the time interval
Second time interval:
Time = 10 sec
Velocity varies uniformly from its maximum value at the beginning of the time interval to 0 at the end of the time interval

324. In a start-and-stop test of an automobile the car was given a constant acceleration for 20 sec. During this time the car traveled 400 ft. During the second time interval the car was stopped in a distance of 80 ft, being given a constant negative acceleration during this period.

a. Draw complete derived curves showing the accelerations, velocities, and distances traveled.

b. Write the equations describing the motion.

325. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body. Write equations for each curve.

First time interval:
Acceleration = 5 fpsps throughout the time interval
Time = 4 sec
Initial velocity = 0
Second time interval:
Acceleration = −20 fpsps throughout the time interval
Final velocity is zero at the end of the interval

326. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body. Write equations for each curve.

First time interval:

Distance traveled = 2000 ft

Time = 20 sec

Velocity varies uniformly from its maximum value at the beginning of the time interval to zero at the end of the time interval

Second time interval:

Velocity varies uniformly from its minimum value at the beginning of the time interval to a maximum value of 50 fps at the end of the time interval

Time = 10 sec

327. A gravity type of package conveyor is designed so that boxes placed on it start from rest and are given a constant acceleration of 4 fpsps during the first time interval. During this interval they travel a distance of 72 ft. During the second time interval of 5 sec they have a constant velocity throughout the interval. What is the total length of the conveyor?

328. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body. Write equations for each curve.

First time interval:

Initial velocity = 10 fps

Velocity varies uniformly from its maximum value at the beginning of the time interval to zero at the end of the time interval

Acceleration = -5 fpsps throughout the time interval

Second time interval:

Distance traveled = 10 ft

Acceleration = 20 fpsps throughout the time interval

329. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the follow-

ing description of the motion of a body. Write equations for each curve.

First time interval:

Acceleration = 6 fpsps throughout the time interval

Time = 4 sec

Initial velocity = 0

Second time interval:

Velocity is constant

Time = 6 sec

Third time interval:

Time = 10 sec

Velocity decreases uniformly to 0 at the end of the time interval

330. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body. Write èquations for each curve.

First time interval:

Initial velocity = 4.8 fps

Velocity decreases uniformly to zero at the end of the time interval

Time = 24 sec

Second time interval:

Velocity = 0 throughout the time interval

Time = 6 sec

Third time interval:

Acceleration = 0.4 fpsps throughout the time interval

Final velocity = 19.2 fps

331. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body. Write equations for each curve.

First time interval:

Distance traveled = 144 ft

Acceleration = 8 fpsps throughout the time interval

Initial velocity = 0

Second time interval:

Acceleration = 0 throughout the time interval

Distance traveled = 96 ft

Third time interval:
Velocity decreases uniformly to zero at the end of the time interval
Time = 10 sec

332. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body. Write equations for each curve.

First time interval:
Distance traveled = 384 ft
Acceleration = -3 fpsps throughout the time interval
Final velocity = 0

Second time interval:
Acceleration = 0 throughout the time interval
Time = 11 sec

Third time interval:
Distance traveled = 160 ft
Acceleration = 5 fpsps throughout the time interval

333. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body.

First time interval: Initial value of the acceleration is zero, final value is fpsps. The acceleration increases uniformly throughout the time interval. The velocity at the end of the first time interval is fps. Initial velocity is fps.

Second time interval: Acceleration is constant throughout the sec interval. Its value is negative and fpsps.

334. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body.

First time interval: Initial value of the acceleration is zero. It increases uniformly to fpsps in sec. The initial velocity is fps.

Second time interval: Initial value of the acceleration is fpsps, but it decreases at a uniform rate to zero in sec.

335. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time curves for the following description of the motion of a body.

First time interval: The initial value of the acceleration is fpsps. It increases at a uniform rate to fpsps in sec. The initial velocity is zero.

Second time interval: Initial value of the acceleration is fpsps, decreasing uniformly to zero in sec.

336. Construct the derived curves showing the distance-time, velocity-time, and the acceleration-time relationships for the following description of the motion of a body.

First time interval: Initial acceleration is fpsps. It decreases at a uniform rate to fpsps at the end of the interval. The initial velocity is fps. The velocity increases fps during the interval.

Second time interval: Initial acceleration is negative fpsps and is changing at a constant rate to zero at the end of sec.

337. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body.

First time interval: Initial velocity is fps. Acceleration has a constant negative value of fpsps for sec.

Second time interval: Final velocity is fps. Acceleration is negative with initial value of fpsps. It changes at a constant rate to zero.

338. Construct the derived curves showing the distance-time, velocity-time, and acceleration-time relationships for the following description of the motion of a body.

First time interval: Initial velocity is fps. Initial acceleration is negative fpsps. The acceleration changes at a uniform rate until its value is negative fpsps at the end of sec.

Second time interval: The acceleration has a negative value which remains constant throughout the interval. The body is brought to rest in sec after the beginning of the first time interval.

339. A given pumping plant pumps a total of 2,500,000 gpd. From 5 to 10 P.M. the pumping rate is gpm. From 10 P.M. to 6 A.M. it pumps at the rate of gpm. Draw the derived curves showing the total quantity pumped in each time period and the rates. What is the pumping rate between 6 A.M. and 5 P.M.?

340. The following data were secured on the motion of a moving body:

First time period: Initial velocity = fps. Initial acceleration = fpsps, changing uniformly to fpsps at the end of the period. Time = sec.

Second time period: Uniform velocity = fps. Distance traveled in this period = ft.

Third time period: The body is brought to rest. Deceleration changes uniformly from 0 to fpsps at the end.

341. A gravity type of package conveyor is constructed so that the boxes handled by it start from rest, are given a uniform acceleration of fpsps, and reach a velocity of fps by the time that they enter the second section of the conveyor. This second section has just enough slope to maintain a uniform velocity for a distance of 150 ft. The third section is level, but the packages receive a deceleration of 6 fpsps due to the friction between the platform and the sliding boxes. The boxes come to rest at the end of this third section.

Draw the series of derived curves describing the above motions. What is the total time and distance that the packages travel?

342. A chute for delivering boxes has three stages of incline, the first two being straight inclines and the third curved. Starting from rest at the top the boxes reach the end of the first stage in 15 sec, and the acceleration is constant. The boxes attain a velocity of 12 fps. During the second stage the velocity is constant for 30 sec, and during the third stage the boxes come to rest with uniformly increasing deceleration (negative acceleration or retardation) in 8 sec. The curved incline starts tangent to the second incline. Draw the complete curves for each stage, showing all equations.

343. An automobile starts from rest and increases its velocity until mph is reached at the end of sec:

During this period the acceleration increases uniformly from fpsps to a maximum. The gas is then shut off, and the car slows down with uniform deceleration of fpsps for a distance of ft. The brakes are then applied, giving the car a negative acceleration which changes uniformly from a maximum of fpsps to 0 in sec.

Draw complete curves showing the distance, time, velocity, and acceleration relations for the automobile during these time periods.

344. A switching locomotive started a train of cars and got up to a speed of mph in sec, the acceleration having been increasing uniformly from 0. The cars were then run at the above speed over a stretch of track miles long, after which the brakes were applied, causing uniform deceleration for a distance of ft, when the train was brought to a stop.

Find the distance, time, and acceleration relations during these periods.

345. An elevator in an office building starts upward from the ground floor, is accelerated so as to attain a speed of fps in sec, and is brought to a stop in the next sec. The acceleration varies uniformly from maximum to 0 in the first time period and varies uniformly from maximum deceleration to 0 in the last time period. The distance from the ground floor to the second floor is ft, and each floor above this is ft, from floor to floor.

a. Draw the complete set of derived curves.

b. Opposite what floor did the elevator stop?

346. Draw the motion curves for the conditions stated in Prob. 129, page 263.

347. A car-pulling capstan used to switch cars at a steel mill is fitted with a drum in. in diameter including cable diameter. A steel cable is used for pulling the cars. Cars are drawn a maximum distance of 360 ft. When the cars are started, the acceleration of the drum varies uniformly from 0 to 1.2 rpm per second in 18 sec. The acceleration now decreases at a uniform rate to 0 in 12 sec. The speed is constant after this.

Draw the angular space, velocity, and acceleration-time curves for the drum and write their equations. How long does it take to haul the cars the 240 ft?

348. One of the sliding parts in an automatic machine travels a total distance of in. in sec. The block starts from rest and reaches its maximum velocity in sec. The acceleration varies uniformly from its greatest value at the start of the movement to 0 as the maximum velocity is reached. The velocity remains uniform for sec, and then the block is brought to rest. The deceleration increases at a uniform rate from 0 throughout the period of retardation.

Compute the maximum velocity, the maximum values of the accelerations, and the distances traveled during each time interval, using the laws of derived curves. Draw the complete series of curves.

Write the equations of the acceleration-time curves; then integrate these differential equations; and obtain the equations of the velocity-time and space-time curves. Substitute the limits, and check the previous values.

349. A block in a machine travels a total distance of in. in sec. The motion of the block is as follows:

Initial velocity, 0.

Initial acceleration, its maximum value.

One-fourth total distance, uniformly varying acceleration, decreasing uniformly to zero as maximum velocity is reached.

Five-eighths total distance, uniform speed.

One-eighth total distance, uniform deceleration.

Final velocity, 0.

350. A revolving table in a machine has a diameter of in. The motion of the table is as follows:

Initial velocity, 0.

One-third revolution, constant acceleration, for 0.6 sec.

One-half revolution, uniform velocity.

One-sixth revolution, constant deceleration.

Final velocity, 0.

Draw the complete set of derived curves describing the motion of the table.

351. A small stream is subject to sudden but short floods. Its normal rate of discharge is cfs. The following description gives the data on a typical flood.

The initial flow is normal. The rate of change of discharge increases uniformly from zero to cfs per hour at the end of hr. The rate of change of discharge now decreases from the above to zero at a uniform rate in hr. The increased rate of discharge is maintained for hr; then the return to normal flow is made in a similar manner to the rise, except that the final periods take 50 per cent longer than the corresponding periods of the initial increase and the final period is the longest of all.

Determine the total number of acre-feet of water that have passed through the stream channel from the start to the finish of the flood. Draw the derived curves showing the variations of the rate of change of discharge with time, the rate of discharge and time, and the total quantity and time.

352. A locomotive used in transcontinental service was given a test and showed the following action in three successive time stages. Draw derived curves to show the results.

First time interval: Time = sec. Initial velocity is 0. Final velocity = fps. Acceleration starts at 0 and increases uniformly.

Second time interval: Distance = ft. Velocity is constant.

Third time interval: Time = sec. The locomotive comes to a stop. The deceleration varies uniformly from its maximum at the beginning of the period to zero at the end.

353. An interurban train can maintain a uniform speed of 75 mph between two stations. The starting acceleration varies uniformly from the initial (and maximum) value to zero at the end of 45 sec. The train maintains its maximum running speed, 75 mph, until it is 2800 ft from the next station. At this point the power is shut off and the train coasts for 1600 ft. During this interval the speed is reduced by air and rolling friction to 60 mph. (Retardation is constant.) The brakes are now applied, and the train is brought to a complete stop, with uniformly increasing retardation.

a. Compute the total distance between stations if the total

elapsed time is 10 min. Determine the value of the maximum retardation and the time required during the coasting and braking period.

b. Draw the complete set of derived curves that completely describe the operating performance of this train; also write the equations describing the motion; and check the numerical results by the equations.

CENTROIDS AND MOMENT OF INERTIA

Study Chap. 10, Topics 10.7–10.14, inclusive, before starting the solution of any problems on centroids and moment of inertia. Arrange diagrams and calculations in accordance with the specifications in Chap. 3, Topic 3.11, and Forms 228–233, inclusive, in the Workbook.

354. A special tee has the following dimensions: length of flange, in.; thickness, in.; length of stem, in.; thickness in.; sides not tapered. Locate the centroid of the tee with reference to the upper edge of the flange. Use Form 230 in the Workbook.

355. A standard structural-steel angle has these dimensions: long leg, in. over all; short leg, in. over all; thickness of each leg, in. Determine the position of the centroid of the section with reference to the outer edge of each leg.

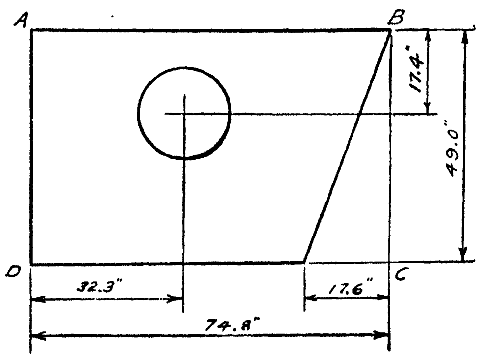


Fig. 117.

356. A steel plate was originally in the shape of a rectangle *ABCD*. The ends *AD* and *BC* were 49.0 in., and the sides *AB*

an CD were 74.8 in. A point E was laid off on the long side CD, 18.4 in. from the corner C. The end of the plate was then sheared off along the line BE. A hole 18.2 in. in diameter was cut out, its center being 32.3 in. from side and 17.4 in. from the end. Locate the centroid of the steel plate (see Fig. 117).

357. The limestone cornice block shown in Fig. 118 was used in the construction of a certain building. No wires or hooks were

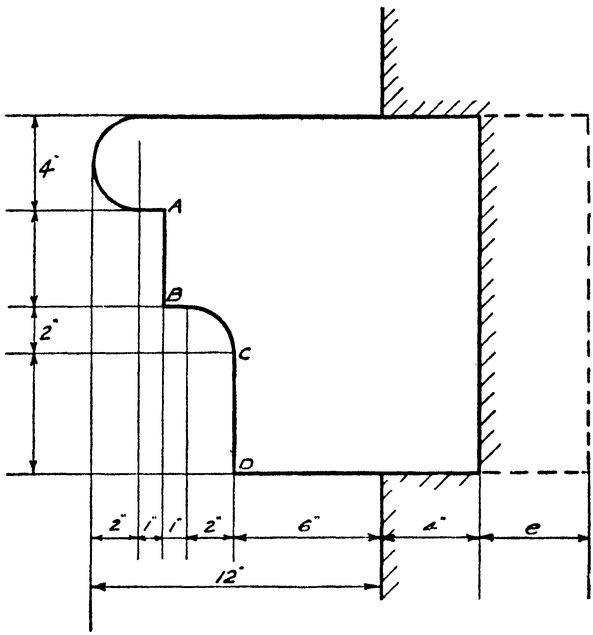


Fig. 118.

used to anchor the block to the walls; and not long after the building was completed, it was discovered that the cornice had tipped down an inch or more. Investigation showed that the center of gravity of the block was outside the point of support. What additional width e should have been called for in the drawings in order to bring the center of gravity of the stone just to the edge of the wall?

It should be noted that the dimension e is not the same as the distance from the edge of the wall to the centroid of the actual block used but it is the additional amount of stone that should have been used in order to have the centroid of the complete

block lie just above the outer edge of the wall. Also note that the block itself is not to be moved, that is, it must still project 12 in. from the wall. Tabulate the computations as indicated in Fig. 22, page 202, or Form 231 in the Workbook.

$AB =$

$CD =$

358. A small solid pulley for use with round belting is to be machined from a disk cut off a piece of steel shafting. It is first turned to form a hub for the set screw; then a V-shaped groove is machined in the rim. All dimensions are as shown in Fig. 119. The pulley bore is to be 0.500 in. Compute the weight of the finished pulley by the use of the theorem of Pappus (see page 358).

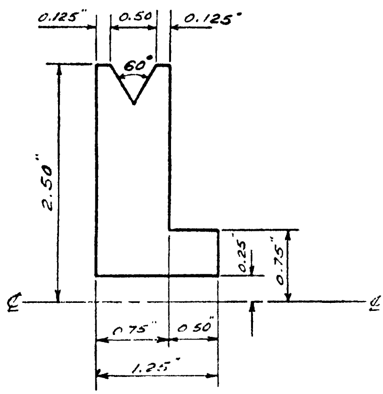


FIG. 119.

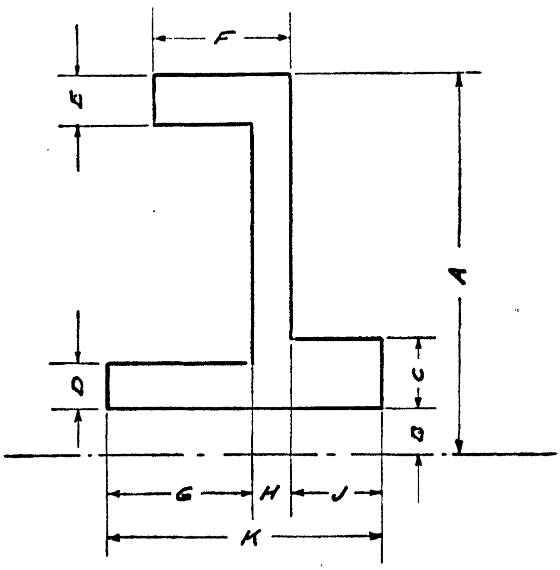


FIG. 120.

359. A cast-iron plate for a screw conveyor box is illustrated in Fig. 120. Compute the weight of the casting, using the theorem

of Pappus to find the volume.

$A =$	$D =$	$H =$
$B =$	$E =$	$J =$
$C =$	$F =$	$K =$
	$G =$	

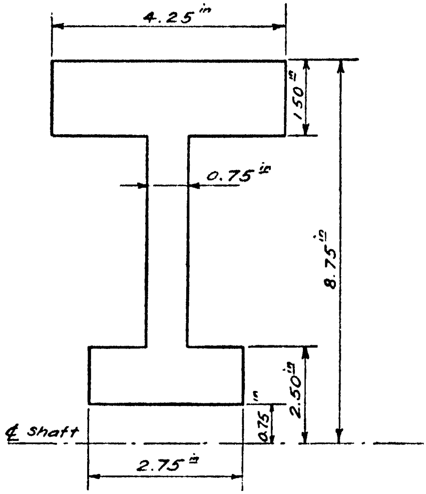


FIG. 121.

360. Half of the cross section of a cast-steel blank for a silent chain gear wheel is shown in Fig. 121. Four circular holes 3 in. in diameter are cast in the web. What is the weight of the blank? Apply the theorem of Pappus to find the volume (see No. 27, page 358).

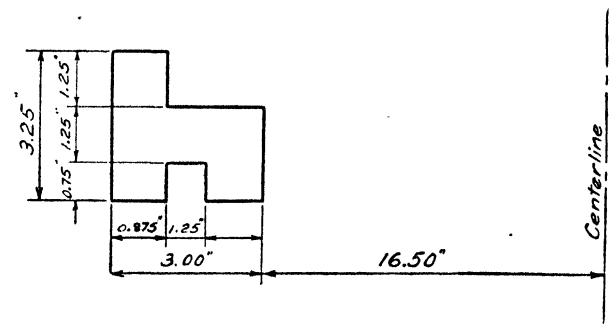


FIG. 122.

361. A section of a bomber gun-turret track is shown in Fig. 122. Use the theorem of Pappus (No. 27, page 358) and compute the volume of the material in the turret track.

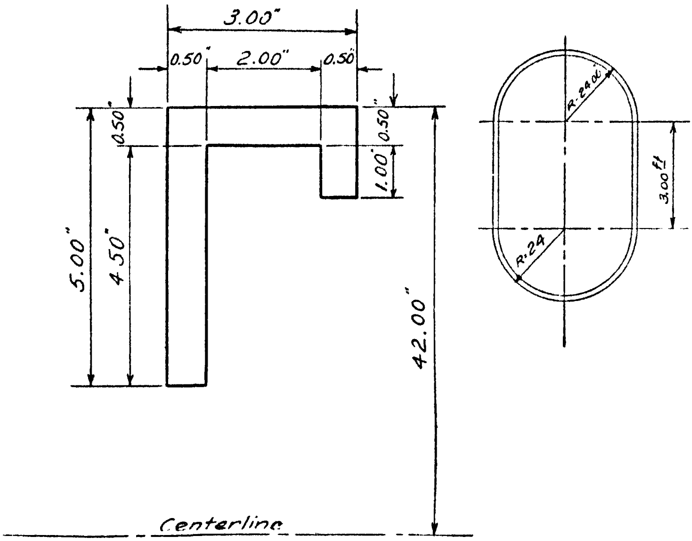


Fig. 123.

362. A portion of the bracing in the bulkhead of a plane is shown in Fig. 123. Use the theorem of Pappus (No. 27, page 358) and compute the volume of the material.

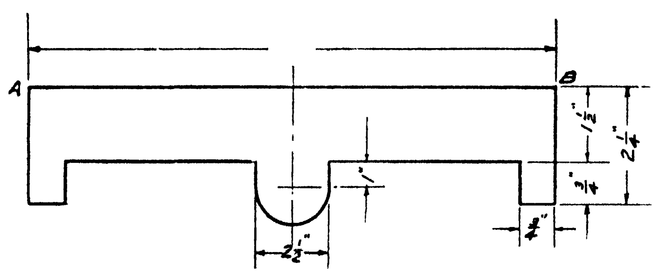


Fig. 124.

363. A cast-iron flywheel ft in outside diameter is made with a rim having the dimensions shown in Fig. 124. Compute its weight through the use of the theorem of Pappus (see No. 27, page 358). $AB =$

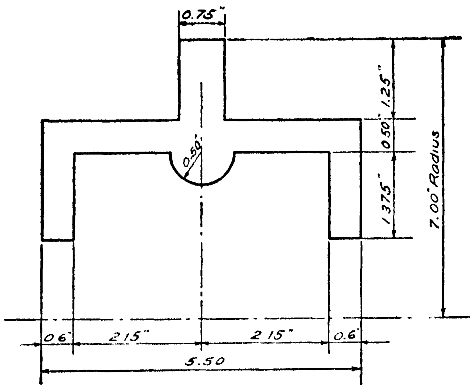


FIG. 125.

364. A cast-ring for a friction clutch has the cross section shown in Fig. 125. Find the weight of the ring, using the theorem of Pappus (see No. 27, page 358).

365. An equal-legged structural-steel angle has the following dimensions: outside length of each leg, in.; thickness of metal, in. Neutral axis is in. from the back of the leg. Compute the moment of inertia of the cross section.

366. An unequal-legged structural-steel angle has the following dimensions: long leg, in.; short leg, in.; thickness of metal, in. The centroid is in. from the back of the short leg and in. from the back of the long leg. Compute the moment of inertia of the cross section with respect to the neutral axis parallel to the short leg.

367. A square hollow cast-iron column has the following dimensions: outside width, in.; thickness of metal, in. Compute the moment of inertia of the cross-sectional area and its radius of gyration.

368. A standard pipe column has the following dimensions: outside diameter, in.; thickness of metal, in. When the pipe column is used as a beam, it is necessary to know the moment of inertia of the cross section. If it is used as a column, the radius of gyration is needed. Find both properties of the column.

369. A certain steel mill rolls an H-column section that has the following dimensions: over-all height, in.; width of flanges, in.; thickness of web, in.; and thickness of flanges, in. The flanges and web have uniform thickness. Compute the moment of inertia of the section with respect to both axes of symmetry. Compute the least radius of gyration.

370. Determine the least radius of gyration of a zee bar having the following dimensions: depth, in.; flanges, in.; thickness, in.

371. Plate-and-angle columns are in common use in building construction. Compute the second moment (moment of inertia) of such a column with respect to

a. The centroidal axis perpendicular to the web.

b. The centroidal axis parallel to the web.

The column is made up as follows:

One web plate, 10 by 0.50 in.

Four angles, 6 by 4 by 0.50 in., with the short leg riveted to the web plate.

Over-all height, 10.50 in.

372. Plate-and-channel columns are sometimes used in building construction. Two channels, flanges pointing out, are riveted to the cover plates at top and bottom.

a. Compute the second moment (moment of inertia) with respect to the centroidal axis that is perpendicular to the channels.

b. Compute the second moment with respect to the centroidal axis parallel to the channels.

The column is made up as follows:

Two channels, 12 in. by 30 lb, placed 6.50 in. back to back.

Two cover plates, 13.5 by 0.50 in.

373. Columns are frequently built up from plates and angles. Determine the moment of inertia about both the principal axes and the least radius of gyration for such a column if the shapes used are as given below.

One web plate, 24 by 0.50 in.

Four flange angles, 5 by 3.50 by 0.50 in., placed so longer leg is perpendicular to web plate.

Two flange plates, 11 by 0.50 in.

Over-all height of the section is 25.50 in.
Tabulate all calculations. Use Form 232 in the Workbook.

374. A standard form of plate girder is built up from plates and angles. Determine the moment of inertia of a riveted plate girder with respect to its horizontal neutral axis when made up as follows:

One web plate, 28 by 0.50 in.

Four flange angles, 5 by 3.50 by 0.50 in., long leg perpendicular to the plate.

Height over all 29.50 in.

Two flange plates, 12 by 0.50 in.

375. Find the moment of inertia and radius of gyration with respect to both principal axes for the following channel column.

Two channels, 12 in. by 40 lb.

Back to back of channels, 8.50 in.

Two web plates riveted to back of channels, 11 by 0.375 in.

Two flange plates, 16 by 0.75 in.

376. Find the moment of inertia of a plate girder built up from the following shapes with respect to the neutral axis perpendicular to the web plate. Tabulate the calculations. Use Form 232 in the Workbook.

One web plate, 36 by 0.50 in.

Four angles, 6 by 6 by 0.75 in.

Two flange plates, 14 by 1 in.

Two outside flange plates, 12 by 0.75 in., placed symmetrically.

Total height, 40 in.

377. The use of welded connections in building construction has led to the development of economical designs that were not possible with riveted connections. "Battledeck floor construction" is one of these new developments. This type of floor is made by laying steel plates over light I beams, then welding the edges of the plates to the top of the beams and to each other. In computing the properties of this combination of shapes, a portion of each plate and the I beam are treated as a composite section. The plate is considered as carrying a portion of the bending stress. Thus it is necessary to locate the neutral axis of the composite beam and to compute the moment of

inertia with respect to its centroidal axis. Make these computations for the following typical battledeck floors:

a. Plate, 20 by 0.50 in., I beam, 10 in. by 40 lb.

b. Plate, 24 by in., I beam, in. by lb.

378. A track for a traveling crane is made of a channel, lying flat with flanges pointing down, supported by a I beam with its web vertical, the upper flange of the I beam being riveted centrally to the web of the channel. Locate the horizontal neutral axis of the two combined, and find the moment of inertia with respect to this axis.

379. Plate-and-channel columns are widely used in building construction. They consist of two standard steel channels, with flanges pointing outward, and a steel cover plate riveted to one pair of flanges. The other pair of flanges are stiffened with lattice bars. Ignore the effect of the latticing, and compute the properties for a column made of the following shapes:

Two channels, 10 in. by 25 lb.

Back to back of channels, 5.5 in.

One cover plate, 12 by 0.50 in.

a. Compute the location of the neutral axis with respect to the upper edge of the cover plate.

b. Compute the moment of inertia with respect to this axis.

c. Compute the moment of inertia with respect to the axis perpendicular to the cover plate.

d. Compute the least radius of gyration.

380. An inclined bridge-truss post is made up of two I beams and a plate in. wide by in. thick, riveted to the upper flanges of the I beams. The section is symmetrical about the vertical axis, and the I beams are spaced horizontally in. apart center to center. Locate the horizontal neutral axis of the section, and find the moment of inertia of its area with respect to both principal axes.

381. The cross section of the frame of a shear press is as shown in Fig. 126.

a. Locate its centroid.

b. Compute the moment of inertia of the section with respect to the centroidal axis.

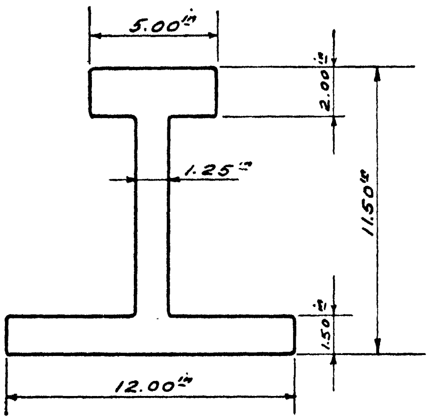


FIG. 126.

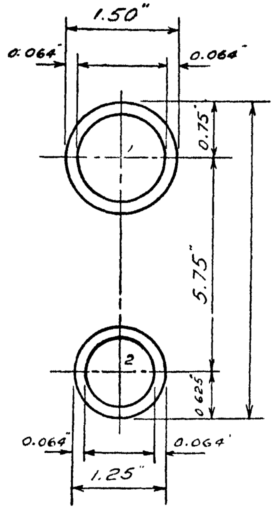


FIG. 127.

382. A type of tubular spar construction is shown in Fig. 127. Compute the location of the horizontal centroidal axis and the value of the second moment with respect to this axis.

383. A crane runway is made up of two standard channels and an I beam. The I beam is in the center with the web vertical, and the channels are laid one on top and one at the bottom, symmetrical about the vertical axis and with the flanges pointing down in both cases.

If the I beam is and the channels are each, find the centroid of the cross section of the runway and the moment of inertia of its area.

384. *a.* Compute the location of the centroidal axis for the area shown in Prob. 39, page 243.

b. Compute the moment of inertia with respect to both centroidal axes.

385. A special column was designed for a certain large office building. It was built up from plates and angles as follows:

Web plate, 14 by 1.25 in.

Four angles, 6 by 6 by 0.625 in.

One cover plate, 16 by 2.25 in. symmetrical about the center line of the web.

One cover plate, 24 by 2.75 in., placed so that one edge was in line with the short cover plate.

a. Determine the location of the centroidal axis perpendicular to the web plates, the second moment of the cross section with respect to this axis, and the radius of gyration. Tabulate the calculations. Use Form 233 in the Workbook.

b. Determine the location of the centroidal axis parallel to the web plates, the second moment of the cross section with respect to this axis, and the radius of gyration. Tabulate the calculations. Use Form 233 in the Workbook.

386. When the steel framing for a certain large office building was designed, it was found necessary to use a number of unsymmetrical columns built up from plates and angles. A typical column was made of the following shapes:

Web plate, 12 by 0.75 in.

Four angles, 7 by 3.50 by 0.75 in., the short leg being riveted to the web plate and extending 0.25 in. beyond its edge.

Two cover plates, 20 by 1.50 in.

Total height perpendicular to cover plates, 15.50 in.

The column was symmetrical about the axis parallel to the cover plates, but the web plate was shifted 2.00 in. to one side of the middle of the cover plates.

a. Determine the location of the centroidal axis that is parallel to the web plate.

b. Compute the moment of inertia and the radius of gyration with reference to the above axis.

c. Compute the moment of inertia and radius of gyration with reference to the axis that is parallel to the cover plates. Tabulate calculations. Use Form 233 in the Workbook.

387. Another column in the building mentioned above had the following shapes in it:

Web plate, 8 by 0.50 in.

Four angles, 5 by 3.50 by 0.50 in., the short leg being riveted to the web plate and extending 0.25 in. beyond its edge.

One cover plate, 12 by 0.75 in., symmetrical about the center line of web plate.

The other cover plate, 16 by 1.50 in., placed so that one edge was 6 in. from the center line of web plate.

Over-all height perpendicular to the cover plates, 10.75 in.

a. Compute the location of the centroidal axes of the column.

b. Determine the moment of inertia of the column with respect to both principal axes.

c. Compute the radius of gyration with respect to each axis. Tabulate the computations.

CHAPTER 12

MISCELLANEOUS TABLES

TABLE 1.—ABBREVIATIONS AND SYMBOLS

Based upon the "American Standard Abbreviations for Scientific and Engineering Terms," ASA Code Z 10.1—1941 and other recognized lists.

NOTE: Use the same abbreviation for both singular and plural. No periods used except where specifically shown, as in. for inch.

acceleration	accel
acceleration of gravity	*g*
acre	spell out
acre-foot	acre-ft
alternating-current (as adjective)	a-c
ampere	amp
ampere hour	amp-hr
answer	ans.
antilogarithm	antilog
approximate	approx
arc	spell out
area	spell out
average	avg
barrel	bbl
board feet (feet board measure)	fbm
brake horsepower	bhp
brake horsepower-hour	bhp-hr
bushel	bu
calorie	cal
center line	℄
cent	c or ¢
center to center	c to c
center of gravity	cg
centigrade (temperature)	C
centimeter	cm
centimeter-gram-second (system)	cgs
circumference	circum
coefficient	coef
coefficient of friction	*f*
cosine	cos
cotangent	cot
cubic	cu
cubic centimeter	cu cm, cm^3 (liquid, meaning milliliter, ml)
cubic foot	cu ft

TABLE 1.—(*Continued*)

cubic feet per minute	cfm
cubic feet per second	cfs
cubic inch	cu in.
cubic meter	cu m or m^3
cubic millimeter	cu mm or mm^3
cubic yard	cu yd
cylinder	cyl
day	spell out
degree	deg or °
degree centigrade	C
degree Fahrenheit	F
degree Kelvin	K
diameter	diam
direct-current (as adjective)	d-c
dollar	$
dozen	doz
dram	dr
efficiency	eff
electric	elec
electromotive force	emf
elevation	el
equation	eq
Fahrenheit (temperature)	F
feet board measure (board feet)	fbm
feet per minute	fpm
feet per second	fps
feet per second per second	fpsps
fluid	fl
foot	ft
foot-pound	ft-lb
foot-pound-second (system)	fps
free on board	fob
gallon	gal
gallons per minute	gpm
gallons per second	gps
grain	spell out
gram	g
horsepower	hp
horsepower-hour	hp-hr
hour	hr
hundred	C
inch	in.
inch-pound	in.-lb
inches per second	ips
inches per second per second	ipsps
inclusive	incl
inside diameter	ID

TABLE 1.—(*Continued*)

internal	int
kilogram	kg
kilometer	km
kilovolt	kv
kilovolt-ampere	kva
kilowatt	kw
kilowatthour	kwhr
latitude	lat
linear foot	lin ft
liquid	liq
liter	l
logarithm (common)	log
logarithm (natural)	$\log_e$ or ln
longitude	long.
mass	spell out
maximum	max
meter	m
mile	spell out
miles per hour	mph
miles per hour per second	mphps
millimeter	mm
million	spell out
million gallons per day	mgd
millivolt	mv
minimum	min
minute	min
minute (angular measure)	′
miscellaneous	misc
moment of inertia, mass or area	I
month	spell out
number	No.
ohm	spell out
ounce	oz
outside diameter	OD
page	p.
parts per million	ppm
pint	pt
pound	lb
pound-foot	lb-ft
pound-inch	lb-in.
pounds per cubic foot	lb per cu ft
pounds per square foot	psf
pounds per square inch	psi
power factor	spell out or pf
product of inertia	I_{xy}
quart	qt
radian	spell out

TABLE 1.—(*Continued*)

radius	rad or *R*
radius of gyration	*k*
revolutions	rev
revolutions per minute	rpm
revolutions per second	rps
rod	spell out
second	sec
second (angular measure)	″
second-foot (see cubic feet per second)	
sine	sin
specific gravity	sp gr
specification	spec.
square	sq
square centimeter	sq cm or cm^2
square foot	sq ft
square inch	sq in.
square kilometer	sq km or km^2
square meter	sq m or m^2
square millimeter	sq mm or mm^2
standard	std
tangent	tan
temperature	temp
time	spell out
thousand	M
thousand foot-pounds	kip-ft
thousand pound	kip
ton	spell out
ton-mile	spell out
velocity	vel
volt	v
volt-ampere	va
volume	vol
watt	w
watthour	whr
week	spell out
weight	wt
yard	yd
year	yr

TABLE 2.—MATHEMATICAL SYMBOLS

plus (addition)	$+$
minus (subtraction)	$-$
plus or minus	$\pm$
multiplication. Use parenthesis only	$(2.37)(85.3)$
division. Use fraction only	$\frac{15}{24}$
ratio	$\frac{a}{b}$
proportion	$\frac{a}{b} = \frac{c}{d}$
square root	$(385)^{\frac{1}{2}}$ or $(385)^{0.5}$
cube root	$(627)^{\frac{1}{3}}$ or $(627)^{0.333}$
other powers or roots	$(863)^{\frac{5}{2}}$ or $(825)^{2.5}$
equals	$=$
is not equal to	$\neq$
is less than	$<$
is greater than	$>$
varies as (spell out)	"varies as"
tends toward or approaches	$\rightarrow$
approximately equals	$a = kx$ (approx) see p. 349
parallel or is parallel to	$\parallel$
perpendicular, or is perpendicular to	$\perp$
therefore, hence	$\therefore$
per cent	%
number, when before figures	#
pounds, when after figures	#
infinity	∞
angle	$\measuredangle$
right angle	rt $\measuredangle$
triangle	$\triangle$
circle	$\odot$
pi $= \frac{\text{circumference}}{\text{diameter}} = 3.1416$	π
summation	Σ
differential (in calculus)	d
integral (in calculus)	$\int$

TABLE 3.—MENSURATION
(Plane surfaces)

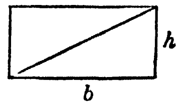
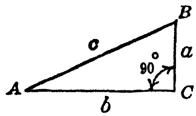
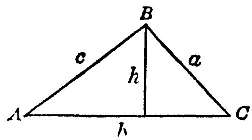
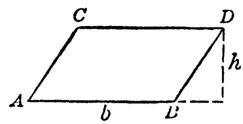
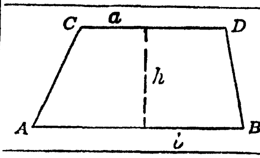
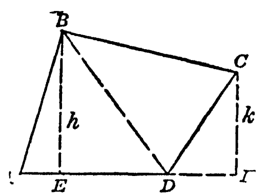
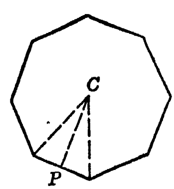
1. *Rectangle* Area = (base) (altitude) Diagonal = $\sqrt{\text{base}^2 + \text{altitude}^2}$	
2. *Right Triangle* Area = $\frac{1}{2}$(base) (altitude) Hypotenuse = $\sqrt{\text{base}^2 + \text{altitude}^2}$ Angles $A + B = 90°$	
3. *Any Triangle* Area = $\frac{1}{2}$(base) (altitude) NOTE: Altitude h perpendicular to base b Angles $A + B + C = 180°$	
4. *Parallelogram* Area = (base) (altitude) NOTE: Altitude h perpendicular to base b Sum of angles = 360°	
5. *Trapezoid* (Sides a and b parallel) Area = $\frac{1}{2}$(sum of parallel sides)(altitude) NOTE: Altitude h perpendicular to a and b	
6. *Trapezium* (Four sides, none parallel) Area: Draw diagonal BD and get sum of areas of triangles ABD and BCD. Or draw altitudes h and k, then area trapezium = area trapezoid EBCF + triangle ABE − triangle DCF	
7. *Regular Polygon* NOTE: A regular polygon has equal sides and equal angles and can be inscribed in or circumscribed about a circle. Area = $\frac{1}{2}\begin{pmatrix}\text{number}\\ \text{of sides}\end{pmatrix}\begin{pmatrix}\text{length of}\\ \text{one side}\end{pmatrix}\begin{pmatrix}\text{distance } CP\\ \text{to center}\end{pmatrix}$	

TABLE 3.—(Plane surfaces —*Continued*)

8. *Circle*

$$\text{Circumference} = \pi\ (\text{diam}) = 2\pi\ (\text{rad})$$

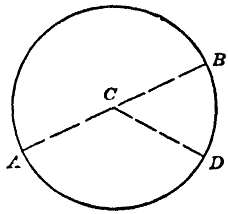
$$\pi = \frac{\text{circumference}}{\text{diameter}} = 3.1416$$

$$\text{Area} = \pi(\text{rad})^2 = \frac{\pi(\text{diam})^2}{4} = 0.7854\ (\text{diam})^2$$

$$1\ \text{radian} = \frac{180^\circ}{\pi} = 57.2958^\circ$$

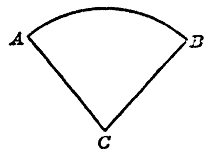
$$\frac{\text{Arc } BD}{\text{Circumference}} = \frac{\text{angle } BCD^\circ}{360^\circ}$$

Also arc BD = (rad) (angle BCD in radians)



9. *Sector of a Circle*

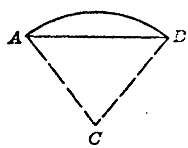
$$\text{Area} = \frac{(\text{arc})\ (\text{rad})}{2} = \frac{\pi(\text{rad})^2\ (\text{angle } ACB^\circ)}{360^\circ} = \frac{(\text{rad})^2\ (\text{angle } ACB \text{ in radians})}{2}$$



10. *Segment of a Circle*

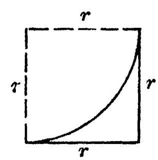
Area of segment = area of sector ABC − area of triangle ABC

$$\text{Also area} = \frac{\text{rad}^2}{2}\left[\frac{\pi(\sphericalangle ACB^\circ)}{180^\circ} - \sin ACB^\circ\right] = \frac{\text{rad}^2}{2}\left(\sphericalangle ACB \text{ in radians} - \sin ACB^\circ\right)$$



11. *Circular Spandrel or Fillet*

$$\text{Area} = \text{area of square} - \frac{\text{area circle}}{4} = \text{rad}^2 - \frac{\pi(\text{rad})^2}{4} = 0.2146\ (\text{rad})^2$$



12. *Ellipse*

$$\text{Area} = \pi(\text{long rad } AC)\ (\text{short rad } CE) = \frac{\pi}{4}\ [\text{long diam } AB]\ [\text{short diam } DE]$$

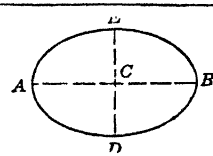
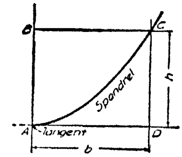
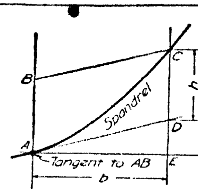


TABLE 3.—(Plane surfaces.—*Continued*)

13. *Parabola*
Rectangular axes, vertex at A
Parabola is tangent to AD at A
Area of section $ABC = \frac{2}{3}bh$
$= \frac{2}{3}(AD)(CD)$
Area of spandrel $ADC = \frac{1}{3}bh$
$= \frac{1}{3}(AD)(DC)$

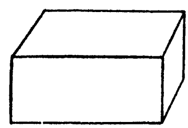


14. *Parabola*
Rectangular axes, vertex unknown
Parabola is tangent to inclined line AB at A
Area section $ABC = \frac{2}{3}bh$
$= \frac{2}{3}(AE)(DC)$
Area spandrel $ADC = \frac{1}{3}bh$
$D = \frac{1}{3}(AE)(DC)$

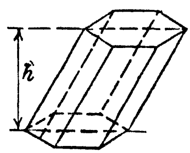


Solids

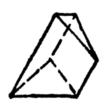
15. *Rectangular Prism*
Volume = (area of base) (altitude)



16. *Any Prism*
Axis either inclined or perpendicular to base
Volume = (area of base) $\left(\begin{matrix}\text{perpendicular} \\ \text{height}\end{matrix}\right)$
or = area of perpendicular cross-section times lateral length



17. *Truncated Prism*
Volume = area of base multiplied by perpendicular distance from base to center of gravity of opposite side



18. *Cylinder*
Axis perpendicular or inclined to base
Volume = (area of base) $\left(\begin{matrix}\text{perpendicular} \\ \text{height}\end{matrix}\right)$
Also when axis is inclined to base,
Volume = (area of section perpendicular to axis) (length of axis)
Area of cylindrical surface = (perimeter of base) (perpendicular height)

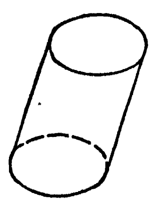
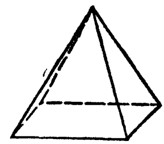


TABLE 3.—(Solids.—*Continued*)

19. *Pyramid*

Axis either inclined or perpendicular to base

Volume $= \frac{1}{3}(\text{area of base})\begin{pmatrix}\text{perpendicular}\\ \text{height}\end{pmatrix}$

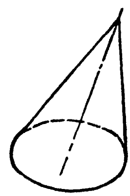


20. *Cone*

Axis either inclined or perpendicular to base

Volume $= \frac{1}{3}(\text{area of base})\begin{pmatrix}\text{perpendicular}\\ \text{height}\end{pmatrix}$

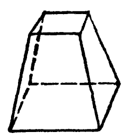
Right cone, area of conical surface =
$\frac{1}{2}$(circumference of base) (slant height)



21. *Frustrum of Pyramid or Cone*

Ends parallel

Volume $= \frac{1}{3}$ perpendicular height times
[area of base + area of top
$+ \sqrt{(\text{area of base})(\text{area of top})}$]



22. *Ungula, slice, of Right Circular Cylinder*

Volume =

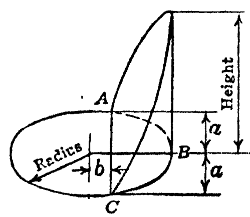
$$\frac{\text{height}}{\text{rad} \pm b}\left[\tfrac{2}{3}a^3 \pm (b)\ (\text{area of base})\right]$$

Use + when base is larger than a half-circle, and − when less.

Area of cylindrical surface =

$$\frac{\text{height}}{\text{rad} \pm b}\left[2a(\text{rad}) \pm (b)\ (\text{arc } ABC)\right]$$

Use ± same as in volume



23. *Sphere*

$$\text{Volume} = \frac{4\pi(\text{rad})^3}{3} = \frac{\pi(\text{diam})^3}{6}$$

Area of surface $= 4\pi(\text{rad})^2$
$= \pi(\text{diam})^2$

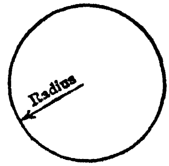
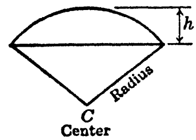


TABLE 3.—(Solids.—*Continued*)

24. *Spherical Sector*

Volume $= \dfrac{2\pi(\text{rad})^2(h)}{3}$

Area of surface = area of conical surface + area spherical segment (see spherical segment)



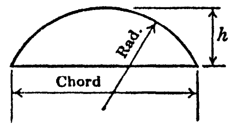
25. *Spherical Segment*

Volume $= \pi h^2 \left(\text{rad} - \dfrac{h}{3}\right)$

$= \pi h \left(\dfrac{\text{chord}^2}{8} + \dfrac{h^2}{6}\right)$

Area of spherical surface =

$2\pi(\text{rad})h = \pi \left(\dfrac{\text{chord}^2}{4} + h^2\right)$



26. *Circular Ring or Torus*

Volume =

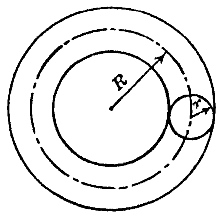
$2\pi^2$ (rad of section, r)^2 (mean rad, R)

$= \dfrac{\pi^2}{4}$ (diam of section)^2 (mean diam)

Area of surface =

$4\pi^2$(mean rad) (rad of section)

$= \pi^2$(mean diam) (diam of section)

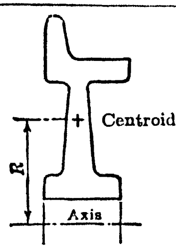


27. *Solids of Revolution* (Theorem of Pappus)

A plane area revolved about an axis in its own plane generates a solid.

Its volume equals the plane area times the length of path followed by the centroid of the area.

Its surface area equals the perimeter of the generating area times the length of path followed by the centroid of the area.



28. *Comparison of Volumes*

Cone, Paraboloid, Sphere, Cylinder having same base diameter and same height

Volumes have following comparative values:

Cone	Paraboloid	Sphere	Cylinder
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1

Cube would be $\dfrac{4}{\pi}$ (vol of cyl)

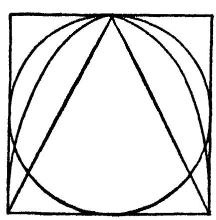


TABLE 4.—CENTROIDS AND SECOND MOMENTS OF SIMPLE SHAPES

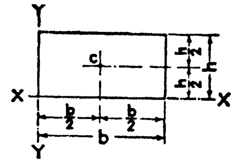
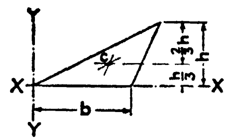
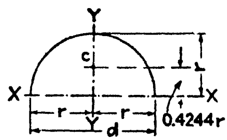
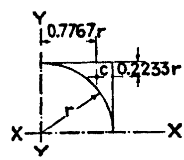
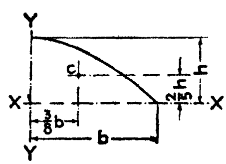
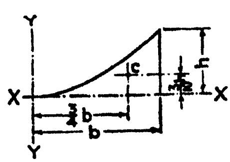
No.	Centroid	Second moment	Shape
1. *Rectangle*	$\bar{y} = \frac{1}{2}h$ $\bar{x} = \frac{1}{2}b$	$I_c = \frac{1}{12}bh^3$ $I_x = \frac{1}{3}bh^3$	
2. *Triangle*	$\bar{y} = \frac{1}{3}h$	$I_c = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$	
3. *Semicircle*	$\bar{y} = 0.4244r$	$I_c = 0.10996r^4$ $I_x = 0.39270r^4$	
4. *Circular Spandrel*	$\bar{y} = 0.2233r$ $\bar{x} = 0.7767r$	$I_c = 0.007524r^4$ $I_x = 0.1370r^4$	
5. *Parabolic Segment*	$\bar{y} = \frac{2}{5}h$ $\bar{x} = \frac{3}{8}b$	$I_c = 0.0724bh^3$ $I_x = 0.15238\ bh^3$	
6. *Parabolic Spandrel*	$\bar{y} = \frac{3}{10}h$ $\bar{x} = \frac{3}{4}b$	$I_c = 0.01763bh^3$ $I_x = 0.04762\ bh^3$	

TABLE 5.—TRIGONOMETRIC FORMULAS

Secondary Functions:

$$\text{Cotangent } A = \frac{\text{adjacent side}}{\text{opposite side}} \qquad \cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}$$

$$\text{Versed sine } A = \frac{\text{hypotenuse} - \text{adjacent side}}{\text{hypotenuse}}$$

$$\text{vers } A = 1 - \cos A$$

$$\text{Secant } A = \frac{\text{hypotenuse}}{\text{adjacent side}} \qquad \sec A = \frac{1}{\cos A} = \frac{\tan A}{\sin A}$$

$$\text{External secant } A = \frac{\text{hypotenuse} - \text{adjacent side}}{\text{adjacent side}}$$

$$\text{exsec } A = \frac{\text{vers } A}{\cos A} = \sec A - 1$$

$$\text{Cosecant } A = \frac{\text{hypotenuse}}{\text{opposite side}} \qquad \text{cosec } A = \frac{1}{\sin A} = \frac{\cot A}{\cos A}$$

$$\text{Coversed sine } A = \frac{\text{hypotenuse} - \text{opposite side}}{\text{hypotenuse}}$$

$$\text{covers } A = 1 - \sin A$$

Single Angles:

$$\sin A = \frac{\cos A}{\cot A} = \cos A \tan A = \sqrt{1 - \cos^2 A}$$

$$\cos A = \frac{\sin A}{\tan A} = \sin A \cot A = \sqrt{1 - \sin^2 A}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{1}{\cot A}$$

Half Angles:

Referring to Fig. 16, if s = half the sum of the sides:

$$\sin \frac{1}{2} A = \sqrt{\frac{(s - AC)(s - AB)}{(AC)(AB)}} = \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{1}{2} A = \sqrt{\frac{s(s - BC)}{(AC)(AB)}} = \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{1}{2} A = \sqrt{\frac{(s - AC)(s - AB)}{s(s - BC)}} = \frac{\sin A}{1 + \cos A}$$

$$\cot \frac{1}{2} A = \frac{\sin A}{1 - \cos A}$$

TABLE 5.—(*Continued*)

Double Angles:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cot 2A = \frac{\cot^2 A - 1}{2 \cot A}$$

Sums and Differences of Angles:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cot (A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$$

Sums and Differences of Functions:

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$

$$\cos B - \cos A = 2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

$$\tan A + \tan B = \frac{\sin (A + B)}{\cos A \cos B}$$

$$\tan A - \tan B = \frac{\sin (A - B)}{\cos A \cos B}$$

$$\cot A + \cot B = \frac{\sin (B + A)}{\sin A \sin B}$$

$$\cot A - \cot B = \frac{\sin (B - A)}{\sin A \sin B}$$

Squares of Functions:

$$\sin^2 A = 1 - \cos^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = 1 - \sin^2 A = \frac{1 + \cos 2A}{2}$$

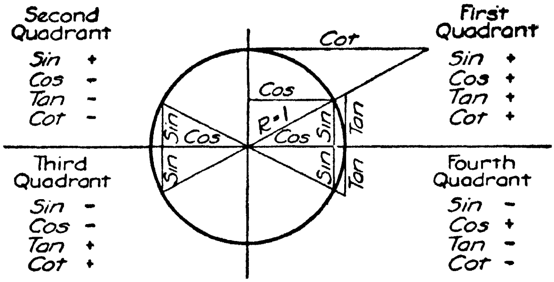
$$\sin^2 A - \sin^2 B = \sin (A + B) \sin (A - B)$$

$$\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$$

$$\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}$$

$$\cot^2 A = \frac{1 + \cos 2A}{1 - \cos 2A}$$

TABLE 6.—SIGNS OF THE TRIGONOMETRIC FUNCTIONS



EQUIVALENTS OF MEASURE

TABLE 7.—LENGTHS*

Inches	Feet	Yards	Rods	Miles	Centi-meters	Meters	Kilo-meters
1	0.08333	0.02778	0.005051	0.0_41578	2.540	0.0254	0.0_4254
12	1	0.3333	0.06061	0.0_31894	30.48	0.3048	0.0_33048
36	3	1	0.1818	0.0_35862	91.44	0.9144	0.0_39144
198	16.5	5.5	1	0.002714	502.9	5.029	0 005029
63360	5280	1760	320	1	160935	1609.4	1.60935
0.3937	0.03281	0.01094	0.001988	0.0_56214	1	0.01	0.00001
39.37	3.2808	1.0936	0.19884	0.0_36214	100	1	0.001
39370	3280.8	1093.6	198.84	0.62137	100000	1000	1

* 0.0_42296 means that there are four ciphers between decimal point and first digit, thus 0.00002296.

TABLE 8.—AREAS*

Square inches	Square feet	Square yards	Square rods	Acres	Square miles	Square meters	Square kilometers
1	0.006944	0.0_37716	0.0_42551	0.0_61594	0.0_92491	0.0_36452	0.0_96452
144	1	0.1111	0.003673	0.0_42296	0.0_73587	0.09290	0.0_79290
1296	9	1	0.03306	0.0_32066	0.0_63228	0.8361	0.0_68361
39204	272.25	30.25	1	0.00625	0.0_59766	25.293	0.0_42529
6272640	43560	4840	160	1	0.001562	4047	0.004047
	27878400	3097600	102400	640	1	2590000	2.59
1550	10.764	1.196	0.03954	0.0_32471	0.0_63861	1	0.0_51
		1195985	39536	247.1	0.3861	1000000	1

* 0.0_42296 means that there are four ciphers between decimal point and first digit, thus 0.00002296.

TABLE 9.—VOLUMES*

Cubic inches	Cubic feet	Cubic yards	U. S. quarts		U. S. gallons		U. S. bushels	Liters
			Liquid	Dry	Liquid	Dry		
1	0.0_35787	0.0_42143	0.01732	0.01488	0.004329	0.003720	0.0_34650	0.01639
1728	1	0.03704	29.92	25.71	7.481	6 429	0.8036	28.32
46656	27	1	807.9	694.3	202.0	173.6	21.70	764.6
57.75	0.03342	0.001238	1	0.8594	0.25	0.2148	0.02686	0 9464
67.20	0.03889	0.001440	1.164	1	0.2909	0.25	0.03125	1.101
231	0.1337	0.004951	4	3.437	1	0.8594	0.1074	3.785
268.8	0.1556	0.005761	4.665	4	1.164	1	0.125	4.405
2150	1.244	0.04609	37.24	32	9 309	8	1	35.24
6102	0.03531	0.001308	1.057	0.9081	0.2642	0 2270	0.2838	1

* 0.0_42296 means that there are four ciphers between decimal point and first digit, thus 0.00002296.

TABLE 10.—WEIGHTS*

Grains	Ounces, avoir-dupois	Pounds, avoir-dupois	Tons, short	Tons, long	Tons, metric	Kilo-grams
1	0.002286	0.0_31429	0.0_77143	0.0_76378	0.0_76480	0.0_46480
437.5	1	0.0625	0.0_43125	0.0_42790	0.0_42835	0.02835
7000	16	1	0.0005	0.0_34464	0.0_34536	0.4536
140_6	32000	2000	1	0.8929	0.9072	907.2
15680_4	35840	2240	1.12	1	1.016	1016
15432356	35274	2205	1.102	0.9842	1	1000
15432	35.27	2.205	0.001102	0.0_39842	0.001	1

* $0\ 0_42296$ means that there are four ciphers between decimal point and first digit, thus 0.00002296.

TABLE 11.—PUMPING RATES

1,000,000 gal. per day (24 hr.)	Gallons per minute	Gallons per second	Cubic feet per minute	Cubic feet per second
1	694.444	11.5741	92.836	1.547
0.001440	1	0.01667	0.1337	0.002228
0.08640	60	1	8.021	0.1337
0.01077	7.481	0.12467	1	0.01667
0.6463	448.86	7.481	60	1

TABLE 12.—VELOCITIES

Feet per second	Feet per minute	Miles per hour	Knots	Meters per second	Meters per minute	Kilometers per hour
1	60	0.6818	0.59209	0.3048	18.29	1.097
0.01667	1	0.01136	0.00987	0.005080	0.3048	0.01829
1.467	88	1	0.86839	0.4470	26.82	1.609
1.68894	101.337	1.15155	1	0.51497	30.898	1.8532
3.281	196.85	2.237	1.942	1	60	3.6
0.05468	3.281	0.03728	0.03237	0.01667	1	0.06
0.9113	54.68	0.6214	0.53960	0.2778	16.67	1

1 knot = 1 nautical mile per hour.
1 nautical mile = 6080.2 ft.

TABLE 13.—ACCELERATIONS

Feet per second per second	Miles per hour per second	Meters per second per second	Kilometers per hour per second
1	0.6818	0.3048	1.09728
1.4667	1	0.44704	1.6093
3.2808	2 237	1	3.6
0.9113	0.6214	0.2778	1

TABLE 14.—POWER*

Foot-pounds per second	Foot-pounds per minute	Horse-power	Watts	Kilowatts
1	60	0.001818	1.3557	0.001356
0.01667	1	0.0_43030	0.0226	0.0_4226
550	33000	1	745.65	0.74565
0.7376	44.257	0.001341	1	0.001
737.6	44256	1.341	1000	1

* 0.0_42296 means that there are four ciphers between decimal point and first digit, thus 0.00002296.

TABLE 15.—PRESSURES*

Pounds per square inch	Pounds per square foot	Short tons per square foot	Atmos-pheres	Col-umns mer-cury at 0°C., inches	Col-umn water at 15°C., feet	Col-umn water at 15°C., inches	Kilo-grams per square centi-meter
1	144	0.0720	0.06804	2.036	2.309	27.70	0.07031
0.00694	1	0.0005	0.0_34725	0.01414	0.01603	0.1924	0.0_34882
13.89	2000	1	0.9450	28.28	32.06	384.8	0.9765
14.70	2116.3	1.058	1	29.92	33.93	407.2	1.0333
0.4912	70.73	0.03536	0.03342	1	1.134	13.61	0.03453
0.4332	62.43	0.03119	0.02947	0.8819	1	12	0.03045
0.03610	5.2023	0.002599	0.00245C	0.07349	0.08333	1	0.002538
14.22	2048	1.024	0.9678	28.96	32.84	394.0	1

* 0.0_42296 means that there are four ciphers between decimal point and first digit, thus 0.00002296.

TABLE 16.—WEIGHTS OF COMMON SUBSTANCES

Substance	Specific gravity	Average weight, lb per cu ft
Acid, muriatic, 40 per cent	1.20	75
Acid, nitric, 91 per cent	1.50	94
Acid, sulphuric, 87 per cent	1.80	112
Air, 0°C., 760 mm	1.0	0.08072
Alcohol, ethyl (100 per cent)	0.789	49
Alcohol, methyl (100 per cent)	0.795	50
Aluminum, alloy, 0.100 lb per cu in	2.768	172.8
Aluminum, cast	2.55 – 2.80	168
Ammonia gas	0.5920	0.0478
Ammonia liquid	0.89	55.6
Antimony	6.62 – 6.72	418
Asbestos	2.1 – 2.8	153
Ashes, cinders	0.64 – 0.72	43
Asphaltum	1.1 – 1.8	87
Babbitt metal, hard	7.34 – 7.75	470
Babbit metal, soft	9.73 –10.67	670
Barley, bulk	0.62	39
Basalt, solid	2.7 – 3.2	182
Basalt, granite, gneiss, crushed, piled	....	96
Benzine	0.73 – 0.75	46
Bismuth	9.70 – 9.90	610
Brass, average cast and rolled plate	8.4	525
Brick, common	1.8 – 2.0	120
Brick, pressed	2.2 – 2.3	140
Brick, pressed, fire	2.3 – 2.4	145
Brick, soft	1.5 – 1.7	100
Bronze, average	8.8	551
Calcium	1.58	98.6
Carbon, amorphous, graphitic	1.88 – 2.25	129
Carbon bisulphide	1.29	80.6
Carbon dioxide	1.5291	0.1230
Carbon monoxide	0.9673	0.0781
Celluloid	1.43	90
Cement, portland, loose	1.44	90
Cement, portland, set	2.7 – 3.2	190
Chalk	1.8 – 2.6	140
Charcoal, pine	0.28 – 0.44	22
Charcoal, oak	0.47 – 0.57	31

TABLE 16.—(*Continued*)

Substance	Specific gravity	Average weight, lb per cu ft
Chlorine	2.49	0.190
Clay, excavated, dry	1.0	63
Clay, excavated, damp, plastic	1.76	110
Clay, marl in bank	1.8 – 2.6	135
Coal, anthracite, block	1.4 – 1.8	95
Coal, bituminous, block	1.2 – 1.5	84
Coal, lignite, block	1.1 – 1.4	80
Coal, anthracite, loose	....	56
Coal, bituminous, loose	....	50
Coke, solid	1.0 – 1.4	81
Coke, loose	....	28
Concrete, reinforced	2.2 – 2.4	150
Concrete, slag, etc	1.9 – 2.3	130
Concrete, cinder, etc	1.5 – 1.7	100
Copper, pure	8.9	554
Copper, wire and rolled sheet	8.91 – 8.95	556
Copper, cast	8.8 – 9.0	552
Copper ore, pyrites	4.1 – 4.3	262
Cork	0.22 – 0.26	15
Corn, shelled	0.73	45
Cotton, flax, hemp	1.47 – 1.50	93
Earth, loose	1.2 – 1.3	76
Earth, packed	1.5	98
Earth, mud	1.7 – 1.8	110
Earth, turf or peat	0.32 – 0.45	25
Emery	4.0	250
Flour, loose	0.40 – 0.50	28
Gas, illuminating, coal	0.35 – 0.45	0.032
Gas, natural	0.57 – 0.74	0.050
Gasoline	0.66 – 0.69	43
German silver	8.58	524
Glass, common	2.40 – 2.80	162
Glass, plate or crown	2.45 – 2.72	161
Glass, crystal	2.90 – 3.00	184
Glass, flint	2.15 – 4.7	210
Glycerine	1.26	79
Gold, cast-hammered	19.25 – 19.35	1205

TABLE 16.—(*Continued*)

Substance	Specific gravity	Average weight, lb per cu ft
Gold coin (U. S.)	17.18 –17.2	1073
Gneiss, serpentine, solid	2.4 – 2.7	165
Granite, syenite, solid	2.5 – 3.1	170
Graphite	1.9 – 2.3	131
Gravel, screened, wet	1.44 – 1.92	112
Gypsum, alabaster	2.3 – 2.8	159
Hay and straw, bales	0.32	20
Hydrogen	0.0693	0.00559
Ice, solid	0.88 – 0.92	56
Ice, crushed	....	25
Iron, gray, cast, 0.260 lb per cu in	7.03 – 7.13	450
Iron, wrought, 0.278 lb per cu in	7.6 – 7.9	480
Iron ore, hematite, loose	....	145
Iron ore, magnetite, in bank	4.9 – 5 2	315
Iron slag	2.5 – 3.0	172
Kerosene	0.78 – 0.82	50
Lead	11.34	710
Lead ore, galena	7.3 – 7.6	465
Leather	0.86 – 1.02	59.5
Lime, quick, loose	....	58
Limestone, solid	2.46 – 2.86	166
Limestone, quarried, crushed	....	95
Marble	2.5 – 2.8	167
Magnesium	1.69 – 1.75	108
Magnesium carbonate	2.4	150
Manganese	7.2 – 8.0	475
Mercury	13.4 –13.6	847
Mica	2.75 – 3.1	183
Mortar, wet	1.4	94
Mortar, hard	1.9	103
Nickel	8.57 – 8.90	546
Nickel, monel metal	8.8 – 9.0	556
Nitrogen	0.9714	0.0783
Oats, bulk	0.51	26
Oil, cottonseed	0.93 – 0.97	60

TABLE 16.—(*Continued*)

Substance	Specific gravity	Average weight, lb per cu ft
Oil, lard	0.90 – 0.97	58
Oil, linseed	0.94	59
Oil, lubricating, mineral	0.90 – 0.93	57
Oil, fuel	0.78 – 0.88	53
Oil, vegetable	0.91 – 0.94	58
Oxygen	1.1056	0.0892
Paper	0.70 – 1.15	58
Paraffin	0.87 – 0.91	55
Petroleum, crude	0.87 – 0.88	55
Pitch	1.07 – 1.15	70
Plaster of paris	1.4 – 2.2	112
Platinum, rolled	21.2 –21.7	1345
Plexiglass, 0.043 lb per cu in	1.19	74.3
Porcelain, china	2.30 – 2.50	150
Porphyry	2.6 – 2.9	172
Potassium	0.865	54
Potatoes, piled	....	42
Pumice, natural	0.37 – 0.90	40
Quartz, flint, solid	2.5 – 2.8	165
Quartz, quarried, crushed	....	95
Resins, rosin, amber	1.07	68
Rubber, caoutchouc, pure	0.93 – 0.96	58
Rubber compound	1.44 – 1.66	105
Salt, block	2.07 – 2.10	130
Salt, granulated, piled	....	47
Saltpeter	1.07	67
Sand, gravel, dry, loose	....	98
Sand, gravel, dry, in bank	1.6 – 1.9	110
Sand, gravel, wet	1.9	120
Sandstone, solid	2.2 – 2.5	147
Sandstone, quarried, broken	....	84
Shale, slate	2.6 – 2.9	175
Shale, crushed, piled	....	92
Silver, cast-hammered	10.4 –10.6	657
Snow, fresh-fallen	0.08 – 0.19	8
Snow, wet, compact	0.24 – 0.56	35
Soapstone, talc	2.6 – 2.8	169
Sodium	0.97 – 0.98	60.7

TABLE 16.—(*Continued*)

Substance	Specific gravity	Average weight, lb per cu ft
Starch	1.53	96
Steel, structural, 0.2833 lb per cu in	7.8 - 7.9	489.6
Steel, cold-drawn	7.83	489
Steel, machine	7.80	487
Steel, tool	7.70 - 7.73	481
Sulphur	1.93 - 2.07	125
Tar	1.0 - 1.20	63
Tile	1.80 - 1.85	116
Tile, hollow	0.42 - 0.72	40
Tin, cast-hammered	7.2 - 7.5	457
Trap rock, solid	2.8 - 3.2	187
Trap rock, crushed, piled	....	110
Tungsten	18.7 -19.2	1183
Turpentine	0.861- 0.867	54
Water, 4°C., max density 8.345 lb per gal	1.0	62.428
Water, 100°C	0.9584	59.830
Water, sea water	1.02 - 1.03	64
Wax, bees	0.95 - 0.98	60
Wheat, bulk	....	48
Wood, U. S. seasoned (moisture, 15 to 20 per cent):		
Ash, white or red	0.62 - 0.75	44
Birch	0.51 - 0.74	40
Cedar, white or red	0.35 - 0.65	35
Cherry	0.67 - 0.73	44
Chestnut	0.56 - 0.66	40
Cypress	0.48 - 0.57	34
Elm, white	0.56 - 0.72	44
Fir, Douglas spruce	0.51 - 0.59	35
Hemlock	0.39 - 0.52	26
Hickory	0.70 - 0.93	51
Lignum vitae	0.75 - 1.30	70
Mahogany	0.56 - 0.85	48
Maple, hard	0.68 - 0.80	46
Oak, live	0.95 - 1.20	64
Oak, red or black	0.65 - 0.75	45
Oak, white	0.70 - 0.80	48
Pine, white	0.38 - 0.48	26
Pine, yellow, long-leaf	0.66 - 0.72	44

TABLE 16.—(*Continued*)

Substance	Specific gravity	Average weight, lb per cu ft
Pine, yellow, short-leaf	0.55 – 0.62	38
Poplar	0.35 – 0.50	27
Redwood, California	0.42	26
Spruce, white	0.40 – 0.46	27
Walnut, black	0.61 – 0.70	40
Wool, compressed, in bales	0.78	48
Zinc, cast	6.9 – 7.05	432
Zinc, rolled	7.0 – 7.2	448

TABLE 17.—WEIGHTS OF FLAT STEEL PER LINEAL FOOT

(Based on 489.6 lb per cu ft)

Width, in.	Thickness, in.									
	1/8	3/16	1/4	5/16	3/8	1/2	5/8	3/4	7/8	1
1/2	0.21	0.31	0.42	0.53	0.63	0.85	1.06	1.28	1.49	1.70
5/8	0.26	0.39	0.53	0.66	0.79	1.06	1.33	1.60	1.86	2.13
3/4	0.31	0.47	0.63	0.79	0.95	1.27	1.59	1.91	2.23	2.55
7/8	0.37	0.55	0.74	0.92	1.11	1.48	1.85	2.23	2.60	2.98
1	0.42	0.63	0.85	1.06	1.28	1.70	2.12	2.55	2.98	3.40
1 1/8	0.47	0.71	0.95	1.20	1.43	1.92	2.39	2.87	3.35	3.83
1 1/4	0.53	0.79	1.06	1.33	1.59	2.12	2.65	3.19	3.72	4.25
1 3/8	0.58	0.87	1.17	1.46	1.76	2.34	2.92	3.51	4.09	4.68
1 1/2	0.63	0.95	1.28	1.59	1.92	2.55	3.19	3.83	4.47	5.10
1 5/8	0.69	1.04	1.38	1.73	2.08	2.72	3.46	4.15	4.84	5.53
1 3/4	0.74	1.15	1.49	1.86	2.23	2.98	3.72	4.47	5.20	5.95
2	0.85	1.28	1.70	2.12	2.55	3.40	4.25	5.10	5.95	6.80
2 1/4	0.96	1.44	1.92	2.39	2.87	3.83	4.78	5.75	6.69	7.65
2 1/2	1.06	1.59	2.12	2.65	3.19	4.25	5.31	6.38	7.44	8.50
2 3/4	1.17	1.75	2.34	2.92	3.51	4.67	5.84	7.02	8.18	9.35
3	1.28	1.91	2.55	3.19	3.83	5.10	6.38	7.65	8.93	10.20
3 1/4	1.38	2.07	2.76	3.45	4.15	5.53	6.91	8.29	9.67	11.05
3 1/2	1.49	2.23	2.98	3.72	4.47	5.95	7.44	8.93	10.41	11.90
3 3/4	1.59	2.39	3.19	3.99	4.78	6.38	7.97	9.57	11.16	12.75
4	1.70	2.55	3.40	4.25	5.10	6.80	8.50	10.20	11.90	13.60
4 1/2	1.91	2.87	3.83	4.78	5.74	7.65	9.57	11.48	13.39	15.30
5	2.13	3.19	4.25	5.31	6.38	8.50	10.63	12.75	14.87	17.00
5 1/2	2.33	3.51	4.67	5.84	7.02	9.35	11.69	14.03	16.36	18.70
6	2.55	3.83	5.10	6.38	7.65	10.20	12.75	15.30	17.85	20.40
6 1/2	2.76	4.14	5.53	6.90	8.29	11.05	13.81	16.58	19.34	22.10
7	2.97	4.46	5.95	7.44	8.93	11.90	14.87	17.85	20.83	23.80
7 1/2	3.18	4.78	6.36	7.97	9.57	12.75	15.94	19.13	22.32	25.50
8	3.40	5.10	6.80	8.50	10.20	13.60	17.00	20.40	23.80	27.20
8 1/2	3.61	5.42	7.22	9.03	10.84	14.44	18.06	21.68	25.30	28.90
9	3.82	5.74	7.65	9.56	11.48	15.30	19.13	22.96	26.78	30.60
9 1/2	4.30	6.06	8.08	10.10	12.12	16.16	20.19	24.23	28.26	32.30
10	4.25	6.38	8.50	10.62	12.75	17.00	21.25	25.50	29.75	34.00
10 1/2	4.46	6.70	8.92	11.16	13.39	17.85	22.32	26.78	31.24	35.70
11	4.67	7.02	9.34	11.68	14.03	18.70	23.38	28.05	32.72	37.40
12	5.10	7.65	10.20	12.75	15.30	20.40	25.50	30.60	35.70	40.80

For weight of sheets of other metals:

Aluminum alloy, multiply above weights by 0.353.
Cast iron, multiply above weights by 0.92.
Copper, multiply above weights by 1.13.
Cast brass, multiply above weights by 1.07.
Lead, multiply above weights by 1.45.

TABLE 18.—WEIGHTS OF ROUND AND SQUARE STEEL

Pounds per lineal foot

(Based on 489.6 lb per cu ft)

Size, in.	Round	Square	Size, in.	Round	Square
3/16	0.094	0.120	2 15/16	23.04	29.34
7/32	0.128	0.163	3	24.03	30.60
1/4	0.167	0.213			
			3 1/8	26.08	33.20
9/32	0.211	0.269	3 3/16	27.13	34.55
5/16	0.261	0.332	3 1/4	28.20	35.92
11/32	0.316	0.402	3 3/8	30.42	38.73
3/8	0.376	0.478	3 7/16	31.56	40.18
13/32	0.441	0.561	3 1/2	32.71	41.65
7/16	0.511	0.651			
1/2	0.668	0.850	3 5/8	35.09	44.68
			3 11/16	36.31	46.24
9/16	0.845	1.076	3 3/4	37.56	47.82
5/8	1.043	1.328	3 7/8	40.10	51.05
11/16	1.262	1.607	3 15/16	41.40	52.71
3/4	1.502	1.913	4	42.73	54.40
13/16	1 763	2.245			
7/8	2.044	2.603	4 1/8	45.44	57.85
15/16	2.347	2.989	4 3/16	46.83	59.62
1	2.670	3.400	4 1/4	48.24	61.41
			4 3/8	51.11	65.08
1 1/16	3.014	3.838	4 7/16	52.58	66.95
1 1/8	3.379	4.303	4 1/2	54.07	68.85
1 3/16	3.766	4.795			
1 1/4	4.173	5.312	4 5/8	57.12	72.73
1 5/16	4.600	5.857	4 11/16	58.67	74.70
1 3/8	5.049	6.428	4 3/4	60.25	76.71
1 7/16	5.518	7.026	4 7/8	63.46	80.81
1 1/2	6.008	7.650	4 15/16	65.10	82.89
			5	66.76	85.00
1 9/16	6.520	8.301			
1 5/8	7.051	8.978	5 1/8	70.14	89.30
1 11/16	7.604	9.682	5 3/16	71.86	91.49
1 3/4	8.178	10.41	5 1/4	73.60	93.72
1 13/16	8.773	11.17	5 3/8	77.15	98.23
1 7/8	9.388	11.95	5 7/16	78.95	100.5
1 15/16	10.02	12.76	5 1/2	80.77	102.8
			5 5/8	84.49	107.6
2	10.68	13.60	5 11/16	86.38	110.0
2 1/8	12.06	15.35	5 3/4	88.29	112.4
2 3/16	12.78	16.27			
2 1/4	13.52	17.22	5 7/8	92.17	117.4
2 3/8	15.07	19.18	5 15/16	94.14	119.9
2 7/16	15.86	20.20	6	96.14	122.4
2 1/2	16.69	21.25	6 1/2	112.8	143.6
			7	130.9	166.6
2 5/8	18.40	23.43	7 1/2	150.2	191.3
2 11/16	19.29	24.56	8	171.0	217.6
2 3/4	20.20	25.71	8 1/2	193.0	245.6
2 7/8	22.07	28.10	9	216.3	275.4

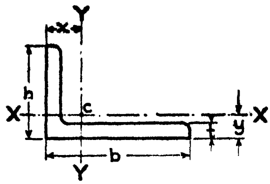


TABLE 19.—PROPERTIES OF SELECTED STANDARD ANGLES

Size					Axis $Y-Y$		Axis $X-X$	
Long leg b in.	Short leg h in.	Thickness t in.	Cross-sectional area sq in.	Weight, lb per lin ft	Second moment I_y in.^{4}	Centroid distance x in.	Second moment I_x in.^{4}	Centroid distance y in.
3.00	2.50	0.25	1.31	4.45	1.17	0.91	0.74	0.66
		0.375	1.92	6.52	1.66	0.96	1.04	0.71
3.50	2.50	0.25	1.44	4.89	1.80	1.11	0.78	0.61
		0.50	2.75	9.34	3.24	1.20	1.36	0.70
3 50	3 50	0.50	3.25	11.04	3.64	1.06		
		0.625	3.98	13 6	4.33	1.10		
4.00	3.00	0.50	3.25	11.1	5.05	1 33	2.42	0.83
		0.625	3.98	13 6	6.03	1.37	2.87	0.87
4.00	3.50	0.50	3.50	11.9	5.32	1.25	3.79	1.00
		0.625	4.30	14.7	6.37	1.29	4.49	1.04
4.00	4.00	0.50	3.75	12.8	5.56	1.18		
		0.625	4.61	15.7	6.66	1.23		
5 00	3.00	0.50	3.75	12.8	9.45	1.75	2.58	0.75
		0.625	4.61	15.7	11.37	1.80	3.06	0.80
5.00	3.50	0.50	4.00	13.6	9.99	1.66	4.05	0.91
		0.75	5.81	19.8	13.92	1.75	5.55	1.00
6.00	3.50	0.50	4.50	15.3	16.59	2.08	4.25	0.83
		0.75	6.56	22.4	23.34	2.18	5.84	0.93
6.00	4.00	0.50	4.75	16.2	17.40	1.99	6.27	0.99
		0.75	6.94	23.6	24.59	2.08	8.68	1.08
6.00	6.00	0.625	7.11	24.2	24.16	1.73		
		0.75	8.44	28.7	28.15	1.78		
7.00	3.50	0.50	5.00	17.0	25.41	2.53	4.41	0.78
		0.75	7.31	24.9	35.99	2.62	6.08	0.87

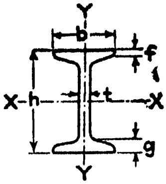


TABLE 20.—PROPERTIES OF SELECTED STANDARD I BEAMS

Dimensions							Axis $X - X$		Axis $Y - Y$	
Depth h in.	Weight, lb per ft	Cross-sectional area, sq in.	Width of flange, b in	Web thickness t, in.	Flange thickness Min. f, in.	Flange thickness Max. g, in.	Second moment I_x, in.⁴	$\sqrt{\frac{I}{A}} = r$ in	Second moment I_y, in.	$\sqrt{\frac{I}{A}} = r$, in.
4	7.7	2.21	2.660	0.190	0.190	0.396	6.0	1.64	0.77	0.59
4	9.5	2.76	2.796	0.326	0.190	0.396	6.7	1.56	0.91	0.58
6	12.5	3.61	3.330	0.230	0.230	0.488	21.8	2.46	1.8	0.72
6	17.25	5.02	3.565	0.465	0.230	0.488	26.0	2.28	2.3	0.68
8	18.4	5.34	4.000	0.270	0.270	0.581	56.9	3.26	3.8	0.84
8	23 0	6.71	4.171	0.441	0.270	0 581	64.2	3.09	4.4	0.81
8	25.5	7.43	4.262	0.532	0.270	0.581	68.1	3.03	4.7	0.80
10	25.4	7.38	4.660	0.310	0.310	0.673	122.1	4.07	6.9	0.97
10	35.0	10.22	4.9[illegible]	0.594	0.310	0.673	145.8	3.78	8.5	0.91
10	40.0	11.69	5.091	0.741	0.310	0.673	158.0	3.68	9.4	0.90
12	31.8	9.26	5.000	0.350	0.350	0.738	215.8	4.83	9.5	1.01
12	40.8	11.84	5.250	0.460	0.460	0.859	268.9	4.77	13.8	1.08
12	50.0	14.57	5.477	0.687	0.460	0.859	301 6	4.55	16.0	1.05

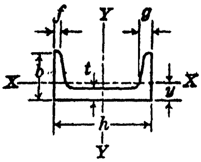


TABLE 21.—PROPERTIES OF SELECTED STANDARD CHANNELS

Dimensions							Axis $Y - Y$		Axis $X - X$		
Depth h in.	Weight, lb per ft	Cross-sectional area, sq in.	Width of flange, b in.	Web thickness t, in.	Flange thickness Min. f, in.	Flange thickness Max. g, in.	Second moment I_y in.^{4}	$\sqrt{\frac{I}{A}} = r$, in.	Second moment I_x, in.	$\sqrt{\frac{I}{A}} = r$, in.	Centroidal distance y in.
4	5.4	1.56	1.580	0.180	0.180	0.413	3.8	1.56	0.32	0.45	0.46
4	7.25	2.12	1.720	0.320	0.180	0.413	4.5	1.47	0.44	0.46	0.46
6	8.2	2.39	1.920	0.200	0.200	0.487	13.0	2.34	0.70	0.54	0.52
6	13.0	3.81	2.157	0.437	0.200	0.487	17.3	2.13	1.1	0.53	0.52
8	11.50	3.36	2.26	0.220	0.220	0.560	32.3	3.10	1.3	0.63	0.58
8	16.25	4.76	2.435	0.395	0.220	0.560	39.8	2.89	1.8	0.61	0.56
8	21.25	6.23	2.619	0.579	0.220	0.560	47.6	2.77	2.2	0.60	0.59
10	15.3	4.47	2.600	0.240	0.240	0.633	66.9	3.87	2.3	0.72	0.64
10	25.0	7.33	2.886	0.526	0.240	0.633	90.7	3.52	3.4	0.68	0.62
10	35.0	10.27	3.180	0.820	0.240	0.633	115.2	3.34	4.6	0.67	0.69
12	20.7	6.03	2.940	0.280	0.280	0.723	128.1	4.61	3.9	0.81	0.70
12	30.0	8.79	3.170	0.510	0.280	0.723	161.2	4.28	5.2	0.77	0.63
12	40.0	11.73	3.415	0.755	0.280	0.723	196.5	4.09	6.6	0.75	0.72

TABLE 22.—MANILA-ROPE DRIVES

Diameter of rope, in.	Weight per foot, lb	Breaking load, lb	Working load, lb
$\frac{5}{8}$	0.13	2,900	78
$\frac{3}{4}$	0.18	4,200	112
$\frac{7}{8}$	0.26	5,700	153
1	0.33	7,500	200
$1\frac{1}{8}$	0.45	9,400	253
$1\frac{1}{4}$	0.50	11,700	312
$1\frac{3}{8}$	0.65	14,000	378
$1\frac{1}{2}$	0.72	16,800	450
$1\frac{5}{8}$	0.85	19,800	527
$1\frac{3}{4}$	0.98	22,900	612
2	1.27	30,000	800

TABLE 23.—WORKING LOADS OF STANDARD HOISTING ROPE
Loads based on one-fifth of breaking strength

Diameter, in.	Approximate weight per foot, lb	Working load, tons of 2000 lb				
		Iron	Cast steel	Extra-strong cast steel	Plow steel	Improved plow steel
$2\frac{3}{4}$	11.95	22.2	42.2	48.6	55.0	63.0
$2\frac{1}{2}$	9.85	18.4	34.0	40.0	46.0	53.0
$2\frac{1}{4}$	8.0	14.4	26.6	32.0	37.0	42.0
2	6.3	11.0	21.2	24.6	28.0	33.0
$1\frac{7}{8}$	5.55	10.0	19.0	22.4	25.0	30.0
$1\frac{3}{4}$	4.85	8.8	17.0	19.8	22.0	27.0
$1\frac{5}{8}$	4.15	7.6	14.4	16.6	19.0	22.0
$1\frac{1}{2}$	3.55	6.6	12.8	14.6	16.0	20.0
$1\frac{3}{8}$	3.0	5.6	11.2	12.8	14.0	17.0
$1\frac{1}{4}$	2.45	4.56	9.4	10.6	12.0	14.0
$1\frac{1}{8}$	2.0	3.72	7.6	8.6	9.4	11.0
1	1.58	2.90	6.0	6.80	7.6	9.0
$\frac{7}{8}$	1.20	2.36	4.6	5.20	5.8	7.0
$\frac{3}{4}$	0.89	1.70	3.5	4.04	4.6	5.3
$\frac{5}{8}$	0.62	1.20	2.5	2.80	3.1	3.8
$\frac{9}{16}$	0.50	0.94	2.0	2.24	2.4	2.9
$\frac{1}{2}$	0.39	0.78	1.68	1.84	2.0	2.4
$\frac{7}{16}$	0.30	0.58	1.30	1.45	1.6	1.9
$\frac{3}{8}$	0.22	0.48	0.96	1.06	1.15	1.35
$\frac{5}{16}$	0.15	0.30	0.62	0.70	0.76	0.9
$\frac{1}{4}$	0.10	0.22	0.44	0.49	0.53	0 63

TABLE 24.—POWER TRANSMITTED BY TURNED STEEL SHAFTING[1]

Diam of shaft, in.	Power transmitted by shaft in hp										
	Speed of shaft in rpm										
	100	150	200	250	300	350	400	450	500	550	600
$1\frac{1}{2}$	3.7	5.6	7.5	9.4	11.2	13.1	15	16.9	18.8	20.5	22
$1\frac{5}{8}$	4.8	7.1	9.5	11.9	14.3	16.6	19	21.0	24.0	26.0	28
$1\frac{3}{4}$	5.9	8.9	11.9	14.9	17.9	21.0	24	27.0	30.0	33.0	36
$1\frac{7}{8}$	7.3	11.0	14.7	18.3	22.0	26.0	29	33.0	37.0	41.0	44
2	8.9	13.3	17.8	22.0	27.0	31.0	35	40.0	44.0	48.0	53
$2\frac{1}{8}$	10.6	16.0	21.0	27.0	32.0	37.0	43	48.0	53.0	58.0	64
$2\frac{1}{4}$	12.6	19.0	25.0	32.0	38.0	44.0	51	57.0	63.0	69.0	76
$2\frac{3}{8}$	14.9	22.0	30.0	37.0	45.0	52.0	60	67.0	74.0	81.0	89
$2\frac{1}{2}$	17.4	26.0	35.0	43.0	52.0	61.0	69	78.0	87.0	96.0	104
$2\frac{5}{8}$	20.0	30.0	40.0	50.0	60.0	71.0	80	90.0	100.0	110.0	120
$2\frac{3}{4}$	23.0	35.0	46.0	58.0	69.0	81.0	92	104.0	115.0	125.0	138
$2\frac{7}{8}$	26.0	40.0	53.0	66.0	79.0	92.0	105	119.0	132.0	145.0	158
3	30.0	45.0	60.0	75.0	90.0	105.0	120	135.0	150.0	165.0	180
$3\frac{1}{8}$	34.0	51.0	68.0	85.0	102.0	119.0	136	152.0	170.0	187.0	203
$3\frac{1}{4}$	38.0	57.0	76.0	95.0	114.0	134.0	153	172.0	191.0	210.0	229
$3\frac{3}{8}$	43.0	64.0	85.0	107.0	128.0	150.0	171	192.0	213.0	234.0	256
$3\frac{1}{2}$	48.0	72.0	95.0	119.0	143.0	167.0	190	214.0	238.0	262.0	286

[1] From Marks' "Mechanical Engineers' Handbook," 4th ed., McGraw-Hill Book Company, Inc., 1941.

TABLE 25.—COEFFICIENTS OF STATIC FRICTION
(Average values)

Materials	Conditions	Static friction f	Static friction θ
General:			
Metals on oak	dry	0.4 –0.6	26–31°
Wood on wood	dry	0.25–0.5	14–26½°
Steel on steel	dry	0.22	12½°
Steel on bronze	dry	0.19	11°
Steel on ice	dry	0.027	1½°
Building materials:			
Stone or brick masonry on mortar	dry	0.70	35°
Stone or brick masonry on mortar	wet	0.30	16½°
Timber on stone	dry	0.40	22°
Iron on masonry	new dry	0.30	16½°
Iron on masonry	rusted on	0.60	31°
Masonry on earth	dry	0.65	33°
Masonry on earth	wet	0.30	16½°
For friction wheels:			Safe working pressure, pounds per linear inch face
Leather fiber on iron	dry	0.31	250
Tarred fiber on iron	dry	0.15	250
Straw fiber on iron	dry	0.25	150
Sulphite fiber on iron	dry	0.33	140
Leather on iron	dry	0.14	150
Cork composition, iron	dry	0.21	50

TABLE 26.—COEFFICIENTS OF SLIDING FRICTION
(Average values)

Materials	Conditions	Sliding friction f	Sliding friction θ
General:			
Metals on oak	dry	0.4–0.6	26–31°
Metals on oak	wet	0.24–0.26	$13\frac{1}{2}$–14°
Metals on oak	soapy	0.2	$11\frac{1}{2}$°
Metals on oak	greased	0.05–0.08	3–$4\frac{1}{2}$°
Wood on wood	dry	0.25–0.5	14–$26\frac{1}{2}$°
Wood on wood	soapy	0.2	$11\frac{1}{2}$°
Leather on oak	....	0.27–0.35	15–$19\frac{1}{2}$°
Leather on metals	dry	0.56	$29\frac{1}{2}$°
Leather on metals	wet	0.36	20°
Leather on metals	greasy	0.23	13°
Leather on metals	oily	0.15	$8\frac{1}{2}$°
Steel on steel	dry	0.15	$8\frac{1}{2}$°
Steel on bronze	dry	0.18	$10\frac{1}{4}$°
Steel on ice	dry	0.014 –0.02	1°
Bronze on bronze	dry	0.20	$11\frac{1}{2}$°
Bronze on cast iron	dry	0.21	12°
Cast iron on cast iron	rough and dry	0.31	17°
Cast iron on cast iron	machined, lubricated	0.15	$8\frac{1}{2}$°
Cast iron on wood	rough and dry	0.49	$26\frac{1}{2}$°
Cast iron on wood	machined, lubricated	0.19	11°
Journal bearings:			
Cast iron on bronze	ordinary lubrication	0.080 –0.260	
Cast iron on bronze	oil bath	0.0012–0.007	
Worm gearing	well greased	0.010 –0.10	
For trains:			
Cast-iron brakes on			
steel tires	10 mph	0.18	
steel tires	50 mph	0.098	
Sleds, wood runners			
on stone surfaces	dry	0.38	21°
on stone surfaces	greased	0.11	6°

TABLE 27.—RESISTANCE TO MOTION OF TRAINS, AUTOS, ETC.
(Average in still air)

Vehicle	Resistance, pounds per ton weight
Trains, starting	20
Train speed 5 mph	3
Train speed 10 mph	4
Train speed 20 mph	6½
Train speed 30 mph	9
Train speed 40 mph	11¼
Train speed 50 mph	13½
Train speed 60 mph	16
Autos, traction resistance:	
Smooth asphalt or concrete pavement	20
Macadam or hard rock pavement	30
Good gravel, well-scraped roads	50
Poor, thick, gravel, sand roads	75–125
Dirt roads, medium condition, dry	60
Muddy roads, passable	250
Air resistance, in pounds, for autos = $0.003V^2A$, where V = speed in miles per hour and A = forward projecting area in square feet. This is in addition to traction resistance.	
Wagons and trucks:	
Asphalt road	30
Macadam	34
Ordinary gravel	38
Dry dirt	50

TABLE 28.—FUNCTIONS AND LOGARITHMS OF π

Functions of π	Logarithm
$\pi = 3.141593$	0.497150
$0.25\pi = 0.785398$	$\bar{1}.895090$
$\pi^2 = 9.869604$	0.994300
$\frac{1}{\pi} = 0.3183099$	$\bar{1}.502850$
$(\pi)^{0.5} = 1.772454$	0.248575

TABLE 29.—DECIMALS OF AN INCH

Fractions of an inch	Decimal value	Fractions of an inch	Decimal value
$\frac{1}{64}$	0.015625	$\frac{33}{64}$	0.515625
$\frac{1}{32}$	0.03125	$\frac{17}{32}$	0.53125
$\frac{3}{64}$	0.046875	$\frac{35}{64}$	0.546875
$\frac{1}{16}$	0.0625	$\frac{9}{16}$	0.5625
$\frac{5}{64}$	0.078125	$\frac{37}{64}$	0.578125
$\frac{3}{32}$	0.09375	$\frac{19}{32}$	0.59375
$\frac{7}{64}$	0.109375	$\frac{39}{64}$	0.609375
$\frac{1}{8}$	0.125	$\frac{5}{8}$	0.625
$\frac{9}{64}$	0.140625	$\frac{41}{64}$	0.640625
$\frac{5}{32}$	0.15625	$\frac{21}{32}$	0.65625
$\frac{11}{64}$	0.171875	$\frac{43}{64}$	0.671875
$\frac{3}{16}$	0.1875	$\frac{11}{16}$	0.6875
$\frac{13}{64}$	0.203125	$\frac{45}{64}$	0.703125
$\frac{7}{32}$	0.21875	$\frac{23}{32}$	0.71875
$\frac{15}{64}$	0.234375	$\frac{47}{64}$	0.734375
$\frac{1}{4}$	0.250	$\frac{3}{4}$	0.750
$\frac{17}{64}$	0.265625	$\frac{49}{64}$	0.765625
$\frac{9}{32}$	0.28125	$\frac{25}{32}$	0.78125
$\frac{19}{64}$	0.296875	$\frac{51}{64}$	0.796875
$\frac{5}{16}$	0.3125	$\frac{13}{16}$	0.8125
$\frac{21}{64}$	0.328125	$\frac{53}{64}$	0.828125
$\frac{11}{32}$	0.34375	$\frac{27}{32}$	0.84375
$\frac{23}{64}$	0.359375	$\frac{55}{64}$	0.859375
$\frac{3}{8}$	0.375	$\frac{7}{8}$	0.875
$\frac{25}{64}$	0.390625	$\frac{57}{64}$	0.890625
$\frac{13}{32}$	0.40625	$\frac{29}{32}$	0.90625
$\frac{27}{64}$	0.421875	$\frac{59}{64}$	0.921875
$\frac{7}{16}$	0.4375	$\frac{15}{16}$	0.9375
$\frac{29}{64}$	0.453125	$\frac{61}{64}$	0.953125
$\frac{15}{32}$	0.46875	$\frac{31}{32}$	0.96875
$\frac{31}{64}$	0.484375	$\frac{63}{64}$	0.984375
$\frac{1}{2}$	0.500	1	1.000

TABLE 30.—DECIMALS OF A FOOT

Decimal of a foot	Inches	Decimal of a foot	Inches	Decimal of a foot	Inches	Decimal of a foot	Inches
0.0052	1/16	0.2552	3 1/16	0.5052	6 1/16	0.7552	9 1/16
0.0104	1/8	0.2604	3 1/8	0.5104	6 1/8	0.7604	9 1/8
0.015625	3/16	0.265625	3 3/16	0.515625	6 3/16	0.765625	9 3/16
0.0208	1/4	0.2708	3 1/4	0.5208	6 1/4	0.7708	9 1/4
0.0260	5/16	0.2760	3 5/16	0.5260	6 5/16	0.7760	9 5/16
0.03125	3/8	0.28125	3 3/8	0.53125	6 3/8	0.78125	9 3/8
0.0365	7/16	0.2865	3 7/16	0.5365	6 7/16	0.7865	9 7/16
0.0417	1/2	0.2917	3 1/2	0.5417	6 1/2	0.7917	9 1/2
0.046875	9/16	0.296875	3 9/16	0.546875	6 9/16	0.796875	9 9/16
0.0521	5/8	0.3021	3 5/8	0.5521	6 5/8	0.8021	9 5/8
0.0573	11/16	0.3073	3 11/16	0.5573	6 11/16	0.8073	9 11/16
0.0625	3/4	0.3125	3 3/4	0.5625	6 3/4	0.8125	9 3/4
0.0677	13/16	0.3177	3 13/16	0.5677	6 13/16	0.8177	9 13/16
0.0729	7/8	0.3229	3 7/8	0.5729	6 7/8	0.8229	9 7/8
0.078125	15/16	0.328125	3 15/16	0.578125	6 15/16	0.828125	9 15/16
0.0833	1	0.3333	4	0.5833	7	0.8333	10
0.0885	1 1/16	0.3385	4 1/16	0.5885	7 1/16	0.8385	10 1/16
0.09375	1 1/8	0.34375	4 1/8	0.59375	7 1/8	0.84375	10 1/8
0.0990	1 3/16	0.3490	4 3/16	0.5990	7 3/16	0.8490	10 3/16
0.1042	1 1/4	0.3542	4 1/4	0.6042	7 1/4	0.8542	10 1/4
0.109375	1 5/16	0.359375	4 5/16	0.609375	7 5/16	0.859375	10 5/16
0.1146	1 3/8	0.3646	4 3/8	0.6146	7 3/8	0.8646	10 3/8
0.1198	1 7/16	0.3698	4 7/16	0.6198	7 7/16	0.8698	10 7/16
0.1250	1 1/2	0.3750	4 1/2	0.6250	7 1/2	0.8750	10 1/2
0.1302	1 9/16	0.3802	4 9/16	0.6302	7 9/16	0.8802	10 9/16
0.1354	1 5/8	0.3854	4 5/8	0.6354	7 5/8	0.8854	10 5/8
0.140625	1 11/16	0.390625	4 11/16	0.640625	7 11/16	0.890625	10 11/16
0.1458	1 3/4	0.3958	4 3/4	0.6458	7 3/4	0.8958	10 3/4
0.1510	1 13/16	0.4010	4 13/16	0.6510	7 13/16	0.9010	10 13/16
0.15625	1 7/8	0.40625	4 7/8	0.65625	7 7/8	0.90625	10 7/8
0.1615	1 15/16	0.4115	4 15/16	0.6615	7 15/16	0.9115	10 15/16
0.1667	2	0.4167	5	0.6667	8	0.9167	11
0.171875	2 1/16	0.421875	5 1/16	0.671875	8 1/16	0.921875	11 1/16
0.1771	2 1/8	0.4271	5 1/8	0.6771	8 1/8	0.9271	11 1/8
0.1823	2 3/16	0.4323	5 3/16	0.6823	8 3/16	0.9323	11 3/16
0.1875	2 1/4	0.4375	5 1/4	0.6875	8 1/4	0.9375	11 1/4
0.1927	2 5/16	0.4427	5 5/16	0.6927	8 5/16	0.9427	11 5/16
0.1979	2 3/8	0.4479	5 3/8	0.6979	8 3/8	0.9479	11 3/8
0.203125	2 7/16	0.453125	5 7/16	0.703125	8 7/16	0.953125	11 7/16
0.2083	2 1/2	0.4583	5 1/2	0.7083	8 1/2	0.9583	11 1/2
0.2135	2 9/16	0.4635	5 9/16	0.7135	8 9/16	0.9635	11 9/16
0.21875	2 5/8	0.46875	5 5/8	0.71875	8 5/8	0.96875	11 5/8
0.2240	2 11/16	0.4740	5 11/16	0.7240	8 11/16	0.9740	11 11/16
0.2292	2 3/4	0.4792	5 3/4	0.7292	8 3/4	0.9792	11 3/4
0.234375	2 13/16	0.484375	5 13/16	0.734375	8 13/16	0.984375	11 13/16
0.2396	2 7/8	0.4896	5 7/8	0.7396	8 7/8	0.9896	11 7/8
0.2448	2 15/16	0.4948	5 15/16	0.7448	9 15/16	0.9948	11 15/16
0.2500	3	0.5000	6	0.7500	9	1.0000	12

TABLE 31.—FUNCTIONS OF NUMBERS

Squares, cubes, square roots, cube roots, and reciprocals.
Circumferences and areas of circles when number is the diameter.

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
1	1	1	1.0000	1.0000	1.000000000	3.1416	0.7854
2	4	8	1.4142	1.2599	0.500000000	6.2832	3.1416
3	9	27	1.7321	1.4422	0.333333333	9.4248	7.0686
4	16	64	2.0000	1.5874	0.250000000	12.5664	12.5664
5	25	125	2.2361	1.7100	0.200000000	15.7080	19.635
6	36	216	2.4495	1.8171	0.166666667	18.850	28 274
7	49	343	2.6458	1.9129	0.142857143	21.991	38.485
8	64	512	2.8284	2.0000	0.125000000	25.133	50.266
9	81	729	3.0000	2.0801	0.111111111	28.274	63 617
10	100	1,000	3.1623	2.1544	0.100000000	31.416	78.540
11	121	1,331	3.3166	2.2240	0.090909091	34.558	95.033
12	144	1,728	3.4641	2.2894	0.083333333	37.699	113.10
13	169	2,197	3.6056	2.3513	0.076923077	40.841	132 73
14	196	2,744	3.7417	2.4101	0.071428571	43.982	153.94
15	225	3,375	3.8730	2.4662	0.066666667	47.124	176.71
16	256	4,096	4.0000	2.5198	0.062500000	50.265	201.06
17	289	4,913	4.1231	2.5713	0.058823529	53.407	226.98
18	324	5,832	4.2426	2.6207	0.055555556	56.549	254.47
19	361	6,859	4.3589	2.6684	0.052631579	59.690	283.53
20	400	8,000	4.4721	2.7144	0.050000000	62.832	314.16
21	441	9,261	4.5826	2.7589	0.047619048	65.973	346.36
22	484	10,648	4.6904	2.8020	0.045454545	69.115	380.13
23	529	12,167	4.7958	2.8439	0.043478261	72.257	415.48
24	576	13,824	4.8990	2.8845	0.041666667	75.398	452.39
25	625	15,625	5.0000	2.9240	0.040000000	78.540	490.87
26	676	17,576	5.0990	2.9625	0.038461538	81.681	530.93
27	729	19,683	5.1962	3.0000	0.037037037	84.823	572.56
28	784	21,952	5.2915	3.0366	0.035714286	87.965	615 75
29	841	24,389	5.3852	3.0723	0.034482759	91.106	660 52
30	900	27,000	5.4772	3.1072	0.033333333	94.248	706.86
31	961	29,791	5.5678	3.1414	0.032258065	97.389	754.77
32	1,024	32,768	5.6569	3.1748	0.031250000	100.53	804.25
33	1,089	35,937	5.7446	3.2075	0.030303030	103.67	855.30
34	1,156	39,304	5.8310	3.2396	0.029411765	106.81	907.92
35	1,225	42,875	5.9161	3.2711	0.028571429	109.96	962.11
36	1,296	46,656	6.0000	3.3019	0.027777778	113.10	1,017.88
37	1,369	50,653	6.0828	3.3322	0.027027027	116.24	1,075.21
38	1,444	54,872	6.1644	3.3620	0.026315789	119.38	1,134.11
39	1,521	59,319	6.2450	3.3912	0.025641026	122.52	1,194.59
40	1,600	64,000	6.3246	3.4200	0.025000000	125.66	1,256.64
41	1,681	68,921	6.4031	3.4482	0.024390244	128.81	1,320.25
42	1,764	74,088	6.4807	3.4760	0.023809525	131.95	1,385.44
43	1,849	79,507	6.5574	3.5034	0.023255814	135.09	1,452.20
44	1,936	85,184	6.6332	3.5303	0.022727273	138.23	1,520.53
45	2,025	91,125	6.7082	3.5569	0.022222222	141.37	1,590.43
46	2,116	97,336	6.7823	3.5830	0.021739130	144.51	1,661.90
47	2,209	103,823	6.8557	3.6088	0.021276600	147.65	1,734.94
48	2,304	110,592	6.9282	3.6342	0.020833333	150.80	1,809.56
49	2,401	117,649	7.0000	3.6593	0.020408163	153.94	1,885.74
50	2,500	125,000	7.0711	3.6840	0.020000000	157.08	1,963.50
51	2,601	132,651	7.1414	3.7084	0.019607843	160.22	2,042.82
52	2,704	140,608	7.2111	3.7325	0.019230769	163.36	2,123.72
53	2,809	148,877	7.2801	3.7563	0.018867925	166.50	2,206.18
54	2,916	157,464	7.3485	3.7798	0.018518519	169.65	2,290.22
55	3,025	166,375	7.4162	3.8030	0.018181818	172.79	2,375.83

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
56	3,136	175,616	7.4833	3.8259	0.017857143	175.93	2,463.01
57	3,249	185,193	7.5498	3.8485	0.017543860	179.07	2,551.76
58	3,364	195,112	7.6158	3.8709	0.017241379	182.21	2,642.08
59	3,481	205,379	7.6811	3.8930	0.016949153	185.35	2,733.97
60	3,600	216,000	7.7460	3.9149	0.016666667	188.50	2,827.43
61	3,721	226,981	7.8102	3.9365	0.016393443	191.64	2,922.47
62	3,844	238,328	7.8740	3.9579	0.016129032	194.78	3,019.07
63	3,969	250,047	7.9373	3.9791	0.015873016	197.92	3,117.25
64	4,096	262,144	8.0000	4.0000	0.015625000	201.06	3,216.99
65	4,225	274,625	8.0623	4.0207	0.015384615	204.20	3,318.31
66	4,356	287,496	8.1240	4.0412	0.015151515	207.34	3,421.19
67	4,489	300,763	8.1854	4.0615	0.014925373	210.49	3,525.65
68	4,624	314,432	8.2462	4.0817	0.014705882	213.63	3,631.68
69	4,761	328,509	8.3066	4.1016	0.014492754	216.77	3,739.28
70	4,900	343,000	8.3666	4.1213	0.014285714	219.91	3,848.45
71	5,041	357,911	8.4261	4.1408	0.014084517	223.05	3,959.19
72	5,184	373,248	8.4853	4.1602	0.013888889	226.19	4,071.50
73	5,329	389,017	8.5440	4.1793	0.013698630	229.34	4,185.39
74	5,476	405,224	8.6023	4.1983	0.013513514	232.48	4,300.84
75	5,625	421,875	8.6603	4.2172	0.013333333	235.62	4,417.86
76	5,776	438,976	8.7178	4.2358	0.013157895	238.76	4,536.46
77	5,929	456,533	8.7750	4.2543	0.012987013	241.90	4,656.63
78	6,084	474,552	8.8318	4.2727	0.012820513	245.04	4,778.36
79	6,241	493,039	8.8882	4.2908	0.012658228	248.19	4,901.67
80	6,400	512,000	8.9443	4.3089	0.012500000	251.33	5,026.55
81	6,561	531,441	9.0000	4.3267	0.012345679	254.47	5,153.00
82	6,724	551,368	9.0554	4.3445	0.012195122	257.61	5,281.02
83	6,889	571,787	9.1104	4.3621	0.012048193	260.75	5,410.61
84	7,056	592,704	9.1652	4.3795	0.011904762	263.89	5,541.77
85	7,225	614,125	9.2195	4.3968	0.011764706	267.04	5,674.50
86	7,396	636,056	9.2736	4.4140	0.011627907	270.18	5,808.80
87	7,569	658,503	9.3274	4.4310	0.011494253	273.32	5,944.68
88	7,744	681,472	9.3808	4.4480	0.011363636	276.46	6,082.12
89	7,921	704,969	9.4340	4.4647	0.011235955	279.60	6,221.14
90	8,100	729,000	9.4868	4.4814	0.011111111	282.74	6,361.73
91	8,281	753,571	9.5394	4.4979	0.010989011	285.88	6,503.88
92	8,464	778,688	9.5917	4.5144	0.010869565	289.03	6,647.61
93	8,649	804,357	9.6437	4.5307	0.010752688	292.17	6,792.91
94	8,836	830,584	9.6954	4.5468	0.010638298	295.31	6,939.78
95	9,025	857,375	9.7468	4.5629	0.010526316	298.45	7,088.22
96	9,216	884,736	9.7980	4.5789	0.010416667	301.59	7,238.23
97	9,409	912,673	9.8489	4.5947	0.010309278	304.73	7,389.81
98	9,604	941,192	9.8995	4.6104	0.010204082	307.88	7,542.96
99	9,801	970,299	9.9499	4.6261	0.010101010	311.02	7,697.69
100	10,000	1,000,000	10.0000	4.6416	0.010000000	314.16	7,853.98
101	10,201	1,030,301	10.0499	4.6570	0.009900990	317.30	8,011.85
102	10,404	1,061,208	10.0995	4.6723	0.009803922	320.44	8,171.28
103	10,609	1,092,727	10.1489	4.6875	0.009708738	323.58	8,332.29
104	10,816	1,124,864	10.1980	4.7027	0.009615385	326.73	8,494.87
105	11,025	1,157,625	10.2470	4.7177	0.009523810	329.87	8,659.01
106	11,236	1,191,016	10.2956	4.7326	0.009433962	333.01	8,824.73
107	11,449	1,225,043	10.3441	4.7475	0.009345794	336.15	8,992.02
108	11,664	1,259,712	10.3923	4.7622	0.009259259	339.29	9,160.88
109	11,881	1,295,029	10.4403	4.7769	0.009174312	342.43	9,331.32
110	12,100	1,331,000	10.4881	4.7914	0.009090909	345.58	9,503.32
111	12,321	1,367,631	10.5357	4.8059	0.009009009	348.72	9,676.89
112	12,544	1,404,928	10.5830	4.8203	0.008928571	351.86	9,852.03
113	12,769	1,442,897	10.6301	4.8346	0.008849558	355.00	10,028.75
114	12,996	1,481,544	10.6771	4.8488	0.008771930	358.14	10,207.03
115	13,225	1,520,875	10.7238	4.8629	0.008695652	361.28	10,386.89

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
116	13,456	1,560,896	10.7703	4.8770	0.008620690	364.42	10,568.32
117	13,689	1,601,613	10.8167	4.8910	0.008547009	367.57	10,751.32
118	13,924	1,643,032	10.8628	4.9049	0.008474576	370.71	10,935.88
119	14,161	1,685,159	10.9087	4.9187	0.008403361	373.85	11,122.02
120	14,400	1,728,000	10.9545	4.9324	0.008333333	376.99	11,309.73
121	14,641	1,771,561	11.0000	4.9461	0.008264463	380.13	11,499.01
122	14,884	1,815,848	11.0454	4.9597	0.008196721	383.27	11,689.87
123	15,129	1,860,867	11.0905	4.9732	0.008130081	386.42	11,882.29
124	15,376	1,906,624	11.1355	4.9866	0.008064516	389.56	12,076.28
125	15,625	1,953,125	11.1803	5.0000	0.008000000	392.70	12,271.85
126	15,876	2,000,376	11.2250	5.0133	0.007936508	395.84	12,468.98
127	16,129	2,048,383	11.2694	5.0265	0.007874016	398.98	12,667.69
128	16,384	2,097,152	11.3137	5.0397	0.007812500	402.12	12,867.96
129	16,641	2,146,689	11.3578	5.0528	0.007751938	405.27	13,069.81
130	16,900	2,197,000	11.4018	5.0658	0.007692308	408.41	13,273.23
131	17,161	2,248,091	11.4455	5.0788	0.007633588	411.55	13,478.22
132	17,424	2,299,968	11.4891	5.0916	0.007575758	414.69	13,684.78
133	17,689	2,352,637	11.5326	5.1045	0.007518797	417.83	13,892.91
134	17,956	2,406,104	11.5758	5.1172	0.007462687	420.97	14,102.61
135	18,225	2,460,375	11.6190	5.1299	0.007407407	424.12	14,313.88
136	18,496	2,515,456	11.6619	5.1426	0.007352941	427.26	14,526.72
137	18,769	2,571,353	11.7047	5.1551	0.007299270	430.40	14,741.14
138	19,044	2,628,072	11.7473	5.1676	0.007246377	433.54	14,957.12
139	19,321	2,685,619	11.7898	5.1801	0.007194245	436.68	15,174.68
140	19,600	2,744,000	11.8322	5.1925	0.007142857	439.82	15,393.80
141	19,881	2,803,221	11.8743	5.2048	0.007092199	442.96	15,614.50
142	20,164	2,863,288	11.9164	5.2171	0.007042254	446.11	15,836.77
143	20,449	2,924,207	11.9583	5.2293	0.006993007	449.25	16,060.61
144	20,736	2,985,984	12.0000	5.2415	0.006944444	452.39	16,286.02
145	21,025	3,048,625	12.0416	5.2536	0.006896552	455.53	16,513.00
146	21,316	3,112,136	12.0830	5.2656	0.006849315	458.67	16,741.55
147	21,609	3,176,523	12.1244	5.2776	0.006802721	461.81	16,971.67
148	21,904	3,241,792	12.1655	5.2896	0.006756757	464.96	17,203.36
149	22,201	3,307,949	12.2066	5.3015	0.006711409	468.10	17,436.62
150	22,500	3,375,000	12.2474	5.3133	0.006666667	471.24	17,671.46
151	22,801	3,442,951	12.2882	5.3251	0.006622517	474.38	17,907.86
152	23,104	3,511,008	12.3288	5.3368	0.006578947	477.52	18,145.84
153	23,409	3,581,577	12.3693	5.3485	0.006535948	480.66	18,385.39
154	23,716	3,652,264	12.4097	5.3601	0.006493506	483.81	18,626.50
155	24,025	3,723,875	12.4499	5.3717	0.006451613	486.95	18,869.19
156	24,336	3,796,416	12.4900	5.3832	0.006410256	490.09	19,113.45
157	24,649	3,869,893	12.5300	5.3947	0.006369427	493.23	19,359.28
158	24,964	3,944,312	12.5698	5.4061	0.006329114	496.37	19,606.68
159	25,281	4,019,679	12.6095	5.4175	0.006289308	499.51	19,855.65
160	25,600	4,096,000	12.6491	5.4288	0.006250000	502.65	20,106.19
161	25,921	4,173,281	12.6886	5.4401	0.006211180	505.80	20,358.31
162	26,244	4,251,528	12.7279	5.4514	0.006172840	508.94	20,611.99
163	26,569	4,330,747	12.7671	5.4626	0.006134969	512.08	20,867.24
164	26,896	4,410,944	12.8062	5.4737	0.006097561	515.22	21,124.07
165	27,225	4,492,125	12.8452	5.4848	0.006060606	518.36	21,382.46
166	27,556	4,574,296	12.8841	5.4959	0.006024096	521.50	21,642.43
167	27,889	4,657,463	12.9228	5.5069	0.005988024	524.65	21,903.97
168	28,224	4,741,632	12.9615	5.5178	0.005952381	527.79	22,167.08
169	28,561	4,826,809	13.0000	5.5288	0.005917160	530.93	22,431.76
170	28,900	4,913,000	13.0384	5.5397	0.005882353	534.07	22,698.01
171	29,241	5,000,211	13.0767	5.5505	0.005847953	537.21	22,965.83
172	29,584	5,088,448	13.1149	5.5613	0.005813953	540.35	23,235.22
173	29,929	5,177,717	13.1529	5.5721	0.005780347	543.50	23,506.18
174	30,276	5,268,024	13.1909	5.5828	0.005747126	546.64	23,778.71
175	30,625	5,359,375	13.2288	5.5934	0.005714286	549.78	24,052.82

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
176	30,976	5,451,776	13.2665	5.6041	0.005681818	552.92	24,328.49
177	31,329	5,545,233	13.3041	5.6147	0.005649718	556.06	24,605.74
178	31,684	5,639,752	13.3417	5.6252	0.005617978	559.20	24,884.56
179	32,041	5,735,339	13.3791	5.6357	0.005586592	562.35	25,164.94
180	32,400	5,832,000	13.4164	5.6462	0.005555556	565.49	25,446.90
181	32,761	5,929,741	13.4536	5.6567	0.005524862	568.63	25,730.43
182	33,124	6,028,568	13.4907	5.6671	0.005494505	571.77	26,015.53
183	33,489	6,128,487	13.5277	5.6774	0.005464481	574.91	26,302.20
184	33,856	6,229,504	13.5647	5.6877	0.005434783	578.05	26,590.44
185	34,225	6,331,625	13.6015	5.6980	0.005405405	581.19	26,880.25
186	34,596	6,434,856	13.6382	5.7083	0.005376344	584.34	27,171.63
187	34,969	6,539,203	13.6748	5.7185	0.005347594	587.48	27,464.59
188	35,344	6,644,672	13.7113	5.7287	0.005319149	590.62	27,759.11
189	35,721	6,751,269	13.7477	5.7388	0.005291005	593.76	28,055.21
190	36,100	6,859,000	13.7840	5.7489	0.005263158	596.90	28,352.87
191	36,481	6,967,871	13.8203	5.7590	0.005235602	600.04	28,652.11
192	36,864	7,077,888	13.8564	5.7690	0.005208333	603.19	28,952.92
193	37,249	7,189,017	13.8924	5.7790	0.005181347	606.33	29,255.30
194	37,636	7,301,384	13.9284	5.7890	0.005154639	609.47	29,559.25
195	38,025	7,414,875	13.9642	5.7989	0.005128205	612.61	29,864.77
196	38,416	7,529,536	14.0000	5.8088	0.005102041	615.75	30,171.86
197	38,809	7,645,373	14.0357	5.8186	0.005076142	618.89	30,480.52
198	39,204	7,762,392	14.0712	5.8285	0.005050505	622.04	30,790.75
199	39,601	7,880,599	14.1067	5.8383	0.005025126	625.18	31,102.55
200	40,000	8,000,000	14.1421	5.8480	0.005000000	628.32	31,415.93
201	40,401	8,120,601	14.1774	5.8578	0.004975124	631.46	31,730.87
202	40,804	8,242,408	14.2127	5.8675	0.004950495	634.60	32,047.39
203	41,209	8,365,427	14.2478	5.8771	0.004926108	637.74	32,365.47
204	41,616	8,489,664	14.2829	5.8868	0.004901961	640.88	32,685.13
205	42,025	8,615,125	14.3178	5.8964	0.004878049	644.03	33,006.36
206	42,436	8,741,816	14.3527	5.9059	0.004854369	647.17	33,329.16
207	42,849	8,869,743	14.3875	5.9155	0.004830918	650.31	33,653.53
208	43,264	8,998,912	14.4222	5.9250	0.004807692	653.45	33,979.47
209	43,681	9,129,329	14.4568	5.9345	0.004784689	656.59	34,306.98
210	44,100	9,261,000	14.4914	5.9439	0.004761905	659.73	34,636.06
211	44,521	9,393,931	14.5258	5.9533	0.004739336	662.88	34,966.71
212	44,944	9,528,128	14.5602	5.9627	0.004716981	666.02	35,298.94
213	45,369	9,663,597	14.5945	5.9721	0.004694836	669.16	35,632.73
214	45,796	9,800,344	14.6287	5.9814	0.004672897	672.30	35,968.09
215	46,225	9,938,375	14.6629	5.9907	0.004651163	675.44	36,305.03
216	46,656	10,077,696	14.6969	6.0000	0.004629630	678.58	36,643.54
217	47,089	10,218,313	14.7309	6.0092	0.004608295	681.73	36,983.61
218	47,524	10,360,232	14.7648	6.0185	0.004587156	684.87	37,325.26
219	47,961	10,503,459	14.7986	6.0277	0.004566210	688.01	37,668.48
220	48,400	10,648,000	14.8324	6.0368	0.004545455	691.15	38,013.27
221	48,841	10,793,861	14.8661	6.0459	0.004524887	694.29	38,359.63
222	49,284	10,941,048	14.8997	6.0550	0.004504505	697.43	38,707.56
223	49,729	11,089,567	14.9332	6.0641	0.004484305	700.58	39,057.07
224	50,176	11,239,424	14.9666	6.0732	0.004464286	703.72	39,408.14
225	50,625	11,390,625	15.0000	6.0822	0.004444444	706.86	39,760.78
226	51,076	11,543,176	15.0333	6.0912	0.004424779	710.00	40,115.00
227	51,529	11,697,083	15.0665	6.1002	0.004405286	713.14	40,470.78
228	51,984	11,852,352	15.0997	6.1091	0.004385965	716.28	40,828.14
229	52,441	12,008,989	15.1327	6.1180	0.004366812	719.42	41,187.07
230	52,900	12,167,000	15.1658	6.1269	0.004347826	722.57	41,547.56
231	53,361	12,326,391	15.1987	6.1358	0.004329004	725.71	41,909.63
232	53,824	12,487,168	15.2315	6.1446	0.004310345	728.85	42,273.27
233	54,289	12,649,337	15.2643	6.1534	0.004291845	731.99	42,638.48
234	54,756	12,812,904	15.2971	6.1622	0.004273504	735.13	43,005.26
235	55,225	12,977,875	15.3297	6.1710	0.004255319	738.27	43,373.61

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
236	55,696	13,144,256	15.3623	6.1797	0.004237288	741.42	43,743.54
237	56,169	13,312,053	15.3948	6.1885	0.004219409	744.56	44,115.03
238	56,644	13,481,272	15.4272	6.1672	0.004201681	747.70	44,488.09
239	57,121	13,651,919	15.4596	6.2058	0.004184100	750.84	44,862.73
240	57,600	13,824,000	15.4919	6.2145	0.004166667	753.98	45,238.93
241	58,081	13,997,521	15.5242	6.2231	0.004149378	757.12	45,616.71
242	58,564	14,172,488	15.5563	6.2317	0.004132231	760.27	45,996.06
243	59,049	14,348,907	15.5885	6.2403	0.004115226	763.41	46,376.98
244	59,536	14,526,784	15.6205	6.2488	0.004098361	766.55	46,759.47
245	60,025	14,706,125	15.6525	6.2573	0.004081633	769.69	47,143.52
246	60,516	14,886,936	15.6844	6.2658	0.004065041	772.83	47,529.16
247	61,009	15,069,223	15.7162	6.2743	0.004048583	775.97	47,916.36
248	61,504	15,252,992	15.7480	6.2828	0.004032258	779.12	48,305.13
249	62,001	15,438,249	15.7797	6.2912	0.004016064	782.26	48,695.47
250	62,500	15,625,000	15.8114	6.2996	0.004000000	785.40	49,087.39
251	63,001	15,813,251	15.8430	6.3080	0.003984064	788.54	49,480.87
252	63,504	16,003,008	15.8745	6.3164	0.003968254	791.68	49,875.92
253	64,009	16,194,277	15.9060	6.3247	0.003952569	794.82	50,272.55
254	64,516	16,387,064	15.9374	6.3330	0.003937008	797.96	50,670.75
255	65,025	16,581,375	15.9687	6.3413	0.003921569	801.11	51,070.52
256	65,536	16,777,216	16.0000	6.3496	0.003906250	804.25	51,471.85
257	66,049	16,974,593	16.0312	6.3579	0.003891051	807.39	51,874.76
258	66,564	17,173,512	16.0624	6.3661	0.003875969	810.53	52,279.24
259	67,081	17,373,979	16.0935	6.3743	0.003861004	813.67	52,685.29
260	67,600	17,576,000	16.1245	6.3825	0.003846154	816.81	53,092.92
261	68,121	17,779,581	16.1555	6.3907	0.003831418	819.96	53,502.11
262	68,644	17,984,728	16.1864	6.3988	0.003816794	823.10	53,912.87
263	69,169	18,191,447	16.2173	6.4070	0.003802281	826.24	54,325.21
264	69,696	18,399,744	16.2481	6.4151	0.003787879	829.38	54,739.11
265	70,225	18,609,625	16.2788	6.4232	0.003773585	832.52	55,154.59
266	70,756	18,821,096	16.3095	6.4312	0.003759398	835.66	55,571.63
267	71,289	19,034,163	16.3401	6.4393	0.003745318	838.81	55,990.25
268	71,824	19,248,832	16.3707	6.4473	0.003731343	841.95	56,410.44
269	72,361	19,465,109	16.4012	6.4553	0.003717472	845.09	56,832.20
270	72,900	19,683,000	16.4317	6.4633	0.003703704	848.23	57,255.53
271	73,441	19,902,511	16.4621	6.4713	0.003690037	851.37	57,680.43
272	73,984	20,123,648	16.4924	6.4792	0.003676471	854.51	58,106.90
273	74,529	20,346,417	16.5227	6.4872	0.003663004	857.65	58,534.94
274	75,076	20,570,824	16.5529	6.4951	0.003649635	860.80	58,964.55
275	75,625	20,796,875	16.5831	6.5030	0.003636364	863.94	59,395.74
276	76,176	21,024,576	16.6132	6.5108	0.003623188	867.08	59,828.49
277	76,729	21,253,933	16.6433	6.5187	0.003610108	870.22	60,262.82
278	77,284	21,484,952	16.6733	6.5265	0.003597122	873.36	60,698.71
279	77,841	21,717,639	16.7033	6.5343	0.003584229	876.50	61,136.18
280	78,400	21,952,000	16.7332	6.5421	0.003571429	879.65	61,575.22
281	78,961	22,188,041	16.7631	6.5499	0.003558719	882.79	62,015.82
282	79,524	22,425,768	16.7929	6.5577	0.003546099	885.93	62,458.00
283	80,089	22,665,187	16.8226	6.5654	0.003533569	889.07	62,901.75
284	80,656	22,906,304	16.8523	6.5731	0.003522127	892.21	63,347.07
285	81,225	23,149,125	16.8819	6.5808	0.003508772	895.35	63,793.97
286	81,796	23,393,656	16.9115	6.5885	0.003496503	898.50	64,242.43
287	82,369	23,639,903	16.9411	6.5962	0.003484321	901.64	64,692.46
288	82,944	23,887,872	16.9706	6.6039	0.003472222	904.78	65,144.07
289	83,521	24,137,569	17.0000	6.6115	0.003460208	907.92	65,597.24
290	84,100	24,389,000	17.0294	6.6191	0.003448276	911.06	66,051.99
291	84,681	24,642,171	17.0587	6.6267	0.003436426	914.20	66,508.30
292	85,264	24,897,088	17.0880	6.6343	0.003424658	917.35	66,966.19
293	85,849	25,153,757	17.1172	6.6419	0.003412969	920.49	67,425.65
294	86,436	25,412,184	17.1464	6.6494	0.003401361	923.63	67,886.68
295	87,025	25,672,375	17.1756	6.6569	0.003389831	926.77	68,349.28

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
296	87,616	25,934,336	17.2047	6.6644	0.003378378	929.91	68,813.45
297	88,209	26,198,073	17.2337	6.6719	0.003367003	933.05	69,279.19
298	88,804	26,463,592	17.2627	6.6794	0.003355705	936.19	69,746.50
299	89,401	26,730,899	17.2916	6.6869	0.003344482	939.34	70,215.38
300	90,000	27,000,000	17.3205	6.6943	0.003333333	942.48	70,685.83
301	90,601	27,270,901	17.3494	6.7018	0.003322259	945.62	71,157.86
302	91,204	27,543,608	17.3781	6.7092	0.003311258	948.76	71,631.45
303	91,809	27,818,127	17.4069	6.7166	0.003301330	951.90	72,106.62
304	92,416	28,094,464	17.4356	6.7240	0.003289474	955 04	72,583.36
305	93,025	28,372,625	17.4642	6.7313	0.003278689	958.19	73,061.66
306	93,636	28,652,616	17.4929	6.7387	0.003267974	961.33	73,541.54
307	94,249	28,934,443	17.5214	6 7460	0 003257329	964.47	74,022.99
308	94,864	29,218,112	17.5499	6.7533	0.003246753	967.61	74,506.01
309	95,481	29,503,629	17.5784	6.7606	0.003236246	970.75	74,990.60
310	96,100	29,791,000	17.6068	6.7679	0.003225806	973.89	75,476.76
311	96,721	30,080,231	17.6352	6.7752	0.003215434	977.04	75,964.50
312	97,344	30,371,328	17.6635	6.7824	0.003205128	980.18	76,453.80
313	97,969	30,664,297	17.6918	6.7897	0.003194888	983.32	76,944.67
314	98,596	30,959,144	17.7200	6.7969	0.003184713	986.46	77,437.12
315	99,225	31,255,875	17.7482	6.8041	0.003174603	989.60	77,931.13
316	99,856	31,554,496	17.7764	6.8113	0.003164557	992.74	78,426.72
317	100,489	31,855,013	17.8045	6.8185	0 003154574	995 88	78,923 88
318	101,124	32,157,432	17.8326	6.8256	0.003144654	999.03	79,422.60
319	101,761	32,461,759	17.8606	6.8328	0.003134796	1,002 17	79,922.90
320	102,400	32,768,000	17.8885	6.8399	0.003125000	1,005.31	80,424.77
321	103,041	33,076,161	17.9165	6.8470	0.003115265	1,008.45	80,928.21
322	103,684	33,386,248	17.9444	6.8541	0.003105590	1,011.59	81,433.22
323	104,329	33,698.267	17 9722	6.8612	0.003095975	1,014.73	81,939.80
324	104,976	34,012,224	18.0000	6.8683	0.003086420	1,017.88	82,447.96
325	105,625	34,328,125	18 0278	6.8753	0.003076923	1,021.02	82,957.68
326	106,276	34,645,976	18 0555	6.8824	0.003067485	1,024 16	83,468 98
327	106,929	34,965,783	18.0831	6.8894	0 003058104	1,027.30	83,981.84
328	107,584	35,287,552	18 1108	6.8964	0.003048780	1,030.44	84,496.28
329	108,241	35,611,289	18.1384	6.9034	0.003039514	1,033.58	85,012.28
330	108,900	35,937,000	18.1659	6.9104	0.003030303	1,036.73	85,529.86
331	109,561	36,264,691	18 1934	6.9174	0.003021148	1,039.87	86,049.01
332	110,224	36,594,368	18.2209	6.9244	0.003012048	1,043.01	86,569.73
333	110,889	36,926,037	18.2483	6.9313	0.003003003	1,046.15	87,092.02
334	111,556	37,259,704	18.2757	6.9382	0.002994012	1,049.29	87,615.88
335	112,225	37,595,375	18.3030	6.9451	0.002985075	1,052.43	88,141.31
336	112,896	37,933,056	18 3303	6.9521	0.002976190	1,055.58	88,668.31
337	113,569	38,272,753	18.3576	6.9589	0 002967359	1,058.72	89,196.88
338	114,244	38,614,472	18.3848	6.9658	0.002958580	1,061.86	89,727.03
339	114,921	38,958,219	18.4120	6.9727	0.002949853	1,065.00	90,258.74
340	115,600	39,304.000	18.4391	6.9795	0.002941176	1,068.14	90,792.03
341	116,281	39,651,821	18.4662	6.9864	0.002932551	1,071.28	91,326.88
342	116,964	40,001,688	18.4932	6.9932	0.002923977	1,074.42	91,863.31
343	117,649	40,353,607	18.5203	7.0000	0.002915452	1,077.57	92,401.31
344	118,336	40,707,584	18.5472	7.0068	0.002906977	1,080.71	92,940.88
345	119,025	41,063,625	18.5742	7.0136	0.002898551	1,083.85	93,482.02
346	119,716	41,421,736	18.6011	7.0203	0.002890173	1,086.99	94,024.73
347	120,409	41,781,923	18.6279	7.0271	0.002881844	1,090.13	94,569.01
348	121,104	42,144,192	18.6548	7.0338	0.002873563	1,093.27	95,114.86
349	121,801	42,508,549	18.6815	7.0406	0.002865330	1,096.42	95,662.28
350	122,500	42,875,000	18.7083	7.0473	0.002857143	1,099.56	96,211.28
351	123,201	43,243,551	18.7350	7.0540	0.002849003	1,102.70	96,761.84
352	123,904	43,614,208	18.7617	7.0607	0.002840909	1,105.84	97,313.97
353	124,609	43,986,977	18.7883	7.0674	0.002832861	1,108.98	97,867.68
354	125,316	44,361,864	18.8149	7.0740	0.002824859	1,112.12	98.422.96
355	126,025	44,738,875	18.8414	7.0807	0.002816901	1,115.26	98.979.80

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
356	126,736	45,118,016	18.8680	7.0873	0.002808989	1,118.41	99,538.22
357	127,449	45,499,293	18.8944	7.0940	0.002801120	1,121.55	100,098.21
358	128,164	45,882,712	18.9209	7.1006	0.002793296	1,124.69	100,659.77
359	128,881	46,268,279	18.9473	7.1072	0.002785515	1,127.83	101,222.90
360	129,600	46,656,000	18.9737	7.1138	0.002777778	1,130.97	101,787.60
361	130,321	47,045,881	19.0000	7.1204	0.002770083	1,134.12	102,353.87
362	131,044	47,437,928	19.0263	7.1269	0.002762431	1,137.26	102,921.72
363	131,769	47,832,147	19.0526	7.1335	0.002754821	1,140.40	103,491.13
364	132,496	48,228,544	19.0788	7.1400	0.002747253	1,143.54	104,062.12
365	133,225	48,627,125	19.1050	7.1466	0.002739726	1,146.68	104,634.67
366	133,956	49,027,896	19.1311	7.1531	0.002732240	1,149.82	105,208.80
367	134,689	49,430,863	19.1572	7.1596	0.002724796	1,152.96	105,784.49
368	135,424	49,836,032	19.1833	7.1661	0.002717391	1,156.11	106,361.76
369	136,161	50,243,409	19.2094	7.1726	0.002710027	1,159.25	106,940.60
370	136,900	50,653,000	19.2354	7.1791	0.002702703	1,162.39	107,521.01
371	137,641	51,064,811	19.2614	7.1855	0.002695418	1,165.53	108,102.99
372	138,384	51,478,848	19.2873	7.1920	0.002688172	1,168.67	108,686.54
373	139,129	51,895,117	19.3132	7.1984	0.002680965	1,171.81	109,271.66
374	139,876	52,313,624	19.3391	7.2048	0.002673797	1,174.96	109,858.35
375	140,625	52,734,375	19.3649	7.2112	0.002666667	1,178.10	110,446.62
376	141,376	53,157,376	19.3907	7.2177	0.002659574	1,181.24	111,036.45
377	142,129	53,582,633	19.4165	7.2240	0.002652520	1,184.38	111,627.86
378	142,884	54,010,152	19.4422	7.2304	0.002645503	1,187.52	112,220.83
379	143,641	54,439,939	19.4679	7.2368	0.002638522	1,190.66	112,815.38
380	144,400	54,872,000	19.4936	7.2432	0.002631579	1,193.80	113,411.49
381	145,161	55,306,341	19.5192	7.2495	0.002624672	1,196.95	114,009.18
382	145,924	55,742,968	19.5448	7.2558	0.002617801	1,200.09	114,608.44
383	146,689	56,181,887	19.5704	7.2622	0.002610966	1,203.23	115,209.27
384	147,456	56,623,104	19.5959	7.2685	0.002604167	1,206.37	115,811.67
385	148,225	57,066,625	19.6214	7.2748	0.002597403	1,209.51	116,415.64
386	148,996	57,512,456	19.6469	7.2811	0.002590674	1,212.66	117,021.18
387	149,769	57,960,603	19.6723	7.2874	0.002583979	1,215.80	117,628.30
388	150,544	58,411,072	19.6977	7.2936	0.002577320	1,218.94	118,236.98
389	151,321	58,863,869	19.7231	7.2999	0.002570694	1,222.08	118,847.24
390	152,100	59,319,000	19.7484	7.3061	0.002564103	1,225.22	119,459.06
391	152,881	59,776,471	19.7737	7.3124	0.002557545	1,228.36	120,072.46
392	153,664	60,236,288	19.7990	7.3186	0.002551020	1,231.50	120,687.42
393	154,449	60,698,457	19.8242	7.3248	0.002544529	1,234.65	121,303.96
394	155,236	61,162,984	19.8494	7.3310	0.002538071	1,237.79	121,922.07
395	156,025	61,629,875	19.8746	7.3372	0.002531646	1,240.93	122,541.75
396	156,816	62,099,136	19.8997	7.3434	0.002525253	1,244.07	123,163.00
397	157,609	62,570,773	19.9249	7.3496	0.002518892	1,247.21	123,785.82
398	158,404	63,044,792	19.9499	7.3558	0.002512563	1,250.35	124,410.21
399	159,201	63,521,199	19.9750	7.3619	0.002506266	1,253.50	125,036.17
400	160,000	64,000,000	20.0000	7.3681	0.002500000	1,256.64	125,663.71
401	160,801	64,481,201	20.0250	7.3742	0.002493766	1,259.78	126,292.81
402	161,604	64,964,808	20.0499	7.3803	0.002487562	1,262.92	126,923.48
403	162,409	65,450,827	20.0749	7.3864	0.002481390	1,266.06	127,555.73
404	163,216	65,939,264	20.0998	7.3925	0.002475248	1,269.20	128,189.55
405	164,025	66,430,125	20.1246	7.3986	0.002469136	1,272.35	128,824.93
406	164,836	66,923,416	20.1494	7.4047	0.002463054	1,275.49	129,461.89
407	165,649	67,419,143	20.1742	7.4108	0.002457002	1,278.63	130,100.42
408	166,464	67,917,312	20.1990	7.4169	0.002450980	1,281.77	130,740.52
409	167,281	68,417,929	20.2237	7.4229	0.002444988	1,284.91	131,382.19
410	168,100	68,921,000	20.2485	7.4290	0.002439024	1,288.05	132,025.43
411	168,921	69,426,531	20.2731	7.4350	0.002433090	1,291.20	132,670.24
412	169,744	69,934,528	20.2978	7.4410	0.002427184	1,294.34	133,316.63
413	170,569	70,444,997	20.3224	7.4470	0.002421308	1,297.48	133,964.58
414	171,396	70,957,944	20.3470	7.4530	0.002415459	1,300.62	134,614.10
415	172,225	71,473,375	20.3715	7.4590	0.002409639	1,303.76	135,265.20

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
416	173,056	71,991,296	20.3961	7.4650	0.002403846	1,306.90	135,917.86
417	173,889	72,511,713	20.4206	7.4710	0.002398082	1,310.04	136,572.10
418	174,724	73,034,632	20.4450	7.4770	0.002392344	1,313.19	137,227.91
419	175,561	73,560,059	20.4695	7.4829	0.002386635	1,316.33	137,885.29
420	176,400	74,088,000	20.4939	7.4889	0.002380952	1,319.47	138,544.24
421	177,241	74,618,461	20.5183	7.4948	0.002375297	1,322.61	139,204.76
422	178,084	75,151,448	20.5426	7.5007	0.002369668	1,325.75	139,866.85
423	178,929	75,686,967	20.5670	7.5067	0.002364066	1,328.89	140,530.51
424	179,776	76,225,024	20.5913	7.5126	0.002358491	1,332.04	141,195.74
425	180,625	76,765,625	20.6155	7.5185	0.002352941	1,335.18	141,862.54
426	181,476	77,308,776	20.6398	7.5244	0.002347418	1,338.32	142,530.92
427	182,329	77,854,483	20.6640	7.5302	0.002341920	1,341.46	143,200.86
428	183,184	78,402,752	20.6882	7.5361	0.002336449	1,344.60	143,872.38
429	184,041	78,953,589	20.7123	7.5420	0.002331002	1,347.74	144,545.46
430	184,900	79,507,000	20.7364	7.5478	0.002325581	1,350.88	145,220.12
431	185,761	80,062,991	20.7605	7.5537	0.002320186	1,354.03	145,896.35
432	186,624	80,621,568	20.7846	7.5595	0.002314815	1,357.17	146,574.15
433	187,489	81,182,737	20.8087	7.5654	0.002309469	1,360.31	147,253.52
434	188,356	81,746,504	20.8327	7.5712	0.002304147	1,363.45	147,934.46
435	189,225	82,312,875	20.8567	7.5770	0.002298851	1,366.59	148,616.97
436	190,096	82,881,856	20.8806	7.5828	0.002293578	1,369.73	149,301.05
437	190,969	83,453,453	20.9045	7.5886	0.002288330	1,372.88	149,986.70
438	191,844	84,027,672	20.9284	7.5944	0.002283105	1,376.02	150,673.93
439	192,721	84,604,519	20.9523	7.6001	0.002277904	1,379.16	151,362.72
440	193,600	85,184,000	20.9762	7.6059	0.002272727	1,382.30	152,053.08
441	194,481	85,766,121	21.0000	7.6117	0.002267574	1,385.44	152,745.02
442	195,364	86,350,888	21.0238	7.6174	0.002262443	1,388.58	153,438.53
443	196,249	86,938,307	21.0476	7.6232	0.002257336	1,391.73	154,133.60
444	197,136	87,528,384	21.0713	7.6289	0.002252252	1,394.87	154,830.25
445	198,025	88,121,125	21.0950	7.6346	0.002247191	1,398.01	155,528.47
446	198,916	88,716,536	21.1187	7.6403	0.002242152	1,401.15	156,228.26
447	199,809	89,314,623	21.1424	7.6460	0.002237136	1,404.29	156,929.62
448	200,704	89,915,392	21.1660	7.6517	0.002232143	1,407.43	157,632.55
449	201,601	90,518,849	21.1896	7.6574	0.002227171	1,410.58	158,337.06
450	202,500	91,125,000	21.2132	7.6631	0.002222222	1,413.72	159,043.13
451	203,401	91,733,851	21.2368	7.6688	0.002217295	1,416.86	159,750.77
452	204,304	92,345,408	21.2603	7.6744	0.002212389	1,420.00	160,459.99
453	205,209	92,959,677	21.2838	7.6801	0.002207506	1,423.14	161,170.77
454	206,116	93,576,664	21.3073	7.6857	0.002202643	1,426.28	161,883.13
455	207,025	94,196,375	21.3307	7.6914	0.002197802	1,429.42	162,597.05
456	207,936	94,818,816	21.3542	7.6970	0.002192982	1,432.57	163,312.55
457	208,849	95,443,993	21.3776	7.7026	0.002188184	1,435.71	164,029.62
458	209,764	96,071,912	21.4009	7.7082	0.002183406	1,438.85	164,748.26
459	210,681	96,702,579	21.4243	7.7138	0.002178649	1,441.99	165,468.47
460	211,600	97,336,000	21.4476	7.7194	0.002173913	1,445.13	166,190.25
461	212,521	97,972,181	21.4709	7.7250	0.002169197	1,448.27	166,913.60
462	213,444	98,611,128	21.4942	7.7306	0.002164502	1,451.42	167,638.53
463	214,369	99,252,847	21.5174	7.7362	0.002159827	1,454.56	168,365.02
464	215,296	99,897,344	21.5407	7.7418	0.002155172	1,457.70	169,093.08
465	216,225	100,544,625	21.5639	7.7473	0.002150538	1,460.84	169,822.72
466	217,156	101,194,696	21.5870	7.7529	0.002145923	1,463.98	170,553.92
467	218,089	101,847,563	21.6102	7.7584	0.002141328	1,467.12	171,286.70
468	219,024	102,503,232	21.6333	7.7639	0.002136752	1,470.26	172,021.05
469	219,961	103,161,709	21.6564	7.7695	0.002132196	1,473.41	172,756.97
470	220,900	103,823,000	21.6795	7.7750	0.002127660	1,476.55	173,494.45
471	221,841	104,487,111	21.7025	7.7805	0.002123142	1,479.69	174,233.51
472	222,784	105,154,048	21.7256	7.7860	0.002118644	1,482.83	174,974.14
473	223,729	105,823,817	21.7486	7.7915	0.002114165	1,485.97	175,716.35
474	224,676	106,496,424	21.7715	7.7970	0.002109705	1,489.12	176,460.12
475	225,625	107,171,875	21.7945	7.8025	0.002105263	1,492.26	177,205.46

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
476	226,576	107,850,176	21.8174	7.8079	0.002100840	1,495.40	177,952.37
477	227,529	108,531,333	21.8403	7.8134	0.002096436	1,498.54	178,700.86
478	228,484	109,215,352	21.8632	7.8188	0.002092050	1,501.68	179,450.91
479	229,441	109,902,239	21.8861	7.8243	0.002087683	1,504.82	180,202.54
480	230,400	110,592,000	21.9089	7.8297	0.002083333	1,507.96	180,955.74
481	231,361	111,284,641	21.9317	7.8352	0.002079002	1,511.11	181,710.50
482	232,324	111,980,168	21.9545	7.8406	0.002074689	1,514.25	182,466.84
483	233,289	112,678,587	21.9773	7.8460	0.002070393	1,517.39	183,224.75
484	234,256	113,379,904	22.0000	7.8514	0.002066116	1,520.53	183,984.23
485	235,225	114,084,125	22.0227	7.8568	0.002061856	1,523.67	184,745.28
486	236,196	114,791,256	22.0454	7.8622	0.002057613	1,526.81	185,507.90
487	237,169	115,501,303	22.0681	7.8676	0.002053388	1,529.96	186,272.10
488	238,144	116,214,272	22.0097	7.8730	0.002049180	1,533.10	187,037.86
489	239,121	116,930,169	22.1133	7.8784	0.002044990	1,536.24	187,805.19
490	240,100	117,649,000	22.1359	7.8837	0.002040816	1,539.38	188,574.10
491	241,081	118,370,771	22.1585	7.8891	0.002036660	1,542.52	189,344.57
492	242,064	119,095,488	22.1811	7.8944	0.002032520	1,545.66	190,116 62
493	243,049	119,823,157	22.2036	7.8998	0.002028398	1,548.81	190,890.24
494	244.036	120,553,784	22.2261	7.9051	0.002024291	1,551.95	191,665.43
495	245.025	121,287,375	22.2486	7.9105	0.002020202	1,555.09	192,442.18
496	246,016	122,023,936	22.2711	7.9158	0.002016129	1,558.23	193,220.51
497	247,009	122,763,473	22.2935	7.9211	0.002012072	1,561.37	194,000.41
498	248.004	123,505,992	22.3159	7.9264	0.002008032	1,564.51	194,781.89
499	249,001	124,251,499	22.3383	7.9317	0.002004008	1,567.65	195,564.93
500	250,000	125,000,000	22.3607	7.9370	0.002000000	1,570.80	196,349.54
501	251,001	125,751,501	22.3830	7.9423	0.001996008	1,573.94	197,135.72
502	252,004	126,506,008	22.4054	7.9476	0.001992032	1,577.08	197,923.48
503	253,009	127,263,527	22.4277	7.9528	0.001988072	1,580.22	198,712.80
504	254,016	128.024,064	22.4499	7.9581	0.001984127	1,583.36	199,503.70
505	255,025	128,787,625	22.4722	7.9634	0.001980198	1,586.50	200,296.17
506	256,036	129,554,216	22.4944	7.9686	0.001976285	1,589.65	201,090.20
507	257,049	130,323,843	22.5167	7.9739	0.001972387	1,592.79	201,885.81
508	258,064	131,096,512	22.5389	7.9791	0.001968504	1.595.93	202,682.99
509	259,081	131,872,229	22.5610	7.9843	0.001964637	1.599.07	203,481.74
510	260,100	132,651,000	22.5832	7.9896	0.001960785	1,602.21	204,282.06
511	261,121	133,432,831	22.6053	7.9948	0.001956947	1,605.35	205,083.95
512	262,144	134,217,728	22.6274	8.0000	0.001953125	1,608.50	205,887.42
513	263,169	135,005,697	22.6495	8.0052	0.001949318	1,611.64	206,692.45
514	264,196	135,796,744	22.6716	8.0104	0.001945525	1,614.78	207,499.05
515	265,225	136,590,875	22.6936	8.0156	0.001941748	1,617.92	208,307.23
516	266,256	137,388,096	22.7156	8.0208	0.001937984	1,621.06	209,116.97
517	267,289	138,188,413	22.7376	8.0260	0.001934236	1,624.20	209,928.29
518	268,324	138,991,832	22.7596	8.0311	0.001930502	1,627.34	210,741.18
519	269,361	139,798,359	22.7816	8.0363	0.001926782	1,630.49	211,555.63
520	270,400	140,608,000	22.8035	8.0415	0.001923077	1,633.63	212,371.66
521	271,441	141,420,761	22.8254	8.0466	0.001919386	1,636.77	213,189.26
522	272,484	142,236,648	22.8473	8.0517	0.001915709	1,639.91	214,008.43
523	273,529	143,055,667	22.8692	8.0569	0.001912046	1,643.05	214,829.17
524	274,576	143,877,824	22.8910	8.0620	0.001908397	1,646.19	215,651.49
525	275,625	144,703,125	22.9129	8.0671	0.001904762	1,649.34	216,475.37
526	276,676	145,531,576	22.9347	8.0723	0.001901141	1,652.48	217,300.82
527	277,729	146,363,183	22.9565	8.0774	0.001897533	1,655.62	218,127.85
528	278,784	147,197,952	22.9783	8.0825	0.001893939	1,658.76	218,956.44
529	279,841	148,035,889	23.0000	8.0876	0.001890359	1,661.90	219,786.61
530	280,900	148,877,000	23.0217	8.0927	0.001886792	1,665.04	220,618.34
531	281,961	149,721,291	23.0434	8.0978	0.001883239	1,668.19	221,451.65
532	283,024	150,568,768	23.0651	8.1028	0.001879699	1,671.33	222,286.53
533	284,089	151,419,437	23.0868	8.1079	0.001876173	1,674.47	223,122.98
534	285,156	152,273,304	23.1084	8.1130	0.001872659	1,677.61	223,961.00
535	286,225	153,130,375	23.1301	8.1180	0.001869159	1,680.75	224,800.59

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
536	287,296	153,990,656	23.1517	8.1231	0.001865672	1,683.89	225,641.75
537	288,369	154,854,153	23.1733	8.1281	0.001862197	1,687.03	226,484.48
538	289,444	155,720,872	23.1948	8.1332	0.001858736	1,690.18	227,328.79
539	290,521	156,590,819	23.2164	8.1382	0.001855288	1,693.32	228,174.66
540	291,600	157,464,000	23.2379	8.1433	0.001851852	1,696.46	229,022.10
541	292,681	158,340,421	23.2594	8.1483	0.001848429	1,699.60	229,871.12
542	293,764	159,220,088	23.2809	8.1533	0.001845018	1,702.74	230,721.71
543	294,849	160,103,007	23.3024	8.1583	0.001841621	1,705.88	231,573.86
544	295,936	160,989,184	23.3238	8.1633	0.001838235	1,709.03	232,427.59
545	297,025	161,878,625	23.3452	8.1683	0.001834862	1,712.17	233,282.89
546	298,116	162,771,336	23.3666	8.1733	0.001831502	1,715.31	234,139.73
547	299,209	163,667,323	23.3880	8.1783	0.001828154	1,718.45	234,998.20
548	300,304	164,566,592	23.4094	8.1833	0.001824818	1,721.59	235,858.21
549	301,401	165,469,149	23.4307	8.1882	0.001821494	1,724.73	236,719.79
550	302,500	166,375,000	23.4521	8.1932	0.001818182	1,727.88	237,582.94
551	303,601	167,284,151	23.4734	8.1982	0.001814882	1,731.02	238,447.67
552	304,704	168,196,608	23.4947	8.2031	0.001811594	1,734.16	239,313.96
553	305,809	169,112,377	23.5160	8.2081	0.001808318	1,737.30	240,181.83
554	306,916	170,031,464	23.5372	8.2130	0.001805054	1,740.44	241,051.26
555	308,025	170,953,875	23.5584	8.2180	0.001801802	1,743.58	241,922.27
556	309,136	171,879,616	23.5797	8.2229	0.001798561	1,746.72	242,794.85
557	310,249	172,808,693	23.6008	8.2278	0.001795332	1,749.87	243,668.99
558	311,364	173,741,112	23.6220	8.2327	0.001792115	1,753.01	244,544.71
559	312,481	174,676,879	23.6432	8.2377	0.001788909	1,756.15	245,422.00
560	313,600	175,616,000	23.6643	8.2426	0.001785714	1,759.29	246,300.86
561	314,721	176,558,481	23.6854	8.2475	0.001782531	1,762.43	247,181.30
562	315,844	177,504,328	23.7065	8.2524	0.001779359	1,765.57	248,063.30
563	316,969	178,453,547	23.7276	8.2573	0.001776199	1,768.72	248,946.87
564	318,096	179,406,144	23.7487	8.2621	0.001773050	1,771.86	249,832.01
565	319,225	180,362,125	23.7697	8.2670	0.001769912	1,775.00	250,718.73
566	320,356	181,321,496	23.7908	8.2719	0.001766784	1,778.14	251,607.01
567	321,489	182,284,263	23.8118	8.2768	0.001763668	1,781.28	252,496.87
568	322,624	183,250,432	23.8328	8.2816	0.001760563	1,784.42	253,388.30
569	323,761	184,220,009	23.8537	8.2865	0.001757469	1,787.57	254,281.29
570	324,900	185,193,000	23.8747	8.2913	0.001754386	1,790.71	255,175.86
571	326,041	186,169,411	23.8956	8.2962	0.001751313	1,793.85	256,072.00
572	327,184	187,149,248	23.9165	8.3010	0.001748252	1,796.99	256,969.71
573	328,329	188,132,517	23.9374	8.3059	0.001745201	1,800.13	257,868.99
574	329,476	189,119,224	23.9583	8.3107	0.001742160	1,803.27	258,769.85
575	330,625	190,109,375	23.9792	8.3155	0.001739130	1,806.41	259,672.27
576	331,776	191,102,976	24.0000	8.3203	0.001736111	1,809.56	260,576.26
577	332,929	192,100,033	24.0208	8.3251	0.001733102	1,812.70	261,481.83
578	334,084	193,100,552	24.0416	8.3300	0.001730104	1,815.84	262,388.96
579	335,241	194,104,539	24.0624	8.3348	0.001727116	1,818.98	263,297.67
580	336,400	195,112,000	24.0832	8.3396	0.001724138	1,822.12	264,207.94
581	337,561	196,122,941	24.1039	8.3443	0.001721170	1,825.27	265,119.79
582	338,724	197,137,368	24.1247	8.3491	0.001718213	1,828.41	266,033.21
583	339,889	198,155,287	24.1454	8.3539	0.001715266	1,831.55	266,948.20
584	341,056	199,176,704	24.1661	8.3587	0.001712329	1,834.69	267,864.76
585	342,225	200,201,625	24.1868	8.3634	0.001709402	1,837.83	268,782.89
586	343,396	201,230,056	24.2074	8.3682	0.001706485	1,840.97	269,702.59
587	344,569	202,262,003	24.2281	8.3730	0.001703578	1,844.11	270,623.86
588	345,744	203,297,472	24.2487	8.3777	0.001700680	1,847.26	271,546.70
589	346,921	204,336,469	24.2693	8.3825	0.001697793	1,850.40	272,471.12
590	348,100	205,379,000	24.2899	8.3872	0.001694915	1,853.54	273,397.10
591	349,281	206,425,071	24.3105	8.3919	0.001692047	1,856.68	274,324.66
592	350,464	207,474,688	24.3311	8.3967	0.001689189	1,859.82	275,253.78
593	351,649	208,527,857	24.3516	8.4014	0.001686341	1,862.96	276,184.48
594	352,836	209,584,584	24.3721	8.4061	0.001683502	1,866.10	277,116.75
595	354,025	210,644,875	24.3926	8.4108	0.001680672	1,869.25	278,050.58

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
596	355,216	211,708,736	24.4131	8.4155	0.001677852	1,872.39	278,985.99
597	356,409	212,776,173	24.4336	8.4202	0.001675042	1,875.53	279,922.97
598	357,604	213,847,192	24.4540	8.4249	0.001672241	1,878.67	280,861.52
599	358,801	214,921,799	24.4745	8.4296	0.001669449	1,881.81	281,801.65
600	360,000	216,000,000	24.4949	8.4343	0.001666667	1,884.95	282,743.34
601	361,201	217,081,801	24.5153	8.4390	0.001663894	1,888.10	283,686.60
602	362,404	218,167,208	24.5357	8.4437	0.001661130	1,891.24	284,631.44
603	363,609	219,256,227	24.5561	8.4484	0.001658375	1,894.38	285,577.84
604	364,816	220,348,864	24.5764	8.4530	0.001655629	1,897.52	286,525.82
605	366,025	221,445,125	24.5967	8.4577	0.001652893	1,900.66	287,475.36
606	367,236	222,545,016	24.6171	8.4623	0.001650165	1,903.80	288,426.48
607	368,449	223,648,543	24.6374	8.4670	0.001647446	1,906.95	289,379.17
608	369,664	244,755,712	24.6577	8.4716	0.001644737	1,910.09	290,333.43
609	370,881	225,866,529	24.6779	8.4763	0.001642036	1,913.23	291,289.26
610	372,100	226,981,000	24.6982	8.4809	0.001639344	1,916.37	292,246.66
611	373,321	228,099,131	24.7184	8.4856	0.001636661	1,919.51	293,205.63
612	374,544	229,220,928	24.7386	8.4902	0.001633987	1,922.65	294,166.17
613	375,769	230,346,397	24.7588	8.4948	0.001631321	1,925.80	295,128.28
614	376,996	231,475,544	24.7790	8.4994	0.001628664	1,928.94	296,091.97
615	378,225	232,608,375	24.7992	8.5040	0.001626016	1,932.08	297,057.22
616	379,456	233,744,896	24.8193	8.5086	0.001623377	1,935.22	298,024.05
617	380,689	234,885,113	24.8395	8.5132	0.001620746	1,938.36	298,992.44
618	381,924	236,029,032	24.8596	8.5178	0.001618123	1,941.50	299,962.41
619	383,161	237,176,659	24.8797	8.5224	0.001615509	1,944.64	300,933.95
620	384,400	238,328,000	24.8998	8.5270	0.001612903	1,947.79	301,907.05
621	385,641	239,483,061	24.9199	8.5316	0.001610306	1,950.93	302,881.73
622	386,884	240,641,848	24.9399	8.5362	0.001607717	1,954.07	303,857.98
623	388,129	241,804,367	24.9600	8.5408	0.001605136	1,957.21	304,835.80
624	389,376	242,970,624	24.9800	8.5453	0.001602564	1,960.35	305,815.20
625	390,625	244,140,625	25.0000	8.5499	0.001600000	1,963.49	306,796.16
626	391,876	245,314,376	25.0200	8.5544	0.001597444	1,966.64	307,778.69
627	393,129	246,491,883	25.0400	8.5590	0.001594896	1,969.78	308,762.79
628	394,384	247,673,152	25.0599	8.5635	0.001592357	1,972.92	309,748.47
629	395,641	248,858,189	25.0799	8.5681	0.001589825	1,976.06	310,735.71
630	396,900	250,047,000	25.0998	8.5726	0.001587302	1,979.20	311,724.53
631	398,161	251,239,591	25.1197	8.5772	0.001584786	1,982.34	312,714.92
632	399,424	252,435,968	25.1396	8.5817	0.001582278	1,985.49	313,706.88
633	400,689	253,636,137	25.1595	8.5862	0.001579779	1,988.63	314,700.40
634	401,956	254,840,104	25.1794	8.5907	0.001577287	1,991.77	315,695.50
635	403,225	256,047,875	25.1992	8.5952	0.001574803	1,994.91	316,692.17
636	404,496	257,259,456	25.2190	8.5997	0.001572327	1,998.05	317,690.42
637	405,769	258,474,853	25.2389	8.6043	0.001569859	2,001.19	318,690.23
638	407,044	259,694,072	25.2587	8.6088	0.001567398	2,004.33	319,691.61
639	408,321	260,917,119	25.2784	8.6132	0.001564945	2,007.48	320,694.56
640	409,600	262,144,000	25.2982	8.6177	0.001562500	2,010.62	321,699.09
641	410,881	263,374,721	25.3180	8.6222	0.001560062	2,013.76	322,705.18
642	412,164	264,609,288	25.3377	8.6267	0.001557632	2,016.90	323,712.85
643	413,449	265,847,707	25.3574	8.6312	0.001555210	2,020.04	324,722.09
644	414,736	267,089,984	25.3772	8.6357	0.001552795	2,023.18	325,732.89
645	416,025	268,336,125	25.3969	8.6401	0.001550388	2,026.33	326,745.27
646	417,316	269,586,136	25.4165	8.6446	0.001547988	2,029.47	327,759.22
647	418,609	270,840,023	25.4362	8.6490	0.001545595	2,032.61	328,774.74
648	419,904	272,097,792	25.4558	8.6535	0.001543210	2,035.75	329,791.83
649	421,201	273,359,449	25.4755	8.6579	0.001540832	2,038.89	330,810.49
650	422,500	274,625,000	25.4951	8.6624	0.001538462	2,042.03	331,830.72
651	423,801	275,894,451	25.5147	8.6668	0.001536098	2,045.18	332,852.53
652	425,104	277,167,808	25.5343	8.6713	0.001533742	2,048.32	333,875.90
653	426,409	278,445,077	25.5539	8.6757	0.001531394	2,051.46	334,900.85
654	427,716	279,726,264	25.5734	8.6801	0.001529052	2,054.60	335,927.36
655	429,025	281,011,375	25.5930	8.6845	0.001526718	2,057.74	336,955.45

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
656	430,336	282,300,416	25.6125	8.6890	0.001524390	2,060.88	337,985.10
657	431,649	283,593,393	25.6320	8.6934	0.001522070	2,064.03	339,016.33
658	432,964	284,890,312	25.6515	8.6978	0.001519757	2,067.17	340,049.13
659	434,281	286,191,179	25.6710	8.7022	0.001517451	2,070 31	341,083.50
660	435,600	287,496,000	25.6905	8.7066	0.001515152	2,073.45	342,119.44
661	436,921	288,804,781	25.7099	8.7110	0.001512859	2,076.59	343,156.95
662	438,244	290,117,528	25.7294	8.7154	0.001510574	2,079.73	344,196.03
663	439,569	291,434,247	25.7488	8.7198	0.001508296	2,082.87	345,236.69
664	440,896	292,754,944	25.7682	8.7241	0.001506024	2,086.02	346,278.91
665	442,225	294,079,625	25.7876	8.7285	0.001503759	2,089.16	347,322.70
666	443,556	295,408,296	25.8070	8.7329	0.001501502	2,092.30	348,368.07
667	444,889	296,740,963	25.8263	8.7373	0.001499250	2,095.44	349,415.00
668	446,224	298,077,632	25.8457	8.7416	0.001497006	2,098.58	350,463.51
669	447,561	299,418,309	25.8650	8.7460	0.001494768	2,101.73	351,513.59
670	448,900	300,763,000	25.8844	8.7503	0.001492537	2,104.87	352,565.24
671	450,241	302,111,711	25.9037	8.7547	0.001490313	2,108.01	353,618.45
672	451,584	303,464,448	25.9230	8.7590	0.001488095	2,111.15	354,673.24
673	452,929	304,821,217	25.9422	8.7634	0.001485884	2,114.29	355,729.60
674	454,276	306,182,024	25.9615	8.7677	0.001483680	2,117.43	356,787.54
675	455,625	307,546,875	25.9808	8.7721	0.001481481	2,120.58	357,847.04
676	456,976	308,915,776	26.0000	8.7764	0.001479290	2,123.72	358,908.11
677	458,329	310,288,733	26.0192	8.7807	0.001477105	2,126.86	359,970.75
678	459,684	311,665,752	26.0384	8.7850	0.001474926	2,130.00	361,034.97
679	461,041	313,046,839	26.0576	8.7893	0.001472754	2,133.14	362,100.75
680	462,400	314,432,000	26.0768	8.7937	0.001470588	2,136.28	363,168.11
681	463,761	315,821,241	26.0960	8.7980	0.001468429	2,139.42	364,237.04
682	465,124	317,214,568	26.1151	8.8023	0.001466276	2,142.56	365,307.54
683	466,489	318,611,987	26.1343	8.8066	0.001464129	2,145.71	366,379.60
684	467,856	320,013,504	26.1534	8.8109	0.001461988	2,148.85	367,453.24
685	469,225	321,419,125	26.1725	8.8152	0.001459854	2,151.99	368,528.45
686	470,596	322,828,856	26.1916	8.8194	0.001457726	2,155.13	369,605.23
687	471,969	324,242,703	26.2107	8.8237	0.001455604	2,158.27	370,683.59
688	473,344	325,660,672	26.2298	8.8280	0.001453488	2,161.41	371,763.51
689	474,721	327,082,769	26.2488	8.8323	0.001451379	2,164.56	372,845.00
690	476,100	328,509,000	26.2679	8.8366	0.001449275	2,167.70	373,928.07
691	477,481	329,939,371	26.2869	8.8408	0.001447178	2,170.84	375,012.70
692	478,864	331,373,888	26.3059	8.8451	0.001445087	2,173.98	376,098.91
693	480,249	332,812,557	26.3249	8.8493	0.001443001	2,177.12	377,186.68
694	481,636	334,255,384	26.3439	8.8536	0.001440922	2,180.26	378,276.03
695	483,025	335,702,375	26.3629	8.8578	0.001438849	2,183.41	379,366.95
696	484,416	337,153,536	26.3818	8.8621	0.001436782	2,186.55	380,459.44
697	485,809	338,608,873	26.4008	8.8663	0.001434720	2,189.69	381,553.50
698	487,204	340,068,392	26.4197	8.8706	0.001432665	2,192.83	382,649.13
699	488,601	341,532,099	26.4386	8.8748	0.001430615	2,195.97	383,746.33
700	490,000	343,000,000	26.4575	8.8790	0.001428571	2,199.11	384,845.10
701	491,401	344,472,101	26.4764	8.8833	0.001426534	2,202.26	385,945.44
702	492,804	345,948,408	26.4953	8.8875	0.001424501	2,205.40	387,047.36
703	494,209	347,428,927	26.5141	8.8917	0.001422475	2,208.54	388,150.84
704	495,616	348,913,664	26.5330	8.8959	0.001420455	2,211.68	389,255.90
705	497,025	350,402,625	26.5518	8.9001	0.001418440	2,214.82	390,362.52
706	498,436	351,895,816	26.5707	8.9043	0.001416431	2,217.96	391,470.72
707	499,849	353,393,243	26.5895	8.9085	0.001414427	2,221.10	392,580.49
708	501,264	354,894,912	26.6083	8.9127	0.001412429	2,224.25	393,691.82
709	502,681	356,400,829	26.6271	8.9169	0.001410437	2,227.39	394,804.73
710	504,100	357,911,000	26.6458	8.9211	0.001408451	2,230.53	395,919.21
711	505,521	359,425,431	26.6646	8.9253	0.001406470	2,233.67	397,035.26
712	506,944	360,944,128	26.6833	8.9295	0.001404494	2,236.81	398,152.89
713	508,369	362,467,097	26.7021	8.9337	0.001402525	2,239.96	399,272.08
714	509,796	363,994,344	26.7208	8.9378	0.001400560	2,243.10	400,392.84
715	511,225	365,525,875	26.7395	8.9420	0.001398601	2,246.24	401,515.18

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
716	512,656	367,061,696	26.7582	8.9462	0.001396648	2,249.38	402,639.08
717	514,089	368,601,813	26.7769	8.9503	0.001394700	2,252.52	403,764.56
718	515,524	370,146,232	26.7955	8.9545	0.001392758	2,255.66	404,891.60
719	516,961	371,694,959	26.8142	8.9587	0.001390821	2,258.80	406,020.22
720	518,400	373,248,000	26.8328	8.9628	0.001388889	2,261.95	407,150.41
721	519,841	374,805,361	26.8514	8.9670	0.001386963	2,265.09	408,282.17
722	521,284	376,367,048	26.8701	8.9711	0.001385042	2,268.23	409,415.50
723	522,729	377,933,067	26.8887	8.9752	0.001383126	2,271.37	410,550.40
724	524,176	379,503,424	26.9072	8.9794	0.001381215	2,274.51	411,686.87
725	525,625	381,078,125	26.9258	8.9835	0.001379310	2,277.65	412,824.91
726	527,076	382,657,176	26.9444	8.9876	0.001377410	2,280.79	413,964.52
727	528,529	384,240,583	26.9629	8.9918	0.001375516	2,283.94	415,105.71
728	529,984	385,828,352	26.9815	8.9959	0.001373626	2,287.08	416,248.46
729	531,441	387,420,489	27.0000	9.0000	0.001371742	2,290.22	417,392.79
730	532,900	389,017,000	27.0185	9.0041	0.001369863	2,293.36	418,538.68
731	534,361	390,617,891	27.0370	9.0082	0.001367989	2,296.50	419,686.15
732	535,824	392,223,168	27.0555	9.0123	0.001366120	2,299.64	420,835.19
733	537,289	393,832,837	27.0740	9.0164	0.001364256	2,302.79	421,985.79
734	538,756	395,446,904	27.0924	9.0205	0.001362398	2,305.93	423,137.97
735	540,225	397,065,375	27.1109	9.0246	0.001360544	2,309.07	424,291.72
736	541,696	398,688,256	27.1293	9.0287	0.001358696	2,312.21	425,447.04
737	543,169	400,315,553	27.1477	9.0328	0.001356852	2,315.35	426,603.94
738	544,644	401,947,272	27.1662	9.0369	0.001355014	2,318.49	427,762.40
739	546,121	403,583,419	27.1846	9.0410	0.001353180	2,321.64	428,922.43
740	547,600	405,224,000	27.2029	9.0450	0.001351351	2,324.78	430,084.03
741	549,081	406,869,021	27.2213	9.0491	0.001349528	2,327.92	431,247.21
742	550,564	408,518,488	27.2397	9.0532	0.001347709	2,331.06	432,411.95
743	552,049	410,172,407	27.2580	9.0572	0.001345895	2,334.20	433,578.27
744	553,536	411,830,784	27.2764	9.0613	0.001344086	2,337.34	434,746.16
745	555,025	413,493,625	27.2947	9.0654	0.001342282	2,340.49	435,915.62
746	556,516	415,160,936	27.3130	9.0694	0.001340483	2,343.63	437,086.64
747	558,009	416,832,723	27.3313	9.0735	0.001338688	2,346.77	438,259.24
748	559,504	418,508,992	27.3496	9.0775	0.001336898	2,349.91	439,433.41
749	561,001	420,189,749	27.3679	9.0816	0.001335113	2,353.05	440,609.16
750	562,500	421,875,000	27.3861	9.0856	0.001333333	2,356.19	441,786.47
751	564,001	423,564,751	27.4044	9.0896	0.001331558	2,359.33	442,965.35
752	565,504	425,259,008	27.4226	9.0937	0.001329787	2,362.48	444,145.80
753	567,009	426,957,777	27.4408	9.0977	0.001328021	2,365.62	445,327.83
754	568,516	428,661,064	27.4591	9.1017	0.001326260	2,368.76	446,511.42
755	570,025	430,368,875	27.4773	9.1057	0.001324503	2,371.90	447,696.59
756	571,536	432,081,216	27.4955	9.1098	0.001322751	2,375.04	448,883.32
757	573,049	433,798,093	27.5136	9.1138	0.001321004	2,378.18	450,071.63
758	574,564	435,519,512	27.5318	9.1178	0.001319261	2,381.33	451,261.51
759	576,081	437,245,479	27.5500	9.1218	0.001317523	2,384.47	452,452.96
760	577,600	438,976,000	27.5681	9.1258	0.001315789	2,387.61	453,645.98
761	579,121	440,711,081	27.5862	9.1298	0.001314060	2,390.75	454,840.57
762	580,644	442,450,728	27.6043	9.1338	0.001312336	2,393.89	456,036.73
763	582,169	444,194,947	27.6225	9.1378	0.001310616	2,397.03	457,234.46
764	583,696	445,943,744	27.6405	9.1418	0.001308901	2,400.18	458,433.77
765	585,225	447,697,125	27.6586	9.1458	0.001307190	2,403.32	459,634.64
766	586,756	449,455,096	27.6767	9.1498	0.001305483	2,406.46	460,837.08
767	588,289	451,217,663	27.6948	9.1537	0.001303781	2,409.60	462,041.10
768	589,824	452,984,832	27.7128	9.1577	0.001302083	2,412.74	463,246.69
769	591,361	454,756,609	27.7308	9.1617	0.001300390	2,415.88	464,453.84
770	592,900	456,533,000	27.7489	9.1657	0.001298701	2,419.02	465,662.57
771	594,441	458,314,011	27.7669	9.1696	0.001297017	2,422.17	466,872.87
772	595,984	460,099,648	27.7849	9.1736	0.001295337	2,425.31	468,084.74
773	597,529	461,889,917	27.8029	9.1775	0.001293661	2,428.45	469,298.18
774	599,076	463,684,824	27.8209	9.1815	0.001291990	2,431.59	470,513.19
775	600,625	465,484,375	27.8388	9.1855	0.001290323	2,434.73	471,729.77

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
776	602,176	467,288,576	27.8568	9.1894	0.001288660	2,437.88	472,947.92
777	603,729	469,097,433	27.8747	9.1933	0.001287001	2,441.02	474,167.65
778	605,284	470,910,952	27.8927	9.1973	0.001285347	2,444.16	475,388.94
779	606,841	472,729,139	27.9106	9.2012	0.001283697	2,447.30	476,611.81
780	608,400	474,552,000	27.9285	9.2052	0.001282051	2,450.44	477,836.24
781	609,961	476,379,541	27.9464	9.2091	0.001280410	2,453.58	479,062.25
782	611,524	478,211,768	27.9643	9.2130	0.001278772	2,456.72	480,289.83
783	613,089	480,048,687	27.9821	9.2170	0.001277139	2,459.87	481,518.97
784	614,656	481,890,304	28.0000	9.2209	0.001275510	2,463.01	482,749.69
785	616,225	483,736,625	28.0179	9.2248	0.001273885	2,466.15	483,981.98
786	617,796	485,587,656	28.0357	9.2287	0.001272265	2,469.29	485,215.84
787	619,369	487,443,403	28.0535	9.2326	0.001270648	2,472.43	486,451.28
788	620,944	489,303,872	28.0713	9.2365	0.001269036	2,475.58	487,688.28
789	622,521	491,169,069	28.0891	9.2404	0.001267427	2,478.72	488,926.85
790	624,100	493,039,000	28.1069	9.2443	0.001265823	2,481.86	490,166.99
791	625,681	494,913,671	28.1247	9.2482	0.001264223	2,485.00	491,408.71
792	627,264	496,793,088	28.1425	9.2521	0.001262626	2,488.14	492,651.99
793	628,849	498,677,257	28.1603	9.2560	0.001261034	2,491.28	493,896.85
794	630,436	500,566,184	28.1780	9.2599	0.001259446	2,494.42	495,143.28
795	632,025	502,459,875	28.1957	9.2638	0.001257862	2,497.57	496,391.27
796	633,616	504,358,336	28.2135	9.2677	0.001256281	2,500.71	497,640.84
797	635,209	506,261,573	28.2312	9.2716	0.001254705	2,503.85	498,891.98
798	636,804	508,169,592	28.2489	9.2754	0.001253133	2,506.99	500,144.69
799	638,401	510,082,399	28.2666	9.2793	0.001251564	2,510.13	501,398.97
800	640,000	512,000,000	28.2843	9.2832	0.001250000	2,513.27	502,654.82
801	641,601	513,922,401	28.3019	9.2870	0.001248439	2,516.42	503,912.25
802	643,204	515,849,608	28.3196	9.2909	0.001246883	2,519.56	505,171.24
803	644,809	517,781,627	28.3373	9.2948	0.001245330	2,522.70	506,431.80
804	646,416	519,718,464	28.3549	9.2986	0.001243781	2,525.84	507,693.94
805	648,025	521,660,125	28.3725	9.3025	0.001242236	2,528.98	508,957.64
806	649,636	523,606,616	28.3901	9.3063	0.001240695	2,532.12	510,222.92
807	651,249	525,557,943	28.4077	9.3102	0.001239157	2,535.26	511,489.77
808	652,864	527,514,112	28.4253	9.3140	0.001237624	2,538.41	512,758.19
809	654,481	529,475,129	28.4429	9.3179	0.001236094	2,541.55	514,028.18
810	656,100	531,441,000	28.4605	9.3217	0.001234568	2,544.69	515,299.74
811	657,721	533,411,731	28.4781	9.3255	0.001233046	2,547.83	516,572.87
812	659,344	535,387,328	28.4956	9.3294	0.001231527	2,550.97	517,847.57
813	660,969	537,367,797	28.5132	9.3332	0.001230012	2,554.12	519,123.84
814	662,596	539,353,144	28.5307	9.3370	0.001228501	2,557.26	520,401.68
815	664,225	541,343,375	28.5482	9.3408	0.001226994	2,560.40	521,681.10
816	665,856	543,338,496	28.5657	9.3447	0.001225490	2,563.54	522,962.08
817	667,489	545,338,513	28.5832	9.3485	0.001223990	2,566.68	524,244.63
818	669,124	547,343,432	28.6007	9.3523	0.001222494	2,569.82	525,528.76
819	670,761	549,353,259	28.6182	9.3561	0.001221001	2,572.96	526,814.46
820	672,400	551,368,000	28.6356	9.3599	0.001219512	2,576.11	528,101.73
821	674,041	553,387,661	28.6531	9.3637	0.001218027	2,579.25	529,390.56
822	675,684	555,412,248	28.6705	9.3675	0.001216545	2,582.39	530,680.97
823	677,329	557,441,767	28.6880	9.3713	0.001215067	2,585.53	531,972.95
824	678,976	559,476,224	28.7054	9.3751	0.001213592	2,588.67	533,266.50
825	680,625	561,515,625	28.7228	9.3789	0.001212121	2,591.81	534,561.62
826	682,276	563,559,976	28.7402	9.3827	0.001210654	2,594.96	535,858.32
827	683,929	565,609,283	28.7576	9.3865	0.001209190	2,598.10	537,156.58
828	685,584	567,663,552	28.7750	9.3902	0.001207729	2,601.24	538,456.41
829	687,241	569,722,789	28.7924	9.3940	0.001206273	2,604.38	539,757.82
830	688,900	571,787,000	28.8097	9.3978	0.001204819	2,607.52	541,060.79
831	690,561	573,856,191	28.8271	9.4016	0.001203369	2,610.66	542,365.34
832	692,224	575,930,368	28.8444	9.4053	0.001201923	2,613.80	543,671.46
833	693,889	578,009,537	28.8617	9.4091	0.001200480	2,616.95	544,979.15
834	695,556	580,093,704	28.8791	9.4129	0.001199041	2,620.09	546,288.40
835	697,225	582,182,875	28.8964	9.4166	0.001197605	2,623.23	547,599.23

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
836	698,896	584,277,056	28.9137	9.4204	0.001196172	2,626.37	548,911.63
837	700,569	586,376,253	28.9310	9.4241	0.001194743	2,629.51	550,225.61
838	702,244	588,480,472	28.9482	9.4279	0.001193317	2,632.66	551,541.15
839	703,921	590,589,719	28.9655	9.4316	0.001191895	2,635.80	552,858.26
840	705,600	592,704,000	28.9828	9.4354	0.001190476	2,638.94	554,176.94
841	707,281	594,823,321	29.0000	9.4391	0.001189061	2,642.08	555,497.20
842	708,964	596,947,688	29.0172	9.4429	0.001187648	2,645.22	556,819.02
843	710,649	599,077,107	29.0345	9.4466	0.001186240	2,648.36	558,142.42
844	712,336	601,211,584	29.0517	9.4503	0.001184834	2,651.50	559,467.39
845	714,025	603,351,125	29.0689	9.4541	0.001183432	2,654.65	560,793.92
846	715,716	605,495,736	29.0861	9.4578	0.001182033	2,657.79	562,122.03
847	717,409	607,645,423	29.1033	9.4615	0.001180638	2,660.93	563,451.71
848	719,104	609,800,192	29.1204	9.4652	0.001179245	2,664.07	564,782.96
849	720,801	611,960,049	29.1376	9.4690	0.001177856	2,667.21	566,115.78
850	722,500	614,125,000	29.1548	9.4727	0.001176471	2,670.35	567,450.17
851	724,201	616,295,051	29.1719	9.4764	0.001175088	2,673.50	568,786.14
852	725,904	618,470,208	29.1890	9.4801	0.001173709	2,676.64	570,123.67
853	727,609	620,650,477	29.2062	9.4838	0.001172333	2,679.78	571,462.77
854	729,316	622,835,864	29 2233	9.4875	0.001170960	2,682.92	572,803.45
855	731,025	625,026,375	29.2404	9.4912	0.001169591	2,686.06	574,145.69
856	732,736	627,222,016	29.2575	9.4949	0.001168224	2,689.20	575,489.51
857	734,449	629,422,793	29.2746	9.4986	0.001166861	2,692.34	576,834.90
858	736,164	631,628,712	29.2916	9.5023	0.001165501	2,695.49	578,181.85
859	737,881	633,839,779	29.3087	9.5060	0.001164144	2,698.63	579,530.38
860	739,600	636,056,000	29.3258	9.5097	0.001162791	2,701.77	580,880.48
861	741,321	638,277,381	29.3428	9.5134	0.001161440	2,704.91	582,232.15
862	743,044	640,503,928	29.3598	9.5170	0.001160093	2,708.05	583,585.39
863	744,769	642,735,647	29.3769	9.5207	0.001158749	2,711.19	584,940.20
864	746,496	644,972,544	29.3939	9.5244	0.001157407	2,714.34	586,296.59
865	748,225	647,214,625	29 4109	9.5281	0.001156069	2,717.48	587,654.54
866	749,956	649,461,896	29.4279	9.5317	0.001154734	2,720.62	589,014.07
867	751,689	651,714,363	29.4449	9.5354	0.001153403	2,723.76	590,375.16
868	753,424	653,972,032	29.4618	9.5391	0.001152074	2,726.90	591,737.83
869	755,161	656,234,909	29.4788	9.5427	0.001150748	2,730.04	593,102.06
870	756,900	658,503,000	29.4958	9.5464	0.001149425	2,733.19	594,467.87
871	758,641	660,776,311	29.5127	9.5500	0.001148106	2,736.33	595,835.25
872	760,384	663,054,848	29.5296	9.5537	0.001146789	2,739.47	597,204.20
873	762,129	665,338,617	29.5466	9.5574	0.001145475	2,742.61	598,574.72
874	763,876	667,627,624	29.5635	9.5610	0.001144165	2,745.75	599,946.81
875	765,625	669,921,875	29.5804	9.5646	0.001142857	2,748.89	601,320.47
876	767,376	672,221,376	29.5973	9.5683	0.001141553	2,752.04	602,695.70
877	769,129	674,526,133	29.6142	9.5719	0.001140251	2,755.18	604,072.50
878	770,884	676,836,152	29.6311	9.5756	0.001138952	2,758.32	605,450.88
879	772,641	679,151,439	29.6479	9.5792	0.001137656	2,761.46	606,830.82
880	774,400	681,472,000	29.6648	9.5828	0.001136364	2,764.60	608,212.34
881	776,161	683,797,841	29.6816	9.5865	0.001135074	2,767.74	609,595.42
882	777,924	686,128,968	29.6985	9.5901	0.001133787	2,770.88	610,980.08
883	779,689	688,465,387	29.7153	9.5937	0.001132503	2,774.03	612,366.31
884	781,456	690,807,104	29.7321	9.5973	0.001131222	2,777.17	613,754.11
885	783,225	693,154,125	29.7489	9.6010	0.001129944	2,780.31	615,143.48
886	784,996	695,506,456	29.7658	9.6046	0.001128668	2,783.45	616,534.42
887	786,769	697,864,103	29.7825	9.6082	0.001127396	2,786.59	617,926.93
888	788,544	700,227,072	29.7993	9.6118	0.001126126	2,789.73	619,321.01
889	790,321	702,595,369	29.8161	9.6154	0.001124859	2,792.88	620,716.66
890	792,100	704,969,000	29.8329	9.6190	0.001123596	2,796.02	622,113.89
891	793,881	707,347,971	29.8496	9.6226	0.001122334	2,799.16	623,512.68
892	795,664	709,932,288	29.8664	9.6262	0.001121076	2,802.30	624,913.04
893	797,449	712,121,957	29.8831	9.6298	0.001119821	2,805.44	626,314.98
894	799,236	714,516,984	29.8998	9.6334	0.001118568	2,808.58	627,718.49
895	801,025	716,917,375	29.9166	9.6370	0.001117318	2,811.72	629,123.56

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum	Area
896	802,816	719,323,136	29.9333	9.6406	0.001116071	2,814.87	630,530.21
897	804,609	721,734,273	29.9500	9.6442	0.001114827	2,818.01	631,938.43
898	806,404	724,150,792	29.9666	9.6477	0.001113586	2,821.15	633,348.22
899	808,201	726,572,699	29.9833	9.6513	0.001112347	2,824.29	634,759.58
900	810,000	729,000,000	30.0000	9.6549	0.001111111	2,827.43	636,172.51
901	811,801	731,432,701	30.0167	9.6585	0.001109878	2,830.58	637,587.01
902	813,604	733,870,808	30.0333	9.6620	0.001108647	2,833.72	639,003.09
903	815,409	736,314,327	30.0500	9.6656	0.001107420	2,836.86	640,420.73
904	817,216	738,763,264	30.0666	9.6692	0.001106195	2,840.00	641,839.95
905	819,025	741,217,625	30.0832	9.6727	0.001104972	2,843.14	643,260.73
906	820,836	743,677,416	30.0998	9.6763	0.001103753	2,846.28	644,683.09
907	822,649	746,142,643	30.1164	9.6799	0.001102536	2,849.42	646,107.01
908	824,464	748,613,312	30.1330	9.6834	0.001101322	2,852.57	647,532.51
909	826,281	751,089,429	30.1496	9.6870	0.001100110	2,855.71	648,959.58
910	828,100	753,571,000	30.1662	9.6905	0.001098901	2,858.85	650,388.22
911	829,921	756,058,031	30.1828	9.6941	0.001097695	2,861.99	651,818.43
912	831,744	758,550,528	30.1993	9.6976	0.001096491	2,865.13	653,250.21
913	833,569	761,048,497	30.2159	9.7012	0.001095290	2,868.27	654,683.56
914	835,396	763,551,944	30.2324	9.7047	0.001094092	2,871.42	656,118.48
915	837,225	766,060,875	30.2490	9.7082	0.001092896	2,874.56	657,554.98
916	839,056	768,575,296	30.2655	9.7118	0.001091703	2,877.70	658,993.04
917	840,889	771,095,213	30.2820	9.7153	0.001090513	2,880.84	660,432.68
918	842,724	773,620,632	30.2985	9.7188	0.001089325	2,883.98	661,873.88
919	844,561	776,151,559	30.3150	9.7224	0.001088139	2,887.12	663,316.66
920	846,400	778,688,000	30.3315	9.7259	0.001086957	2,890.26	664,761.01
921	848,241	781,229,961	30.3480	9.7294	0.001085776	2,893.41	666,206.92
922	850,084	783,777,448	30.3645	9.7329	0.001084599	2,896.55	667,654.41
923	851,929	786,330,46[illegible]	30.3809	9.7364	0.001083423	2,899.69	669,103.47
924	853,776	788,889,024	30.3974	9.7400	0.001082251	2,902.83	670,554.10
925	855,625	791,453,125	30.4128	9.7435	0.001081081	2,905.97	672,006.30
926	857,476	794,022,776	30.4302	9.7470	0.001079914	2,909.12	673,460.08
927	859,329	796,597,983	30.4467	9.7505	0.001078749	2,912.26	674,915.42
928	861,184	799,178,752	30.4631	9.7540	0.001077586	2,915.40	676,372.33
929	863,041	801,765,089	30.4795	9.7575	0.001076426	2,918.54	677,830.82
930	864,900	804,357,000	30.4959	9.7610	0.001075269	2,921.68	679,290.87
931	866,761	806,954,491	30.5123	9.7645	0.001074114	2,924.82	680,752.50
932	868,624	809,557,568	30.5287	9.7680	0.001072961	2,927.96	682,215 69
933	870,489	812,166,237	30.5450	9.7715	0.001071811	2,931.11	683,680.46
934	872,356	814,780,504	30.5614	9.7750	0.001070664	2,934.25	685,146.80
935	874,225	817,400,375	30.5778	9.7785	0.001069519	2,937.39	686,614.71
936	876,096	820,025,856	30.5941	9.7819	0.001068376	2,940.53	688,084.19
937	877,969	822,656,953	30.6105	9.7854	0.001067236	2,943.67	689,555.24
938	879,844	825,293,672	30.6268	9 7889	0.001066098	2,946.81	691,027.86
939	881,721	827,936,019	30.6431	9.7924	0.001064963	2,949.96	692,502.05
940	883,600	830,584,000	30.6594	9.7959	0.001063830	2,953.10	693,977.82
941	885,481	833,237,621	30.6757	9.7993	0.001062699	2,956.24	695,455.15
942	887,364	835,896,888	30.6920	9.8028	0.001061571	2,959.38	696,934.06
943	889,249	838,561,807	30.7083	9.8063	0.001060445	2,962.52	698,414.53
944	891,136	841,232,384	30.7246	9.8097	0.001059322	2,965.66	699,896.58
945	893,025	843,908,625	30.7409	9.8132	0.001058201	2,968.80	701,380.19
946	894,916	846,590,536	30.7571	9.8167	0.001057082	2,971.95	702,865.38
947	896,809	849,278,123	30.7734	9.8201	0.001055966	2,975.09	704,352.14
948	898,704	851,971,392	30.7896	9.8236	0.001054852	2,978.23	705,840.47
949	900,601	854,670,349	30.8058	9.8270	0.001053741	2,981.37	707,330.37
950	902,500	857,375,000	30.8221	9.8305	0.001052632	2,984.51	708,821.84
951	904,401	860,085,351	30.8383	9.8339	0.001051525	2,987.66	710,314.88
952	906,304	862,801,408	30.8545	9.8374	0.001050420	2,990.80	711,809.50
953	908,209	865,523,177	30.8707	9.8408	0.001049318	2,993.94	713,305.68
954	910,116	868,250,664	30.8869	9.8443	0.001048218	2,997.08	714,803.43
955	912,025	870,983,875	30.9031	9.8477	0.001047120	3,000.22	716,302.76

TABLE 31.—(*Continued*)

No.	Square	Cube	Sq Root	Cu Root	Reciprocal	Circum.	Area
956	913,936	873,722,816	30.9192	9.8511	0.001046025	3,003.36	717,803.66
957	915,849	876,467,493	30.9354	9.8546	0.001044932	3,006.50	719,306.12
958	917,764	879,217,912	30.9516	9.8580	0.001043841	3,009.65	720,810 16
959	919,681	881,974,079	30.9677	9.8614	0.001042753	3,012.79	722,3 5.77
960	921,600	884,736,000	30.9839	9.8648	0.001041667	3,015.93	723,822.95
961	923,521	887,503,681	31.0000	9.8683	0.001040583	3,019.07	725,331.70
962	925,444	890,277,128	31.0161	9.8717	0.001039501	3,022.21	726,842.02
963	927,369	893,056,347	31.0322	9.8751	0.001038422	3,025.35	728,353.91
964	929,296	895,841,344	31.0483	9.8785	0.001037344	3,028.50	729,867.37
965	931,225	898,632,125	31.0644	9.8819	0.001036269	3,031.64	731,382.40
966	933,156	901,428,696	31.0805	9.8854	0.001035197	3,034.78	732,899.01
967	935,089	904,231,063	31.0966	9.8888	0.001034126	3,037.92	734,417 18
968	937,024	907,039,232	31.1127	9.8922	0.001033058	3,041.06	735,936.93
969	938,961	909,853,209	31.1288	9.8956	0.001031992	3,044.20	737,458.24
970	940,900	912,673,000	31.1448	9.8990	0.001030928	3,047.34	738,981.13
971	942,841	915,498,611	31.1609	9.9024	0.001029866	3,050.49	740,505.59
972	944,784	918,330,048	31.1769	9.9058	0.001028807	3,053.63	742,031.62
973	946,729	921,167,317	31.1929	9.9092	0.001027749	3,056.77	743,559.22
974	948,676	924,010,424	31.2090	9.9126	0.001026694	3,059.91	745,088.39
975	950,625	926,859,375	31.2250	9.9160	0.001025641	3,063.05	746,619.13
976	952,576	929,714,176	31.2410	9.9194	0.001024590	3,066.19	748,151.44
977	954,529	932,574,833	31.2570	9.9227	0.001023541	3,069.34	749,685.32
978	956,484	935,441,352	31.2730	9.9261	0.001022495	3,072.48	751,220.78
979	958,441	938,313,739	31.2890	9.9295	0.001021450	3,075.62	752,757.80
980	960,400	941,192,000	31.3050	9.9329	0.001020408	3,078.76	754,296.40
981	962,361	944,076,141	31.3209	9.9363	0.001019368	3,081.90	755,836.56
982	964,324	946,966,168	31.3369	9.9396	0.001018330	3,085.04	757,378.30
983	966,289	949,862,087	31.3528	9.9430	0.001017294	3,088.19	758,921.61
984	968,256	952,763,904	31.3688	9.9464	0.001016260	3,091.33	760,466.48
985	970,225	955,671,625	31.3847	9.9497	0.001015228	3,094.47	762,012.93
986	972,196	958,585,256	31.4006	9.9531	0.001014199	3,097 61	763,560.95
987	974,169	961,504,803	31.4166	9.9565	0.001013171	3,100.75	765,110.54
988	976,144	964,430,272	31.4325	9.9598	0.001012146	3,103.89	766,661.70
989	978,121	967,361,669	31.4484	9.9632	0.001011122	3,107.04	768,214.44
990	980,100	970,299,000	31.4643	9.9666	0.001010101	3,110.18	769,768.74
991	982,081	973,242,271	31.4802	9.9699	0.001009082	3,113.32	771,324.61
992	984,064	976,191,488	31.4960	9.9733	0.001008065	3,116.46	772,882.06
993	986,049	979,146,657	31.5119	9.9766	0.001007049	3,119.60	774,441.07
994	988,036	982,107,784	31.5278	9.9800	0.001006036	3,122.74	776,001.66
995	990,025	985,074,875	31.5436	9.9833	0.001005025	3,125.88	777,563.82
996	992,016	988,047,936	31.5595	9.9866	0.001004016	3,129.03	779,127.54
997	994,009	991,026,973	31.5753	9.9900	0.001003009	3,132.17	780,692.84
998	996,004	994,011,992	31.5911	9.9933	0.001002004	3,135.31	782,259.71
999	998,001	997,002,999	31.6070	9.9967	0.001001001	3,138.45	783,828.15
1000	1,000,000	1,000,000,000	31.6228	10.0000	0.001000000	3,141.59	785,398.16

TABLE 32.—FIVE-PLACE TABLE OF COMMON LOGARITHMS

First three digits in number	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
101	00432	00475	00518	00561	00604	00647	00689	00732	00775	00817
102	00860	00903	00945	00988	01030	01072	01115	01157	01199	01242
103	01284	01326	01368	01410	01452	01494	01536	01578	01620	01662
104	01703	01745	01787	01828	01870	01912	01953	01995	02036	02078
105	02119	02160	02202	02243	02284	02325	02366	02407	02449	02490
106	02531	02572	02612	02653	02694	02735	02776	02816	02857	02898
107	02938	02979	03019	03060	03100	03141	03181	03222	03262	03302
108	03342	03383	03423	03463	03503	03543	03583	03623	03663	03703
109	03743	03782	03822	03862	03902	03941	03981	04021	04060	04100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
111	04532	04571	04610	04650	04689	04727	04766	04805	04844	04883
112	04922	04961	04999	05038	05077	05115	05154	05192	05231	05269
113	05308	05346	05385	05423	05461	05500	05538	05576	05614	05652
114	05690	05729	05767	05805	05843	05881	05918	05956	05994	06032
115	06070	06108	06145	06183	06221	06258	06296	06333	06371	06408
116	06446	06483	06521	06558	06595	06633	06670	06707	06744	06781
117	06819	06856	06893	06930	06967	07004	07041	07078	07115	07151
118	07188	07225	07262	07298	07335	07372	07408	07445	07482	07518
119	07555	07591	07628	07664	07770	07737	07773	07809	07846	07882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
121	08279	08314	08350	08386	08422	08458	08493	08529	08565	08600
122	08636	08672	08707	08743	08778	08814	08849	08884	08920	08955
123	08991	09026	09061	09096	09132	09167	09202	09237	09272	09307
124	09342	09377	09412	09447	09482	09517	09552	09587	09621	09656
125	09691	09726	09760	09795	09830	09864	09899	09934	09968	10003
126	10037	10072	10106	10140	10175	10209	10243	10278	10312	10346
127	10380	10415	10449	10483	10517	10551	10585	10619	10653	10687
128	10721	10755	10789	10823	10857	10890	10924	10958	10992	11025
129	11059	11093	11126	11160	11193	11227	11261	11294	11327	11361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
131	11727	11760	11793	11826	11860	11893	11926	11959	11992	12024
132	12057	12090	12123	12156	12189	12222	12254	12287	12320	12352
133	12385	12418	12450	12483	12516	12548	12581	12613	12646	12678
134	12710	12743	12775	12808	12840	12872	12905	12937	12969	13001

TABLE 32.—(*Continued*)

First three digits in number	0	1	2	3	4	5	6	7	8	9
135	13033	13066	13098	13130	13162	13194	13226	13258	13290	13322
136	13354	13386	13418	13450	13481	13513	13545	13577	13609	13640
137	13672	13704	13735	13767	13799	13830	13862	13893	13925	13956
138	13988	14019	14051	14082	14114	14145	14176	14208	14239	14270
139	14301	14333	14364	14395	14426	14457	14489	14520	14551	14582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
141	14922	14953	14983	15014	15045	15076	15106	15137	15168	15198
142	15229	15259	15290	15320	15351	15381	15412	15442	15473	15503
143	15534	15564	15594	15625	15655	15685	15715	15746	15776	15806
144	15836	15866	15897	15927	15957	15987	16017	16047	16077	16107
145	16137	16167	16197	16227	16256	16286	16316	16346	16376	16406
146	16435	16465	16495	16524	16554	16584	16613	16643	16673	16702
147	16732	16761	16791	16820	16850	16879	16909	16938	16967	16997
148	17026	17056	17085	17114	17143	17173	17202	17231	17260	17289
149	17319	17348	17377	17406	17435	17464	17493	17522	17551	17580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
151	17898	17926	17955	17984	18013	18041	18070	18099	18127	18156
152	18184	18213	18241	18270	18298	18327	18355	18384	18412	18441
153	18469	18498	18526	18554	18583	18611	18639	18667	18696	18724
154	18752	18780	18808	18837	18865	18893	18921	18949	18977	19005
155	19033	19061	19089	19117	19145	19173	19201	19229	19257	19285
156	19312	19340	19368	19396	19424	19451	19479	19507	19535	19562
157	19590	19618	19645	19673	19700	19728	19756	19783	19811	19838
158	19866	19893	19921	19948	19976	20003	20030	20058	20085	20112
159	20140	20167	20194	20222	20249	20276	20303	20330	20358	20385
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656
161	20683	20710	20737	20763	20790	20817	20844	20871	20898	20925
162	20952	20978	21005	21032	21059	21085	21112	21139	21165	21192
163	21219	21245	21272	21299	21325	21352	21378	21405	21431	21458
164	21484	21511	21537	21564	21590	21617	21643	21669	21696	21722
165	21748	21775	21801	21827	21854	21880	21906	21932	21958	21985
166	22011	22037	22063	22089	22115	22141	22167	22194	22220	22246
167	22272	22298	22324	22350	22376	22401	22427	22453	22479	22505
168	22531	22557	22583	22608	22634	22660	22686	22712	22737	22763
169	22789	22814	22840	22866	22891	22917	22943	22968	22994	23019

TABLE 32.—(*Continued*)

First three digits in number	0	1	2	3	4	5	6	7	8	9
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
171	23300	23325	23350	23376	23401	23426	23452	23477	23502	23528
172	23553	23578	23603	23629	23654	23679	23704	23729	23754	23779
173	23805	23830	23855	23880	23905	23930	23955	23980	24005	24030
174	24055	24080	24105	24130	24155	24180	24204	24229	24254	24279
175	24304	24329	24353	24378	24403	24428	24452	24477	24502	24527
176	24551	24576	24601	24625	24650	24674	24699	24724	24748	24773
177	24797	24822	24846	24871	24895	24920	24944	24969	24993	25018
178	25042	25066	25091	25115	25139	25164	25188	25212	25237	25261
179	25285	25310	25334	25358	25382	25406	25431	25455	25479	25503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
181	25768	25792	25816	25840	25864	25888	25912	25935	25959	25983
182	26007	26031	26055	26079	26102	26126	26150	26174	26198	26221
183	26245	26269	26293	26316	26340	26364	26387	26411	26435	26458
184	26482	26505	26529	26553	26576	26600	26623	26647	26670	26694
185	26717	26741	26764	26788	26811	26834	26858	26881	26905	26928
186	26951	26975	26998	27021	27045	27068	27091	27114	27138	27161
187	27184	27207	27231	27254	27277	27300	27323	27346	27370	27393
188	27416	27439	27462	27485	27508	27531	27554	27577	27600	27623
189	27646	27669	27692	27715	27738	27761	27784	27807	27830	27852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
191	28103	28126	28149	28171	28194	28217	28240	28262	28285	28307
192	28330	28353	28375	28398	28421	28443	28466	28488	28511	28533
193	28556	28578	28601	28623	28646	28668	28691	28713	28735	28758
194	28780	28803	28825	28847	28870	28892	28914	28937	28959	28981
195	29003	29026	29048	29070	29092	29115	29137	29159	29181	29203
196	29226	29248	29270	29292	29314	29336	29358	29380	29403	29425
197	29447	29469	29491	29513	29535	29557	29579	29601	29623	29645
198	29667	29688	29710	29732	29754	29776	29798	29820	29842	29863
199	29885	29907	29929	29951	29973	29994	30016	30038	30060	30081
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620

TABLE 32.—(*Continued*)

First two digits in number	0	1	2	3	4	5	6	7	8	9
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743

TABLE 32.—(*Continued*)

First two digits in number	0	1	2	3	4	5	6	7	8	9
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727

TABLE 32.—(*Continued*)

First two digits in number	0	1	2	3	4	5	6	7	8	9
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957

To get the logarithm of a number: See p. 85 for rules for computation with logarithms.

a. Determine the characteristic (integral part) by inspection (refer to page 84).

b. Find the first two (or for numbers ranging from 100 to 200, the first three) digits in the given number.

c. Follow horizontally across the page to the column headed with a single digit. This is the third (or from 100 to 200, the fourth) digit in the number.

d. The value in this column and on this horizontal line is the mantissa (decimal part) of the required logarithm.

e. Annex this mantissa to the characteristic to get the complete logarithm.

To get the number corresponding to a given logarithm:

a. Hunt in the body of the table until given mantissa is found.

b. Read the figures in the number column on the same line.

c. Annex the single digit at the head of the column in which the mantissa was found.

Interpolation formulas used to extend the range of a table of logarithms.

Symbols used:

x = any five-digit number whose logarithm is desired.
a = the five-digit number next below x.
b = the five-digit number next above x.
m_x = the desired mantissa of x.
m_a = the given mantissa of a.
m_b = the given mantissa of b.

$$\log x = \log a + \left(\frac{x - a}{b - a}\right)(m_b - m_a).$$

$$\log^{-1} x = a + \left\{\frac{m_x - m_a}{m_b - m_a}\right\}(b - a).$$

TABLE 33.—FOUR-PLACE TABLE OF TRIGONOMETRIC FUNCTIONS[1]

TRIGONOMETRIC FUNCTIONS (at intervals of 10′)

Annex—10 in columns marked*.

De-grees	Ra-dians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
0° 00′	0.0000	.0000	∞	1.0000	0.0000	.0000	∞	∞	∞	1.5708	90° 00′
10	0.0029	.0029	7.4637	1.0000	.0000	.0029	7.4637	343.77	2.5363	1.5679	50
20	0.0058	.0058	.7648	1.0000	.0000	.0058	.7648	171.89	.2352	1.5650	40
30	0.0087	.0087	.9408	1.0000	.0000	.0087	.9409	114.59	.0591	1.5621	30
40	0.0116	.0116	8.0658	0.9999	.0000	.0116	8.0658	85.940	1.9342	1.5592	20
50	0.0145	.0145	.1627	.9999	.0000	.0145	.1627	68.750	.8373	1.5563	10
1° 00′	0.0175	.0175	8.2419	.9998	9.9999	.0175	8.2419	57.290	1.7581	1.5533	89° 00′
10	0.0204	.0204	.3088	.9998	.9999	.0204	.3089	49.104	.6911	1.5504	50
20	0.0233	.0233	.3668	.9997	.9999	.0233	.3669	42.964	.6331	1.5475	40
30	0.0262	.0262	.4179	.9997	.9999	.0262	.4181	38.188	.5819	1.5446	30
40	0.0291	.0291	.4637	.9996	.9998	.0291	.4638	34.368	.5362	1.5417	20
50	0.0320	.0320	.5050	.9995	.9998	.0320	.5053	31.242	.4947	1.5388	10
2° 00′	0.0349	.0349	8.5428	.9994	9.9997	.0349	8.5431	28.636	1.4569	1.5359	88° 00′
10	0.0378	.0378	.5776	.9993	.9997	.0378	.5779	26.432	.4221	1.5330	50
20	0.0407	.0407	.6097	.9992	.9996	.0407	.6101	24.542	.3899	1.5301	40
30	0.0436	.0436	.6397	.9990	.9996	.0437	.6401	22.904	.3599	1.5272	30
40	0.0465	.0465	.6677	.9989	.9995	.0466	.6682	21.470	.3318	1.5243	20
50	0.0495	.0494	.6940	.9988	.9995	.0495	.6945	20.206	.3055	1.5213	10
3° 00′	0.0524	.0523	8.7188	.9986	9.9994	.0524	8.7194	19.081	1.2806	1.5184	87° 00′
10	0.0553	.0552	.7423	.9985	.9993	.0553	.7429	18.075	.2571	1.5155	50
20	0.0582	.0581	.7645	.9983	.9993	.0582	.7652	17.169	.2348	1.5126	40
30	0.0611	.0610	.7857	.9981	.9992	.0612	.7865	16.350	.2135	1.5097	30
40	0.0640	.0640	.8059	.9980	.9991	.0641	.8067	15.605	.1933	1.5068	20
50	0.0669	.0669	.8251	.9978	.9990	.0670	.8261	14.294	.1739	1.5039	10
4° 00′	0.0698	.0698	8.8436	.9976	9.9989	.0699	8.8446	14.301	1.1554	1.5010	86° 00′
10	0.0727	.0727	.8613	.9974	.9989	.0729	.8624	13.727	.1376	1.4981	50
20	0.0756	.0756	.8783	.9971	.9988	.0758	.8795	13.197	.1205	1.4952	40
30	0.0785	.0785	.8946	.9969	.9987	.0787	.8960	12.706	.1040	1.4923	30
40	0.0814	.0814	.9104	.9967	.9986	.0816	.9118	12.251	.0882	1.4893	20
50	0.0844	.0843	.9256	.9964	.9985	.0846	.9272	11.826	.0728	1.4864	10
5° 00′	0.0873	.0872	8.9403	.9962	9.9983	.0875	8.9420	11.430	1.0580	1.4835	85° 00′
10	0.0902	.0901	.9545	.9959	.9982	.0904	.9563	11.059	.0437	1.4806	50
20	0.0931	.0929	.9682	.9957	.9981	.0934	.9701	10.712	.0299	1.4777	40
30	0.0960	.0958	.9816	.9954	.9980	.0963	.9836	10.385	.0164	1.4748	30
40	0.0989	.0987	.9945	.9951	.9979	.0992	.9966	10 078	.0034	1.4719	20
50	0.1018	.1016	9.0070	.9948	.9977	.1022	9.0093	9.7882	0.9907	1.4690	10
6° 00′	0.1047	.1045	9.0192	.9945	9.9976	.1051	9.0216	9.5144	0.9784	1.4661	84° 00′
10	0.1076	.1074	.0311	.9942	.9975	.1080	.0336	9.2553	.9664	1.4632	50
20	0.1105	.1103	.0426	.9939	.9973	.1110	.0453	9.0098	.9547	1.4603	40
30	0.1134	.1132	.0539	.9936	.9972	.1139	.0567	8.7769	.9433	1.4574	30
40	0.1164	.1161	.0648	.9932	.9971	.1169	.0678	8.5555	.9322	1.4544	20
50	0.1193	.1190	.0755	.9929	.9969	.1198	.0786	8.3450	.9214	1.4515	10
7° 00′	0.1222	.1219	9.0859	.9925	9.9968	.1228	9.0891	8.1443	0.9109	1.4486	83° 00′
10	0.1251	.1248	.0961	.9922	.9966	.1257	.0995	7.9530	.9005	1.4457	50
20	0.1280	.1276	.1060	.9918	.9964	.1287	.1096	7.7704	.8904	1.4428	40
30	0.1309	.1305	.1157	.9914	.9963	.1317	.1194	7.5958	.8806	1.4399	30
40	0.1338	.1334	.1252	.9911	.9961	.1346	.1291	7.4287	.8709	1.4370	20
50	0.1367	.1363	.1345	.9907	.9959	.1376	.1385	7.2687	.8615	1.4341	10
8° 00′	0.1396	.1392	9.1436	.9903	9.9958	.1405	9.1478	7.1154	0.8522	1.4312	82° 00′
10	0.1425	.1421	.1525	.9899	.9956	.1435	.1569	6.9682	.8431	1.4283	50
20	0.1454	.1449	.1612	.9894	.9954	.1465	.1658	6.8269	.8342	1.4254	40
30	0.1484	.1478	.1697	.9890	.9952	.1495	.1745	6.6912	.8255	1.4224	30
40	0.1513	.1507	.1781	.9886	.9950	.1524	.1831	6.5606	.8169	1.4195	20
50	0.1542	.1536	.1863	.9881	.9948	.1554	.1915	6.4348	.8085	1.4166	10
9° 00′	0.5171	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	1.4137	81° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Ra-dians	De-grees

[1] From Marks' "Mechanical Engineers' Handbook."

TABLE 33.—*(Continued)*[1]

TRIGONOMETRIC FUNCTIONS

Annex—10 in columns marked*.

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
9° 00′	0.1571	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	1.4137	81° 00′
10	0.1600	.1593	.2022	.9872	.9944	.1614	.2078	6.1970	.7922	1.4108	50
20	0.1629	.1622	.2100	.9868	.9942	.1644	.2158	6.0844	.7842	1.4079	40
30	0.1658	.1650	.2176	.9863	.9940	.1673	.2236	5.9758	.7764	1.4050	30
40	0.1687	.1679	.2251	.9858	.9938	.1703	.2313	5.8708	.7687	1.4021	20
50	0.1716	.1708	.2324	.9853	.9936	.1733	.2389	5.7694	.7611	1.3992	10
10° 00′	0.1745	.1736	9.2397	.9848	9.9934	.1763	9.2463	5.6713	0.7537	1.3963	80° 00′
10	0.1774	.1765	.2468	.9843	.9931	.1793	.2536	5.5764	.7464	1.3934	50
20	0.1804	.1794	.2538	.9838	.9929	.1823	.2609	5.4845	.7391	1.3904	40
30	0.1833	.1822	.2606	.9833	.9927	.1853	.2680	5.3955	.7320	1.3875	30
40	0.1862	.1851	.2674	.9827	.9924	.1883	.2750	5.3093	.7250	1.3846	20
50	0.1891	.1880	.2740	.9822	.9922	.1914	.2819	5.2257	.7181	1.3817	10
11° 00′	0.1920	.1908	9.2806	.9816	9.9919	.1944	9.2887	5.1446	0.7113	1.3788	79° 00′
10	0.1949	.1937	.2870	.9811	.9917	.1974	.2953	5.0658	.7047	1.3759	50
20	0.1978	.1965	.2934	.9805	.9914	.2004	.3020	4.9894	.6980	1.3730	40
30	0.2007	.1994	.2997	.9799	.9912	.2035	.3085	4.9152	.6915	1.3701	30
40	0.2036	.2022	.3058	.9793	.9909	.2065	.3149	4.8430	.6851	1.3672	20
50	0.2065	.2051	.3119	.9787	.9907	.2095	.3212	4.7729	.6788	1.3643	10
12° 00′	0.2094	.2079	9.3179	.9781	9.9904	.2126	9.3275	4.7046	0.6725	1.3614	78° 00′
10	0.2123	.2108	.3238	.9775	.9901	.2156	.3336	4.6382	.6664	1.3584	50
20	0.2153	.2136	.3296	.9769	.9899	.2186	.3397	4.5736	.6603	1.3555	40
30	0.2182	.2164	.3353	.9763	.9896	.2217	.3458	4.5107	.6542	1.3526	30
40	0.2211	.2193	.3410	.9757	.9893	.2247	.3517	4.4494	.6483	1.3497	20
50	0.2240	.2221	.3466	.9750	.9890	.2278	.3576	4.3897	.6424	1.3468	10
13° 00′	0.2269	.2250	9.3521	.9744	9.9887	.2309	9.3634	4.3315	0.6366	1.3439	77° 00′
10	0.2298	.2278	.3575	.9737	.9884	.2339	.3691	4.2747	.6309	1.3410	50
20	0.2327	.2306	.3629	.9730	.9881	.2370	.3748	4.2193	.6252	1.3381	40
30	0.2356	.2334	.3682	.9724	.9878	.2401	.3804	4.1653	.6196	1.3352	30
40	0.2385	.2363	.3734	.9717	.9875	.2432	.3859	4.1126	.6141	1.3323	20
50	0.2414	.2391	.3786	.9710	.9872	.2462	.3914	4.0611	.6086	1.3294	10
14° 00′	0.2443	.2419	9.3837	.9703	9.9869	.2493	9.3968	4.0108	0.6032	1.3265	76° 00′
10	0.2473	.2447	.3887	.9696	.9866	.2524	.4021	3.9617	.5979	1.3235	50
20	0.2502	.2476	.3937	.9689	.9863	.2555	.4074	3.9136	.5926	1.3206	40
30	0.2531	.2504	.3986	.9681	.9859	.2586	.4127	3.8667	.5873	1.3177	30
40	0.2560	.2532	.4035	.9674	.9856	.2617	.4178	3.8208	.5822	1.3148	20
50	0.2589	.2560	.4083	.9667	.9853	.2648	.4230	3.7760	.5770	1.3119	10
15° 00′	0.2618	.2588	9.4130	.9659	9.9849	.2679	9.4281	3.7321	0.5719	1.3090	75° 00′
10	0.2647	.2616	.4117	.9652	.9846	.2711	.4331	3.6891	.5669	1.3061	50
20	0.2676	.2644	.4223	.9644	.9843	.2742	.4381	3.6470	.5619	1.3032	40
30	0.2705	.2672	.4269	.9636	.9839	.2773	.4430	3.6059	.5570	1.3003	30
40	0.2734	.2700	.4314	.9628	.9836	.2805	.4479	3.5656	.5521	1.2974	20
50	0.2763	.2728	.4359	.9621	.9832	.2836	.4527	3.5261	.5473	1.2945	10
16° 00′	0.2793	.2756	9.4403	.9613	9.9828	.2867	9.4575	3.4874	0.5425	1.2915	74° 00′
10	0.2822	.2784	.4447	.9605	.9825	.2899	.4622	3.4495	.5378	1.2886	50
20	0.2851	.2812	.4491	.9596	.9821	.2931	.4669	3.4124	.5331	1.2857	40
30	0.2880	.2840	.4533	.9588	.9817	.2962	.4716	3.3759	.5284	1.2828	30
40	0.2909	.2868	.4576	.9580	.9814	.2994	.4762	3.3402	.5238	1.2799	20
50	0.2938	.2896	.4618	.9572	.9810	.3026	.4808	3.3052	.5192	1.2770	10
17° 00′	0.2967	.2924	9.4659	.9563	9.9806	.3057	9.4853	3.2709	0.5147	1.2741	73° 00′
10	0.2996	.2952	.4700	.9555	.9802	.3089	.4898	3.2371	.5102	1.2712	50
20	0.3025	.2979	.4741	.9546	.9798	.3121	.4943	3.2041	.5057	1.2683	40
30	0.3054	.3007	.4781	.9537	.9794	.3153	.4987	3.1716	.5013	1.2654	30
40	0.3083	.3035	.4821	.9528	.9790	.3185	.5031	3.1397	.4969	1.2625	20
50	0.3113	.3062	.4861	.9520	.9786	.3217	.5075	3.1084	.4925	1.2595	10
18° 00′	0.3142	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	1.2566	72° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

[1] From MARKS' "Mechanical Engineers' Handbook."

TABLE 33.—(*Continued*)[1]

TRIGONOMETRIC FUNCTIONS

Annex—10 in columns marked*.

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
18° 00′	0.3142	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	1.2566	72° 00′
10	0.3171	.3118	.4939	.9502	.9778	.3281	.5161	3.0475	.4839	1.2537	50
20	0.3200	.3145	.4977	.9492	.9774	.3314	.5203	3.0178	.4797	1.2508	40
30	0.3229	.3173	.5015	.9483	.9770	.3346	.5245	2.9887	.4755	1.2479	30
40	0.3258	.3201	.5052	.9474	.9765	.3378	.5287	2.9600	.4713	1.2450	20
50	0.3287	.3228	.5090	.9465	.9761	.3411	.5329	2.9319	.4671	1.2421	10
19° 00′	0.3316	.3256	9.5126	.9455	9.9757	.3443	9.5370	2.9042	0.4630	1.2392	71° 00′
10	0.3345	.3283	.5163	.9446	.9752	.3476	.5411	2.8700	.4589	1.2363	50
20	0.3374	.3311	.5199	.9436	.9748	.3508	.5451	2.8502	.4549	1.2334	40
30	0.3403	.3338	.5235	.9426	.9743	.3541	.5491	2.8239	.4509	1.2305	30
40	0.3432	.3365	.5270	.9417	.9739	.3574	.5531	2.7980	.4469	1.2275	20
50	0.3462	.3393	.5306	.9407	.9734	.3607	.5571	2.7725	.4429	1.2246	10
20° 00′	0.3491	.3420	9.5341	.9397	9.9730	.3640	9.5611	2.7475	0.4389	1.2217	70° 00′
10	0.3520	.3448	.5375	.9387	.9725	.3673	.5650	2.7228	.4350	1.2188	50
20	0.3549	.3475	.5409	.9377	.9721	.3706	.5689	2.6985	.4311	1.2159	40
30	0.3578	.3502	.5443	.9367	.9716	.3739	.5727	2.6746	.4273	1.2130	30
40	0.3607	.3529	.5477	.9356	.9711	.3772	.5766	2.6511	.4234	1.2101	20
50	0.3636	.3557	.5510	.9346	.9706	.3805	.5804	2.6279	.4196	1.2072	10
21° 00′	0.3665	.3584	9.5543	.9336	9.9702	.3839	9.5842	2.6051	0.4158	1.2043	69° 00′
10	0.3694	.3611	.5576	.9325	.9697	.3872	.5879	2.5826	.4121	1.2014	50
20	0.3723	.3638	.5609	.9315	.9692	.3906	.5917	2.5605	.4083	1.1985	40
30	0.3752	.3665	.5641	.9304	.9687	.3939	.5954	2.5386	.4046	1.1956	30
40	0.3782	.3692	.5673	.9293	.9682	.3973	.5991	2.5172	.4009	1.1926	20
50	0.3811	.3719	.5704	.9283	.9677	.4006	.6028	2.4960	.3972	1.1897	10
22° 00′	0.3840	.3746	9.5736	.9272	9.9672	.4040	9.6064	2.4751	0.3936	1.1868	68° 00′
10	0.3869	.3773	.5767	.9261	.9667	.4074	.6100	2.4545	.3900	1.1839	50
20	0.3898	.3800	.5798	.9250	.9661	.4108	.6136	2.4342	.3864	1.1810	40
30	0.3927	.3827	.5828	.9239	.9656	.4142	.6172	2.4142	.3828	1.1781	30
40	0.3956	.3854	.5859	.9228	.9651	.4176	.6208	2.3945	.3792	1.1752	20
50	0.3985	.3881	.5889	.9216	.9646	.4210	.6243	2.3750	.3757	1.1723	10
23° 00′	0.4014	.3907	9.5919	.9205	9.9640	.4245	9.6279	2.3559	0.3721	1.1694	37° 00′
10	0.4043	.3934	.5948	.9194	.9635	.4279	.6314	2.3369	.3686	1.1665	50
20	0.4072	.3961	.5978	.9182	.9629	.4314	.6348	2.3183	.3652	1.1636	40
30	0.4102	.3987	.6007	.9171	.9624	.4348	.6383	2.2998	.3617	1.1606	30
40	0.4131	.4014	.6036	.9159	.9618	.4383	.6417	2.2817	.3583	1.1577	20
50	0.4160	.4041	.6065	.9147	.9613	.4417	.6452	2.2637	.3548	1.1548	10
24° 00′	0.4189	.4067	9.6093	.9135	9.9607	.4452	9.6486	2.2460	0.3514	1.1519	36° 00′
10	0.4218	.4094	.6121	.9124	.9602	.4487	.6520	2.2286	.3480	1.1490	50
20	0.4247	.4120	.6149	.9112	.9596	.4522	.6553	2.2113	.3447	1.1461	40
30	0.4276	.4147	.6177	.9100	.9590	.4557	.6587	2.1943	.3413	1.1432	30
40	0.4305	.4173	.6205	.9088	.9584	.4592	.6620	2.1775	.3380	1.1403	20
50	0.4334	.4200	.6232	.9075	.9579	.4628	.6654	2.1609	.3346	1.1374	10
25° 00′	0.4363	.4226	9.6259	.9063	9.9573	.4663	9.6687	2.1445	0.3313	1.1345	35° 00′
10	0.4392	.4253	.6286	.9051	.9567	.4699	.6720	2.1283	.3280	1.1316	50
20	0.4422	.4279	.6313	.9038	.9561	.4734	.6752	2.1123	.3248	1.1286	40
30	0.4451	.4305	.6340	.9026	.9555	.4770	.6785	2.0965	.3215	1.1257	30
40	0.4480	.4331	.6366	.9013	.9549	.4806	.6817	2.0809	.3183	1.1228	20
50	0.4509	.4358	.6392	.9001	.9543	.4841	.6850	2.0655	.3150	1.1199	10
26° 00′	0.4538	.4384	9.6418	.8988	9.9537	.4877	9.6882	2.0503	0.3118	1.1170	64° 00′
10	0.4567	.4410	.6444	.8975	.9530	.4913	.6914	2.0353	.3086	1.1141	50
20	0.4596	.4436	.6470	.8962	.9524	.4950	.6946	2.0204	.3054	1.1112	40
30	0.4625	.4462	.6495	.8949	.9518	.4986	.6977	2.0057	.3023	1.1083	30
40	0.4654	.4488	.6521	.8936	.9512	.5022	.7009	1.9912	.2991	1.1054	20
50	0.4683	.4514	.6546	.8923	.9505	.5059	.7040	1.9768	.2960	1.1025	10
27° 00′	0.4712	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	1.0996	33° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

[1] From Marks' "Mechanical Engineers' Handbook."

TABLE 33.—(*Continued*)[1]

TRIGONOMETRIC FUNCTIONS

Annex—10 in. columns marked*.

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
27° 00′	0.4712	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	1.0996	63° 00′
10	0.4741	.4566	.6595	.8897	.9492	.5132	.7103	1.9486	.2897	1.0966	50
20	0.4771	.4592	.6620	.8884	.9486	.5169	.7134	1.9347	.2866	1.0937	40
30	0.4800	.4617	.6644	.8870	.9479	.5206	.7165	1.9210	.2835	1.0908	30
40	0.4829	.4643	.6668	.8857	.9473	.5243	.7196	1.9074	.2804	1.0879	20
50	0.4858	.4669	.6692	.8843	.9466	.5280	.7226	1.8940	.2774	1.0850	10
28° 00′	0.4887	.4695	9.6716	.8829	9.9459	.5317	9.7257	1.8807	0.2743	1.0821	62° 00′
10	0.4916	.4720	.6740	.8816	.9453	.5354	.7287	1.8676	.2713	1.0792	50
20	0.4945	.4746	.6763	8802	.9446	.5392	.7317	1.8546	.2683	1.0763	40
30	0.4974	.4772	.6787	.8788	.9439	.5430	.7348	1.8418	.2652	1.0734	30
40	0.5003	.4797	.6810	.8774	.9432	.5467	.7378	1.8291	.2622	1.0705	20
50	0.5032	.4823	.6833	.8760	.9425	.5505	.7408	1.8165	.2592	1.0676	10
29° 00′	0.5061	.4848	9.6856	.8746	9.9418	.5543	9.7438	1.8040	0.2562	1.0647	61° 00′
10	0.5091	.4874	.6878	.8732	.9411	.5581	.7467	1.7917	.2533	1.0617	50
20	0.5120	.4899	.6901	.8718	.9404	.5619	.7497	1.7796	.2503	1.0588	40
30	0.5149	.4924	.6923	.8704	.9397	.5658	.7526	1.7675	.2474	1.0559	30
40	0.5178	.4950	.6946	.8689	.9390	.5696	.7556	1.7556	.2444	1.0530	20
50	0.5207	.4975	.6968	.8675	.9383	.5735	.7585	1.7437	.2415	1.0501	10
30° 00′	0.5236	.5000	9.6990	.8660	9.9375	.5774	9.7614	1.7321	0.2386	1.0472	60° 00′
10	0.5265	.5025	.7012	.8646	.9368	.5812	.7644	1.7205	.2356	1.0443	50
20	0.5294	.5050	.7033	.8631	.9361	.5851	.7673	1.7090	.2327	1.0414	40
30	0.5323	.5075	.7055	.8616	.9353	.5890	.7701	1.6977	.2299	1.0385	30
40	0.5352	.5100	.7076	.8601	.9346	.5930	.7730	1.6864	.2270	1.0356	20
50	0.5381	.5125	.7097	.8587	.9338	.5969	.7759	1.6753	.2241	1.0327	10
31° 00′	0.5411	.5150	9.7118	.8572	9.9331	.6009	9.7788	1.6643	0.2212	1.0297	59° 00′
10	0.5440	.5175	.7139	.8557	.9323	.6048	.7816	1.6534	.2184	1.0268	50
20	0.5469	.5200	.7160	.8542	.9315	.6088	.7845	1.6426	.2155	1.0239	40
30	0.5498	.5225	.7181	.8526	.9308	.6128	.7873	1.6319	.2127	1.0210	30
40	0.5527	.5250	.7201	.8511	.9300	.6168	.7902	1.6212	.2098	1.0181	20
50	0.5556	.5275	.7222	.8496	.9292	.6208	.7930	1.6107	.2070	1.0152	10
32° 00′	0.5585	.5299	9.7242	.8480	9.9284	.6249	9.7958	1.6003	0.2042	1.0123	58° 00′
10	0.5614	.5324	.7262	.8465	.9276	.6289	.7986	1.5900	.2014	1.0094	50
20	0.5643	.5348	.7282	.8450	.9268	.6330	.8014	1.5798	.1986	1.0065	40
30	0.5672	.5373	.7302	.8434	.9260	.6371	.8042	1.5697	.1958	1.0036	30
40	0.5701	.5398	.7322	.8418	.9252	.6412	.8070	1.5597	.1930	1.0007	20
50	0.5730	.5422	.7342	.8403	.9244	.6453	.8097	1.5497	.1903	0.9977	10
33° 00′	0.5760	.5446	9.7361	.8387	9.9236	.6494	9.8125	1.5399	0.1875	0.9948	57° 00′
10	0.5789	.5471	.7380	.8371	.9228	.6536	.8153	1.5301	.1847	0.9919	50
20	0.5818	.5495	.7400	.8355	.9219	.6577	.8180	1.5204	.1820	0.9890	40
30	0.5847	.5519	.7419	.8339	.9211	.6619	.8208	1.5108	.1792	0.9861	30
40	0.5876	.5544	.7438	.8323	.9203	.6661	.8235	1.5013	.1765	0.9832	20
50	0.5905	.5568	.7457	.8307	.9194	.6703	.8263	1.4919	.1737	0.9803	10
34° 00′	0.5934	.5592	9.7476	.8290	9.9186	.6745	9.8290	1.4826	0.1710	0.9774	56° 00′
10	0.5963	.5616	.7494	.8274	.9177	.6787	.8317	1.4733	.1683	0.9745	50
20	0.5992	.5640	.7513	.8258	.9169	.6830	.8344	1.4641	.1656	0.9716	40
30	0.6021	.5664	.7531	.8241	.9160	.6873	.8371	1.4550	.1629	0.9687	30
40	0.6050	.5688	.7550	.8225	.9151	.6916	.8398	1.4460	.1602	0.9657	20
50	0.6080	.5712	.7568	.8208	.9142	.6959	.8425	1.4370	.1575	0.9628	10
35° 00′	0.6109	.5736	9.7586	.8192	9.9134	.7002	9.8452	1.4281	0.1548	0.9599	55° 00′
10	0.6138	.5760	.7604	.8175	.9125	.7046	.8479	1.4193	.1521	0.9570	50
20	0.6167	.5783	.7622	0.8158	.9116	.7089	.8506	1.4106	.1494	0.9541	40
30	0.6196	.5807	.7640	.8141	.9107	.7133	.8533	1.4019	.1467	0.9512	30
40	0.6225	.5831	.7657	.8124	.9098	.7177	.8559	1.3934	.1441	0.9483	20
50	0.6254	.5854	.7675	.8107	.9089	.7221	.8586	1.3848	.1414	0.9454	10
36° 00′	0.6283	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	0.9425	54° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

[1] From MARKS' "Mechanical Engineers' Handbook."

TABLE 33.—(*Continued*)[1]

TRIGONOMETRIC FUNCTIONS[2]

Annex—10 in columns marked*.

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log *	Nat.	Log.*	Nat.	Log.*	Nat	Log.		
36° 00′	0.6283	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	0.9425	54° 00′
10	0.6312	.5901	.7710	.8073	.9070	.7310	.8639	1.3680	.1361	0.9396	50
20	0.6341	.5925	.7727	.8056	.9061	.7355	.8666	1.3597	.1334	0.9367	40
30	0.6370	.5948	.7744	.8039	.9052	.7400	.8692	1.3514	.1308	0.9338	30
40	0.6400	.5972	.7761	.8021	.9042	.7445	.8718	1.3432	.1282	0.9308	20
50	0.6429	.5995	.7778	.8004	.9033	.7490	.8745	1.3351	.1255	0.9279	10
37° 00′	0.6458	.6018	9.7795	.7986	9.9023	.7536	9.8771	1.3270	0.1229	0.9250	53° 00′
10	0.6487	.6041	.7811	.7969	.9014	.7581	.8797	1.3190	.1203	0.9221	50
20	0.6516	.6065	.7828	.7951	.9004	.7627	.8824	1.3111	.1176	0.9192	40
30	0.6545	.6088	.7844	.7934	.8995	.7673	.8850	1.3032	.1150	0.9163	30
40	0.6574	.6111	.7861	.7916	.8985	.7720	.8876	1.2954	.1124	0.9134	20
50	0.6603	.6134	.7877	.7898	.8975	.7766	.8902	1.2876	.1098	0.9105	10
38° 00′	0.6632	.6157	9.7893	.7880	9.8965	.7813	9.8928	1.2799	0.1072	0.9076	52° 00′
10	0.6661	.6180	.7910	.7862	.8955	.7860	.8954	1.2723	.1046	0.9047	50
20	0.6690	.6202	.7926	.7844	.8945	.7907	.8980	1.2647	.1020	0.9018	40
30	0.6720	.6225	.7941	.7826	.8935	.7954	.9006	1.2572	.0994	0.8988	30
40	0.6749	.6248	.7957	.7808	.8925	.8002	.9032	1.2497	.0968	0.8959	20
50	0.6778	.6271	.7973	.7790	.8915	.8050	.9058	1.2423	.0942	0.8930	10
39° 00′	0.6807	.6293	9.7989	.7771	9.8905	.8098	9.9084	1.2349	0.0916	0.8901	51° 00′
10	0.6836	.6316	.8004	.7753	.8895	.8146	.9110	1.2276	.0890	0.8872	50
20	0.6865	.6338	.8020	.7735	.8884	.8195	.9135	1.2203	.0865	0.8843	40
30	0.6894	.6361	.8035	.7716	.8874	.8243	.9161	1.2131	.0839	0.8814	30
40	0.6923	.6383	.8050	.7698	.8864	.8292	.9187	1.2059	.0813	0.8785	20
50	0.6952	.6406	.8066	.7679	.8853	.8342	.9212	1.1988	.0788	0.8756	10
40° 00′	0.6981	.6428	9.8081	.7660	9.8843	.8391	9.9238	1.1918	0.0762	0.8727	50° 00′
10	0.7010	.6450	.8096	.7642	.8832	.8441	.9264	1.1847	.0736	0.8698	50
20	0.7039	.6472	.8111	.7623	.8821	.8491	.9289	1.1778	.0711	0.8668	40
30	0.7069	.6494	.8125	.7604	.8810	.8541	.9315	1.1708	.0685	0.8639	30
40	0.7098	.6517	.8140	.7585	.8800	.8591	.9341	1.1640	.0659	0.8610	20
50	0.7127	.6539	.8155	.7566	.8789	.8642	.9366	1.1571	.0634	0.8581	10
41° 00′	0.7156	.6561	9.8169	.7547	9.8778	.8693	9.9392	1.1504	0.0608	0.8552	49° 00′
10	0.7185	.6583	.8184	.7528	.8767	.8744	.9417	1.1436	.0583	0.8523	50
20	0.7214	.6604	.8198	.7509	.8756	.8796	.9443	1.1369	.0557	0.8494	40
30	0.7243	.6626	.8213	.7490	.8745	.8847	.9468	1.1303	.0532	0.8465	30
40	0.7272	.6648	.8227	.7470	.8733	.8899	.9494	1.1237	.0506	0.8436	20
50	0.7301	.6670	.8241	7451	.8722	.8952	.9519	1.1171	.0481	0.8407	10
42° 00′	0.7330	.6691	9.8255	.7431	9.8711	.9004	9.9544	1.1106	0.0456	0.8378	48° 00′
10	0.7359	.6713	.8269	.7412	.8699	.9057	.9570	1.1041	.0430	0.8348	50
20	0.7389	.6734	.8283	.7392	.8688	.9110	.9595	1.0977	.0405	0.8319	40
30	0.7418	.6756	.8297	.7373	.8676	.9163	.9621	1.0913	.0379	0.8290	30
40	0.7447	.6777	.8311	.7353	.8665	.9217	.9646	1.0850	.0354	0.8261	20
50	0.7476	.6799	.8324	.7333	.8653	.9271	.9671	1.0786	.0329	0.8232	10
43° 00′	0.7505	.6820	9.8338	.7314	9.8641	.9325	9.9697	1.0724	0.0303	0.8203	47° 00′
10	0.7534	.6841	.8351	.7294	.8629	.9380	.9722	1.0661	.0278	0.8174	50
20	0.7563	.6862	.8365	.7274	.8618	.9435	.9747	1.0599	.0253	0.8145	40
30	0.7592	.6884	.8378	.7254	.8606	.9490	.9772	1.0538	.0228	0.8116	30
40	0.7621	.6905	.8391	.7234	.8594	.9545	.9798	1.0477	.0202	0.8087	20
50	0.7650	.6926	.8405	.7214	.8582	.9601	.9823	1.0416	.0177	0.8058	10
44° 00′	0.7679	.6947	9.8418	.7193	9.8569	.9657	9.9848	1.0355	0.0152	0.8029	46° 00′
10	0.7709	.6967	.8431	.7173	.8557	.9713	.9874	1.0295	.0126	0.7999	50
20	0.7738	.6988	.8444	.7153	.8545	.9770	.9899	1.0235	.0101	0.7970	40
30	0.7767	.7009	.8457	.7133	.8532	.9827	.9924	1.0176	.0076	0.7941	30
40	0.7796	.7030	.8469	.7112	.8520	.9884	.9949	1.0117	.0051	0.7912	20
50	0.7825	.7050	.8482	.7092	.8507	.9942	.9975	1.0058	.0025	0.7883	10
45° 00′	0.7854	.7071	9.8495	.7071	9.8495	1.0000	0.0000	1.0000	0.0000	0.7854	45° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

[1] From MARKS' "Mechanical Engineers' Handbook."

[2] For functions of angles 45 to 90°, use the names of the functions at bottom of page and read up in the right hand column of angles.

TABLE 34.—NATURAL LOGARITHMS

(Also known as *hyperbolic* or *Naperian logarithms*)

Conversion constants		Numbers over 10	Numbers less than 1.0
The base, e = 2.718281828 $\log_e 10$ = 2.30258509 $\log_{10} e$ = 0.43429448	n	$\log_e (10)^n$ $= n(\log_e 10)$ $= n(2.302585)$	$\log_e (0.1)^n$ $= n(\log_e 0.1)$ $= n(0.697415 - 3)$
Below are given the logarithms of numbers from 1 to 10 to the base, e, correct to five decimal places. Moving the decimal point in the number n places to the right is equivalent to adding n times (2.30258509) to the logarithm when the number is greater than 10 or n times (0.69741491 − 3) to the logarithm when the number is less than 1	1	2.302585	0.697415 − 3
	2	4.605170	0.394830 − 5
	3	6.907755	0.0922447 − 7
	4	9.210340	0.789660 − 10
	5	11.512926	0.487074 − 12
	6	13.815511	0.184489 − 14
	7	16.118096	0.881904 − 17
	8	18.420681	0.579319 − 19
	9	20.723266	0.276734 − 21

N	0	1	2	3	4	5	6	7	8	9
1.0	0.0 0000	0995	1980	2956	3922	4879	5827	6766	7696	8618
1.1	9531	*0436	*1333	*2222	*3103	*3976	*4842	*5700	*6551	*7395
1.2	0.1 8232	9062	9885	*0701	*1511	*2314	*3111	*3902	*4686	*5464
1.3	0.2 6236	7003	7763	8518	9267	*0010	*0748	*1481	*2208	*2930
1.4	0.3 3647	4359	5066	5767	6464	7156	7844	8526	9204	9878
1.5	0.4 0547	1211	1871	2527	3178	3825	4469	5108	5742	6373
1.6	7000	7623	8243	8858	9470	*0078	*0682	*1282	*1879	*2473
1.7	0.5 3063	3649	4232	4812	5389	5962	6531	7098	7661	8222
1.8	8779	9333	9884	*0432	*0977	*1519	*2058	*2594	*3127	*3658
1.9	0.6 4185	4710	5233	5752	6269	6783	7294	7803	8310	8813
2.0	9315	9813	*0310	*0804	*1295	*1784	*2271	*2755	*3237	*3716
2.1	0.7 4194	4669	5142	5612	6081	6547	7011	7473	7932	8390
2.2	8846	9299	9751	*0200	*0648	*1093	*1536	*1978	*2418	*2855
2.3	0.8 3291	3725	4157	4587	5015	5442	5866	6289	6710	7129
2.4	7547	7963	8377	8789	9200	9609	*0016	*0422	*0826	*1228
2.5	0.9 1629	2028	2426	2822	3216	3609	4001	4391	4779	5166
2.6	5551	5935	6317	6698	7078	7456	7833	8208	8582	8954
2.7	9325	9695	*0063	*0430	*0796	*1160	*1523	*1885	*2245	*2604
2.8	1.0 2962	3318	3674	4028	4380	4732	5082	5431	5779	6126
2.9	6471	6815	7158	7500	7841	8181	8519	8856	9192	9527
3.0	9861	*0194	*0526	*0856	*1186	*1514	*1841	*2168	*2493	*2817
3.1	1.1 3140	3462	3783	4103	4422	4740	5057	5373	5688	6002
3.2	6315	6627	6938	7248	7557	7865	8173	8479	8784	9089
3.3	9392	9695	9996	*0297	*0597	*0896	*1194	*1491	*1788	*2083
3.4	1.2 2378	2671	2964	3256	3547	3837	4127	4415	4703	4990
3.5	5276	5562	5846	6130	6413	6695	6976	7257	7536	7815
3.6	8093	8371	8647	8923	9198	9473	9746	*0019	*0291	*0563
3.7	1.3 0833	1103	1372	1641	1909	2176	2442	2708	2972	3237
3.8	3500	3763	4025	4286	4547	4807	5067	5325	5584	5841
3.9	6098	6354	6609	6864	7118	7372	7624	7877	8128	8379
4.0	8629	8879	9128	9377	9624	9872	*0118	*0364	*0610	*0854
4.1	1.4 1099	1342	1585	1828	2070	2311	2552	2792	3031	3270
4.2	3508	3746	3984	4220	4456	4692	4927	5161	5395	5629
4.3	5862	6094	6326	6557	6787	7018	7247	7476	7705	7933
4.4	8160	8387	8614	8840	9065	9290	9515	9739	9962	0185
4.5	1.5 0408	0630	0851	1072	1293	1513	1732	1951	2170	2388
4.6	2606	2823	3039	3256	3471	3687	3902	4116	4330	4543
4.7	4756	4969	5181	5393	5604	5814	6025	6235	6444	6653
4.8	6862	7070	7277	7485	7691	7898	8104	8309	8515	8719
4.9	8924	9127	9331	9534	9737	9939	*0141	*0342	*0543	*0744

TABLE 34.—(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
5.1	2924	3120	3315	3511	3705	3900	4094	4287	4481	4673
5.2	4866	5058	5250	5441	5632	5823	6013	6203	6393	6582
5.3	6771	6959	7147	7335	7523	7710	7896	8083	8269	8455
5.4	8640	8825	9010	9194	9378	9562	9745	9928	*0111	*0293
5.5	1.7 0475	0656	0838	1019	1199	1380	1560	1740	1919	2098
5.6	2277	2455	2633	2811	2988	3166	3342	3519	3695	3871
5.7	4047	4222	4397	4572	4746	4920	5094	5267	5440	5613
5.8	5786	5958	6130	6302	6473	6644	6815	6985	7156	7326
5.9	7495	7665	7834	8002	8171	8339	8507	8675	8842	9009
6.0	9176	9342	9509	9675	9840	*0006	*0171	*0336	*0500	*0665
6.1	1.8 0829	0993	1156	1319	1482	1645	1808	1970	2132	2294
6.2	2455	2616	2777	2938	3098	3258	3418	3578	3737	3896
6.3	4055	4214	4372	4530	4688	4845	5003	5160	5317	5473
6.4	5630	5786	5942	6097	6253	6408	6563	6718	6872	7026
6.5	7180	7334	7487	7641	7794	7947	8099	8251	8403	8555
6.6	8707	8858	9010	9160	9311	9462	9612	9762	9912	*0061
6.7	1.9 0211	0360	0509	0658	0806	0954	1102	1250	1398	1545
6.8	1692	1839	1986	2132	2279	2425	2571	2716	2862	3007
6.9	3152	3297	3442	3586	3730	3874	4018	4162	4305	4448
7.0	4591	4734	4876	5019	5161	5303	5445	5586	5727	5869
7.1	6009	6150	6291	6431	6571	6711	6851	6991	7130	7269
7.2	7408	7547	7685	7824	7962	8100	8238	8376	8513	8650
7.3	8787	8924	9061	9198	9334	9470	9606	9742	9877	*0013
7.4	2.0 0148	0283	0418	0553	0687	0821	0956	1089	1223	1357
7.5	1490	1624	1757	1890	2022	2155	2287	2419	2551	2683
7.6	2815	2946	3078	3209	3340	3471	3601	3732	3862	3992
7.7	4122	4252	4381	4511	4640	4769	4898	5027	5156	5284
7.8	5412	5540	5668	5796	5924	6051	6179	6306	6433	6560
7.9	6686	6813	6939	7065	7191	7317	7443	7568	7694	7819
8.0	7944	8069	8194	8318	8443	8567	8691	8815	8939	9063
8.1	9186	9310	9433	9556	9679	9802	9924	*0047	*0169	*0291
8.2	2.1 0413	0535	0657	0779	0900	1021	1142	1263	1384	1505
8.3	1626	1746	1866	1986	2106	2226	2346	2465	2585	2704
8.4	2823	2942	3061	3180	3298	3417	3535	3653	3771	3889
8.5	4007	4124	4242	4359	4476	4593	4710	4827	4943	5060
8.6	5176	5292	5409	5524	5640	5756	5871	5987	6102	6217
8.7	6332	6447	6562	6677	6791	6905	7020	7134	7248	7361
8.8	7475	7589	7702	7816	7929	8042	8155	8267	8380	8493
8.9	8605	8717	8830	8942	9054	9165	9277	9389	9500	9611
9.0	9722	9834	9944	*0055	*0166	*0276	*0387	*0497	*0607	*0717
9.1	2.2 0827	0937	1047	1157	1266	1375	1485	1594	1703	1812
9.2	1920	2029	2138	2246	2354	2462	2570	2678	2786	2894
9.3	3001	3109	3216	3324	3431	3538	3645	3751	3858	3965
9.4	4071	4177	4284	4390	4496	4601	4707	4813	4918	5024
9.5	5129	5234	5339	5444	5549	5654	5759	5863	5968	6072
9.6	6176	6280	6384	6488	6592	6696	6799	6903	7006	7109
9.7	7213	7316	7419	7521	7624	7727	7829	7932	8034	8136
9.8	8238	8340	8442	8544	8646	8747	8849	8950	9051	9152
9.9	9253	9354	9455	9556	9657	9757	9858	9958	*0058	*0158
10.0	2.3 0259	0358	0458	0558	0658	0757	0857	0956	1055	1154

$\log_e x = (2.30258509)(\log_{10} x)$ where $2.30258509 = \log_e 10$

$\log_{10} x = (0.43429448)(\log_e x)$ where $0.43429448 = \log_{10} e$

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